Physics 2B Electricity and Magnetism

Dylan Brennan
brennan@physics.ucsd.edu

Winter 2006

Dylan Brennan	DEPARTMENT OF PHYSICS	Winter 2006

Physics 2B(a)
Electricity and Magnetism

WEB:	http://www-physics.ucsd.edu/students/courses/winter2006
INSTRUCTOR:	Dylan Brennan Office: TBA Phone: 858-455-4144 Email: brennan@physics.ucsd.edu Office Hours: Tu 9:00-11:00AM
COURSE COORDINATOR:	Patti Hey 118 Urey Hall Addition Phone: x21468
TEACHING ASSISTANT:	Jui Yu Chiu Office: MH2101 Email: jchiu@physics.ucsd.edu Office Hours: W 1:30-2:30PM, F 11:00AM-12:00PM
LECTURES:	MW 9:00-9:50 AM WLH 2005 Tu 8:00-8:50 AM CENTR 119
QUIZZES:	F 9:00-9:50 AM WLH 2005
DISCUSSION SESSION:	W 5:00-5:50 PM CENTR 119 Discussion sessions are informal classes intended for you to ask questions and get answers about material covered in lectures. You are advised to participate.
FINAL EXAM:	Wed., March 22, 2006, 8:00-11:00AM, Location TBA
GRADING:	67% quizzes and 33% final exam. There will be a curve.
TEXT:	Richard Wolfson, Jay M. Pasachoff, Physics For Scientists and Engineers, Third Edition, Addison Wesley
HOMEWORK:	Homework will not be graded. A list of suggested problems from each chapter will be posted on the course website. Questions on the quizzes and final exam will resemble the homework problems, and you are encouraged to address any questions you have about homework problems in the discussion sessions.

Dylan P. Brennan
brennan@physics.ucsd.edu
455-4144 @ General Atomics

Office Hours: Tu 9:00 - 11:00 AM
Location TBA

TA
Jui Yu Chiu
jchiu@physics.ucsd.edu

Office Hours:
W 1:30-2:30 PM
F 11:00 AM - 12:00 PM
MH2101

TENTATIVE COURSE OUTLINE

LECTURE NUMBER (Start Date)	TOPIC	QUIZ SUBJECT (Date)	ASSIGNED PROBLEMS
1-3 (Jan. 9)	Electric Charge and Field, Coulomb's Law, Ch. 23	Chapter 23 (Jan. 13)	Ch. 23: 1, 11, 19, 22, 24, 30, 39, 46, 48, 59, 68, 78
4-6 (Jan. 16)	Gauss's Law, Ch. 24	Chapter 24 (Jan. 20)	Ch. 24: 9, 10, 19, 28, 31, 49, 54, 63
7-9 (Jan. 23)	Electric Potential, Ch. 25	Chapter 25 (Jan. 27)	Ch. 25: 8, 11, 28, 35, 39, 50
10-12 (Jan. 30)	Capacitance, Ch 26 Current, Ch. 27	Chapters 26, 27 (Feb. 3)	Ch. 26: 15, 36, 37, 46, 54 Ch. 27: 10, 36, 39, 50, 71
13-15 (Feb. 6)	Curcuits, Ch. 28	Chapter 28 (Feb. 10)	Ch. 28: 22, 25, 26, 29, 31, 43, 55
16-18 (Feb. 13)	Magnetic Field, Ch. 29	Chapter 29 (Feb. 17)	Ch. 29: 13, 22, 27, 36, 38, 53
19-21 (Feb. 20)	Sources of Magnetic Field, Ch. 30	Chapter 30 (Feb. 24)	Ch. 30: 10, 15, 17, 23, 24, 35, 36, 40, 45, 57
22-24 (Feb. 27)	Induction, Ch. 31 Inductors, Ch. 32	Chapters 30, 31 (Mar. 3)	Ch. 31: 13, 14, 17, 21, 25, 27, 29, 32, 34, 35 Ch. 32: 5, 9, 22, 36, 55, 66, 71
25-27 (Mar. 6)	Alternating Currents, Ch. 33	Chapters 32, 33 (Mar. 10)	Ch. 33: 5, 11, 18, 23, 30, 39, 45, 60
28-30 (Mar. 13)	Maxwell's Equations, Ch. 34	No Quiz	Ch. 34: 1, 2, 3, 4, 5

Charge, Coulomb's Law, and Electric Field

Lectures 1, 2, 3

The Forces of Nature

The Strong Force: (which your book calls the color force
because there are three charges: *red, green,* and *blue)*.
This is the force between quarks and gluons that holds atomic
atomic nuclei together.

