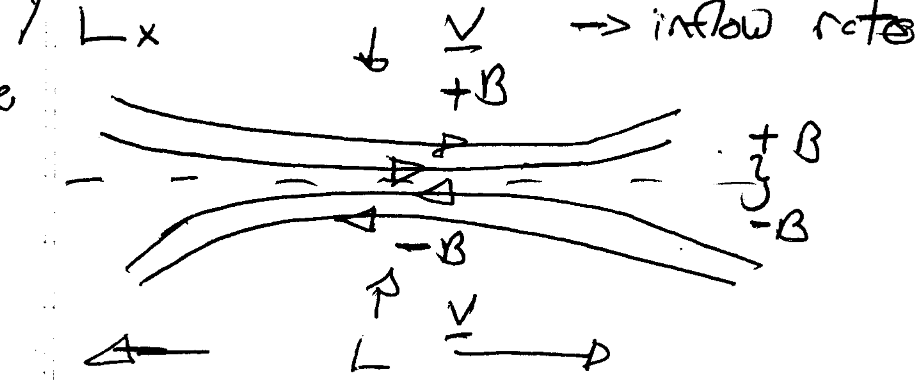
⇒ Reconnection and Tearing Instability

→ Reconnection : A Brief Introduction

- Reconnection :

  - (rapid) dissipation of magnetic energy associated with merger of oppositely directed lines of B-field

i.e



$J = \frac{c}{4\pi}\left(\frac{2B}{\Delta}\right)$

↳ sheet thickness

  - thin current layer/sheet forms at merger layer ; Dissipation of current layer sets $\left\{\begin{array}{c}\text{reconnection}\\ \text{dissipation}\end{array}\right\}$ rate $(\Delta L)\,\eta J^2$.

→ observe:

  - Reconnection is multi-scale process (i.e. L, Δ)
    ⟷ how dynamics determines small scale upon which η must be important.

    Here → Δ (scaling, etc.) ?

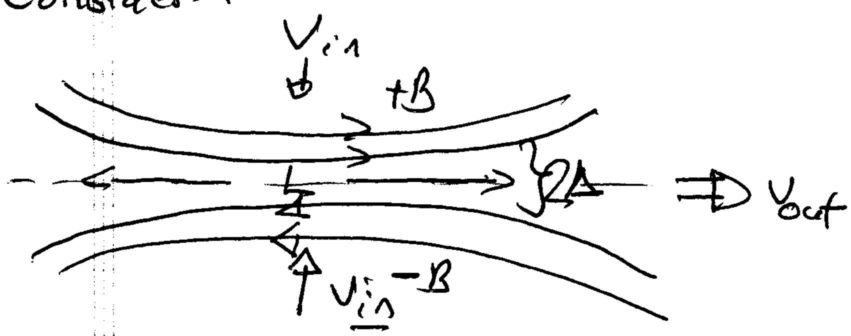
— reconnection can be:

a) — driven (inflow V) : Sweet-Parker, etc.

b) — spontaneous : Tearing Mode

a)
— Sweet - Parker Reconnection

Consider:



Δ ? V_{out} ?
V_{in} ? $\rightarrow$ steady flow.

Leverage:
- mass balance
- momentum balance
- energy balance

($\nabla \cdot \mathbf{V} = 0$)

$\Rightarrow$ ① $V_{in} L = \Delta\, V_{out}$ mass balance

② Energy Balance

inflow B-energy = dissipation

$$V_{in} L (B^2/8\pi) = (2\Delta L)\,\frac{J^2}{\sigma}$$

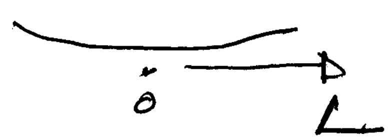
$$J = \frac{c}{4\pi}\frac{\partial B}{\partial \Delta} = \frac{c}{4\pi}\frac{B}{\Delta}$$