The Electromagnetic Force: about 100 times weaker than the strong force;
two charges: *plus* and *minus*.

The Weak Force: about 20 orders of magnitude weaker than electricity;
responsible for nuclear beta decay, changes neutrons
to protons and *vice versa.* This is how neutrinos interact.

Gravitation: the weakest force by far! About 39 orders of magnitude weaker
than electricity!

Overview of Electromagnetism

Greeks note that rubbing materials can cause them to attract/repel and they note that certain stones from “Magnesia” attract iron.

18th century “parlor tricks”

Franklin figures out that there are two kinds of electric charge.

19th century: Faraday performs revealing experiments
Maxwell discovers equations which unify electricity and magnetism and lead to the prediction of electromagnetic waves and the development of Einstein’s relativity.

20th century: Relativity, the photoelectric effect, quantum mechanics, all lead to the development of quantum electrodynamics.
Unification of the electromagnetic and weak interactions.

21st century: ?

Technological Revolution

Coulomb's Law

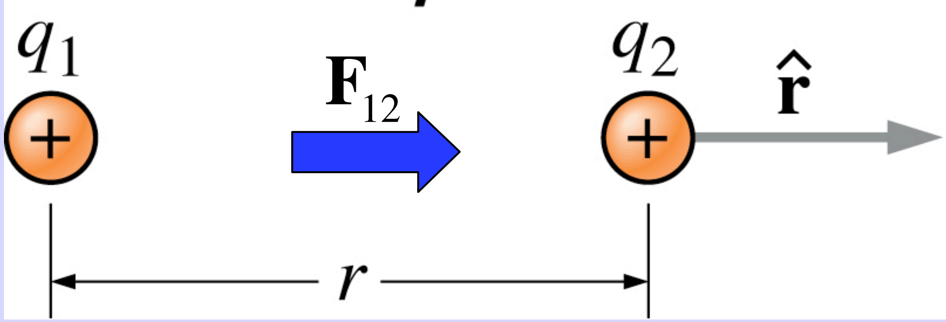
Like charges repel, opposite charges attract; ***net*** electric charge is conserved.

We will label the two kinds of charge as either ***positive*** or ***negative***.

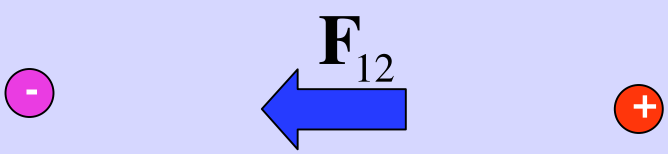
Then the force between two ***point*** charges is:

$$\mathbf{F}_{12} = \frac{k q_1 q_2}{r^2}\hat{\mathbf{r}}$$

the force exerted *on* charge q_2 *by* the charge q_1



unit vector points ***from*** q_1 ***towards*** q_2



$$\hat{\mathbf{r}} = \text{unit vector} = \frac{\mathbf{r}}{|\mathbf{r}|}$$

Coulomb's Constant

$$k \equiv \frac{1}{4\pi\varepsilon_0} \approx 9.0 \times 10^9 \ \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

Where the unit of charge that we will use is the Coulomb:

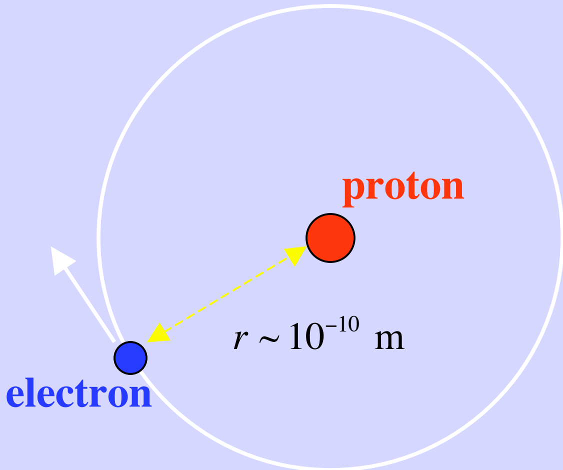
$$1\ \mathrm{C} \approx 6.241 \times 10^{18} \text{ elementary charges}$$

"permittivity" constant $$\varepsilon_0 \approx 8.85 \times 10^{-12} \ \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}$$

The Shocking Truth About Electricity

The electric force is phenomenally strong!