$$V_{in} \cancel{L}\, (\cancel{B^2}/8\pi) = \frac{c^2}{16\pi^2}\frac{(2\Delta L)}{\sigma}\,\frac{\cancel{B^2}}{\Delta^2}$$

$$V_{in} = \frac{c^2}{2\pi\sigma}\frac{(1)}{\Delta} = \frac{\eta}{\Delta}$$

③ Momentum balance

$$\int_0^L \left\{ \rho v \frac{\partial v}{\partial x} = -\frac{\partial p}{\partial x} \right\}$$



$$\rho \frac{V_{out}^2}{2} = p\Big|_L^0 = \Delta p \, ; \; \Delta p = p_0 - p_\infty$$

$$\therefore \rho V_{out}^2 = 2\Delta p = 2\frac{B^2}{8\pi}$$

i.e. pressure excess is just magnetic pressure

but

$$V_{in} = \eta/\Delta \qquad \frac{\eta}{\Delta} = \frac{\Delta}{L} V_A$$

$$\Rightarrow \boxed{\begin{aligned} \Delta &= \left(\frac{\eta L}{V_A}\right)^{1/2} = \frac{L}{R_m^{1/2}} \\ V_{in} &= \frac{V_A}{R_m^{1/2}} \end{aligned}}$$

→ long, thin sheets.

→ velocity to dissipate B via η

$$R_m = \frac{V_A L}{\eta} \sim \frac{V_A/L}{\eta/L^2}$$

→ Magnetic Reynolds #

$1/R_m$ is fundamental smallness parameter of the problem, i.e. 'problem' exists only since η is small.

Note:

→ V_{in} explicitly depends on η, though via fractional power.

→ ? Weaker dependence on η ? — { Petschek, 3D, ⋮ }

→ Sweet - Parker rate recently verified experimentally (H. Ji, et. al. PRL '98).

→ anomalous η ?

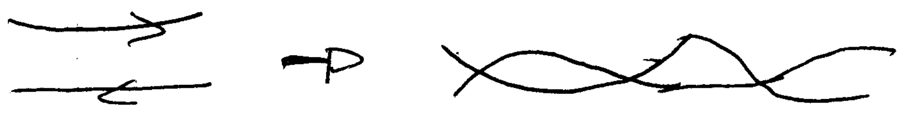
→ ion-acoustic instability (J large)

→ turbulence (3D)

→ etc

→ why 'reconnection'

island chain

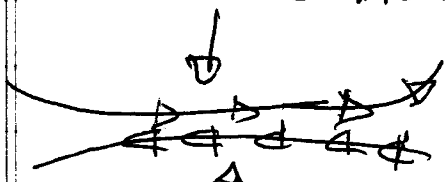


etc.

· ▷ Tearing Mode

- Common element of magnetic null line (though field $\perp$ to plane of reconnection present) between re-connection and resonant modes

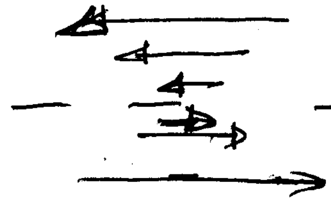
i.e.



driven reconnection

i.e.

reconnection layer

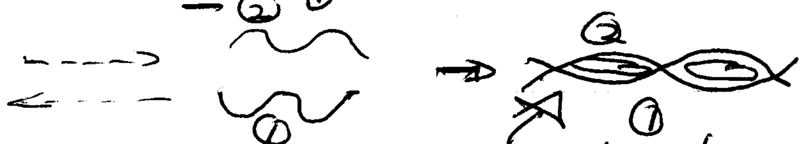


$\underline{k}\cdot\underline{B}=0$
$=0$
resonance

instability $\Rightarrow$ spontaneous reconnection

finite m mode

i.e.



island chain

Note: – Resonant instability and $\tilde{\psi}(x)$ $(\tilde{B}_r(x))$ even parity in x $(\tilde{\psi}(-x)=\tilde{\psi}(x)$, with $\tilde{\psi}(0)\neq 0)$ $\Rightarrow$ "tearing"

- while possible to have ∇p-driven tearing, etc. (i.e. electromagnetic resistive interchanges), simplest tearing is current gradient driven, i.e. $\beta\to 0$.

THE PROBLEM: In ideal MHD,

$$\hat{E}_{\parallel} = 0 \Rightarrow \gamma\vec{\psi} = \nabla_{\parallel}\vec{\phi} = ik_{\parallel}\vec{\phi}$$
$$= i\left(\frac{k_{\theta}x}{L_s}\right)\vec{\phi}$$

Now, $x \to 0 \Rightarrow \gamma\vec{\psi} \Rightarrow 0$. As $\vec{\psi}(0) \neq 0 \Rightarrow$ $\gamma = 0$! Thus can't have tearing parity instability in ideal MHD. Obvious also, as field-fluid decoupling necessary for reconnection. Thus, need add resistivity

$$\hat{E}_{\parallel} = \eta\vec{J}$$

i.e. tearing mode as resistive internal kink $\Rightarrow \gamma = \gamma(\eta) \to$ slower than ideal

BUT, obviously resistivity significant only near resonant surface, treat problem a la'

- 'ideal' MHD "exterior" ($x \neq 0 \to \eta$ doesn't matter)

$$\begin{aligned} B\cdot\nabla J &= 0 \\ E_{\parallel} &= 0 \end{aligned} \Rightarrow \nabla^2\hat{\psi} + \frac{L_s}{B_0 x}\frac{d\langle J\rangle}{dr}\hat{\psi} = 0$$

note singularity!

- resistive MHD interior, i.e. $x \approx 0$.

i.e. $\gamma \hat{\psi} - B_0 \nabla_\parallel \hat{\phi} = \eta \nabla^2 \hat{\psi}$

$\gamma \nabla^2 \hat{\phi} = i k_\parallel B_0 \nabla^2 \hat{\psi}$

- will <u>match</u> interior and exterior.

Important:

tearing instability — current gradient <u>drive</u>; resistivity <u>trigger</u>

- Exterior Solution

$$\nabla_{\perp r}^2 \hat{\psi} - k_\theta^2 \hat{\psi} + \frac{k_\theta}{B_0 x} \frac{\partial J_0}{\partial r} \hat{\psi} = 0$$

singularity at resonant surface

if $\langle J \rangle' = 0 \Rightarrow \nabla^2 \hat{\psi} = 0$

$\psi = e^{-|k_\theta x|}$ ($\to 0$ at $\pm\infty$)

$\langle J \rangle' \neq 0 \Rightarrow$?

Now, to tear need: $\begin{cases} \hat{\psi}(0) \neq 0 \\ \hat{J}(0) = \partial_x^2 \psi\big|_0 \neq 0 \end{cases}$

i.e. must draw current on resonant surface to tear i.e.  $\hat{J}_z(0) \neq 0$

$$\nabla^2\psi = \hat{J}_z(0)$$

$$\therefore \quad \Delta' = \frac{1}{\psi(0)} \frac{\partial\psi}{\partial x}\Bigg|_{x\to 0_-}^{x\to 0_+} \neq 0$$

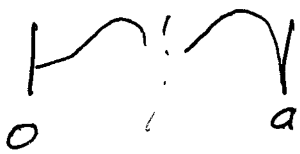
measure of
surface current (on resonant surface)

Now

→ will show need $\Delta' > 0$ for tearing mode

N.B. $\Delta' < 0$ ($\Rightarrow -2|k_\theta| = \Delta'$) for $\langle J_\parallel \rangle' = 0$. Current gradient needed for instability.

→ clear Δ' determined entirely by exterior solution.

i.e. 

- satisfy b.c. at a and integrate to $\underline{k}\cdot\underline{B}_0 = 0$ singularity
- satisfy b.c. at 0 and integrate to $\underline{k}\cdot\underline{B} = 0$ singularity

⇒ determine Δ'

→ guidelines : – $\Delta' > 0$ only for low m.
(i.e. $m = 2, 3 \ldots$)

– $\langle J_{\parallel}\rangle'$ steep favor tearing.
L_s large (weak shear)

→ obvious that resistive layer is, in essence, boundary layer between exterior regions.