Consider a comparison between the electric and gravitational forces acting between the electron and the proton in a hydrogen atom.



Gravitational Force

$$F_G = G\frac{M_p m_e}{r^2} \sim 10^{-47} \text{ Joule m}^{-1}$$

Electric Force

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{Q_p q_e}{r^2} \sim 10^{-8} \text{ Joule m}^{-1}$$

$$M_p \approx 938.26 \text{ MeV/c}^2$$
$$m_e \approx 0.511 \text{ MeV/c}^2$$

$$Q_p = +1 \text{ elementary charge} \approx +1.6022 \times 10^{-19} \text{ Coulomb}$$
$$q_e = -1 \text{ elementary charge} \approx -1.6022 \times 10^{-19} \text{ Coulomb}$$

The Electric Field

It is convenient to define the ELECTRIC FIELD at a point as the force per unit (positive) charge, so we can determine the force on any magnitude of charge at that point as qE:

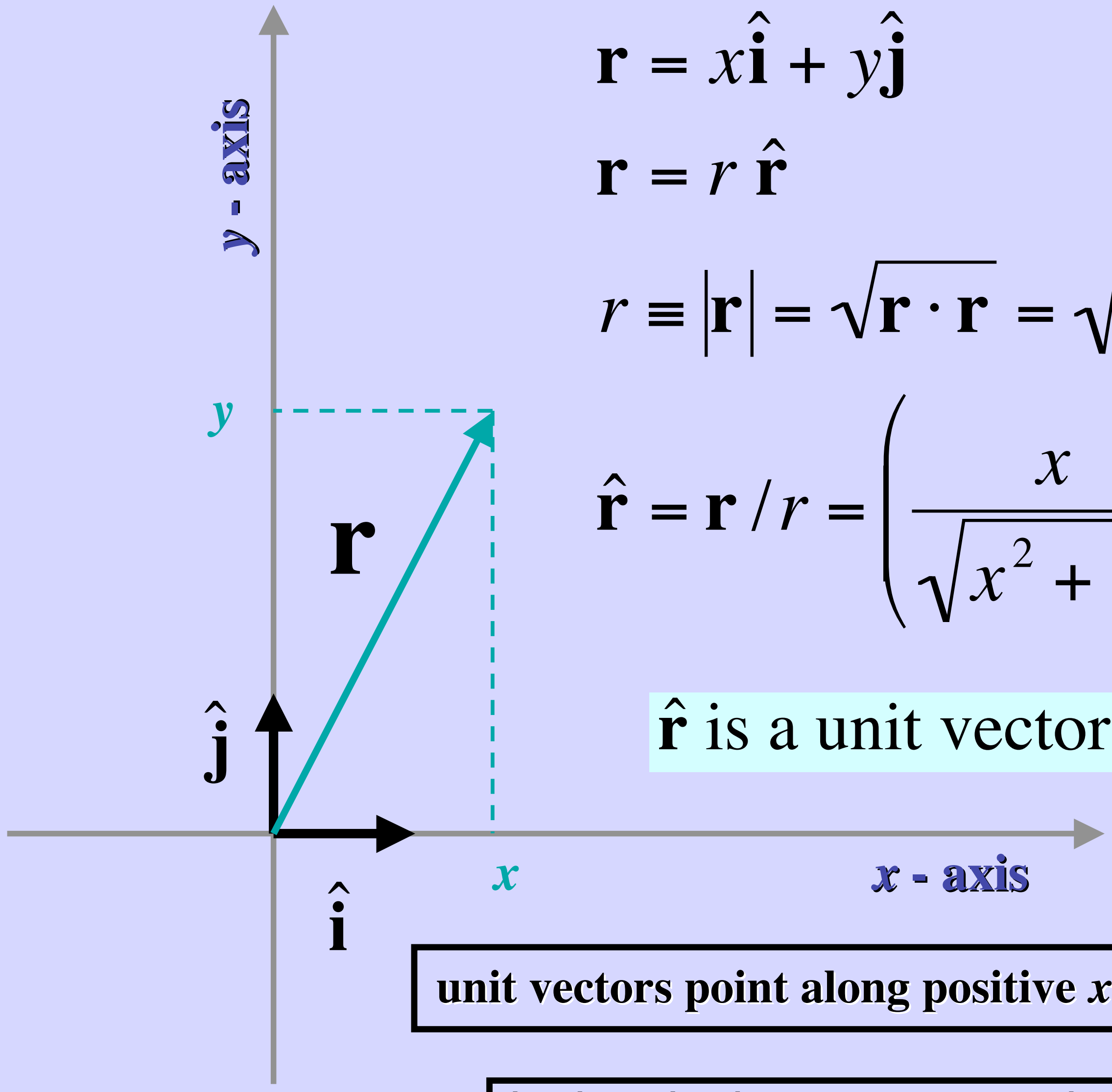
What are the dimensions of Electric Field?