– Interior / Layer

→ Scaling Treatment

$$\gamma \psi - B_0 \nabla_{\parallel} \phi = \eta \nabla^2 \psi$$

$$\gamma \nabla^2 \phi = i k_{\parallel} B_0 \nabla_{\perp}^2 \psi + \cancel{\tilde{B}_r \frac{\partial}{\partial r} \langle J_{\parallel} \rangle}$$

$$\nabla_{\perp}^2 \sim \partial_x^2$$

$$\gamma \vec{\psi} - i k_{\parallel} B_0 \vec{\phi} = \eta \psi''$$

$$\gamma \phi'' = i k_{\parallel} B_0 \psi''$$

$$\therefore \quad \gamma \phi'' = \frac{k_{\parallel}^2 B_0^2}{\eta} \phi + i k_{\parallel} B_0 \frac{\gamma}{\eta} \psi$$

so, for layer width w: asymptotic balance $\Rightarrow$

$$\frac{\gamma}{w^2} \sim \frac{k_\theta^2 B_0^2}{L_s^2 \eta} w^2$$

$$\Rightarrow \quad w \cong \left(\frac{L_s^2 \gamma \eta}{k_\theta^2 B_0^2} \right)^{1/4}$$

usual resistive layer scaling,

Then, integrating Ohms Law thru layer

$$\gamma w \psi - \cancel{i \frac{k_\theta w^2}{L_s^2} B_0 \phi} = \eta \Delta' \psi$$

$$\gamma = \eta \Delta' / w$$

$\left\{ \begin{array}{l} \Delta' > 0 \text{ for} \\ \text{instability} \end{array} \right.$

$$\Rightarrow \quad \gamma = \eta \Delta' \left[\left(\frac{k_\theta^2 B_0^2}{L_s^2 \gamma \eta} \right)^{1/4} \right]$$

$$\gamma^{5/4} = \eta^{3/4} \Delta' \left[k_\theta^2 \frac{B_0^2}{(Rq)^2} \hat{s}^2 \right]^{1/4} \qquad \rho_0 \equiv 1$$

$$\Rightarrow \boxed{\begin{aligned}\gamma &\cong \eta^{3/5}\,\Delta'^{4/5}\left[k_\theta^2\,\omega_A^2\,\delta'^2\right]^{1/5}\\ &\cong (\Delta' a)^{4/5}\left[k_\theta a\right]^{2/5}\left(\frac{V_A}{L_s}\right)^{2/5}\left(\frac{\eta}{a^2}\right)^{3/5}\end{aligned}}$$

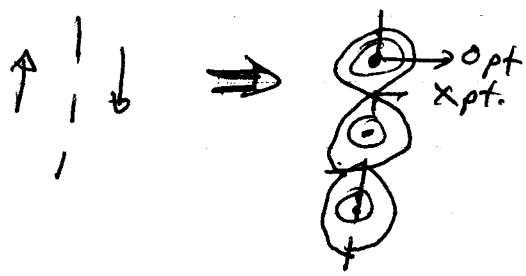
→ usual hybrid γ

→ big crank gets factor of .6

→ linear theory valid till island ~ tearing layer width

→ What Does Tearing Mode Do?

Answer: Forms an island



filaments the current sheet!

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}$$

$$\frac{dx}{dy} = \frac{B_x}{B_y} = \frac{\tilde{B}\sin ky}{B_0 \frac{x}{L_s}}$$

$$B_0 \frac{x^2}{L_s} = -\frac{\tilde{B}}{k}\cos ky + C \Rightarrow$$

$$w_I \sim \left(\frac{L_s}{k_\theta}\,\frac{\tilde{B}}{B_0}\right)^{1/2}$$

island width.