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$\left[\left|\mathbf{E}\right|\right] = \frac{\mathrm{N}}{\mathrm{C}} = \frac{\mathrm{J}}{\mathrm{m\,C}} = \frac{\mathrm{Volts}}{\mathrm{m}}$$

$$\mathbf{Volt} = \frac{\mathbf{Joule}}{\mathbf{Coulomb}} = \frac{\mathbf{J}}{\mathbf{C}}$$

Brief Review: Vectors in 2 Dimensions



$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$\mathbf{r} = r\,\hat{\mathbf{r}}$$

$$r \equiv |\mathbf{r}| = \sqrt{\mathbf{r}\cdot\mathbf{r}} = \sqrt{x^2 + y^2}$$

$$\hat{\mathbf{r}} = \mathbf{r}/r = \left(\frac{x}{\sqrt{x^2+y^2}}\right)\hat{\mathbf{i}} + \left(\frac{y}{\sqrt{x^2+y^2}}\right)\hat{\mathbf{j}}$$

$\hat{\mathbf{r}}$ is a unit vector along $\mathbf{r}$, so $\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = 1$

unit vectors point along positive *x* and *y* axes

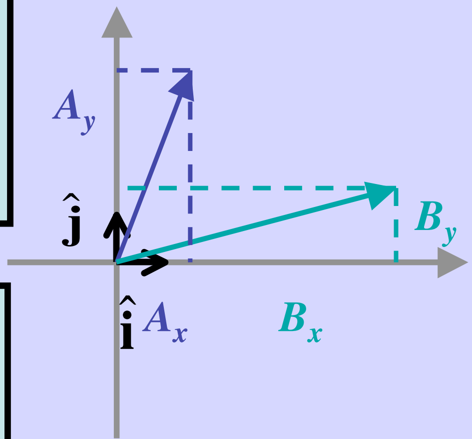
$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = 1 \quad \text{and} \quad \hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = 0$$

inner (scalar) product of two vectors

$$\mathbf{A} = (A_x)\hat{\mathbf{i}} + (A_y)\hat{\mathbf{j}}$$

$$x\text{ - component } A_x = \text{ projection along } x\text{ - axis } = \mathbf{A}\cdot\hat{\mathbf{i}}$$

$$y\text{ - component } A_y = \text{ projection along } y\text{ - axis } = \mathbf{A}\cdot\hat{\mathbf{j}}$$



$$\mathbf{B} = (B_x)\hat{\mathbf{i}} + (B_y)\hat{\mathbf{j}}$$

$$x\text{ - component } B_x = \text{ projection along } x\text{ - axis } = \mathbf{B}\cdot\hat{\mathbf{i}}$$

$$y\text{ - component } B_y = \text{ projection along } y\text{ - axis } = \mathbf{B}\cdot\hat{\mathbf{j}}$$

inner product

$$\mathbf{A}\cdot\mathbf{B} = A_x B_x + A_y B_y$$

$$= |\mathbf{A}||\mathbf{B}|\cos\theta$$

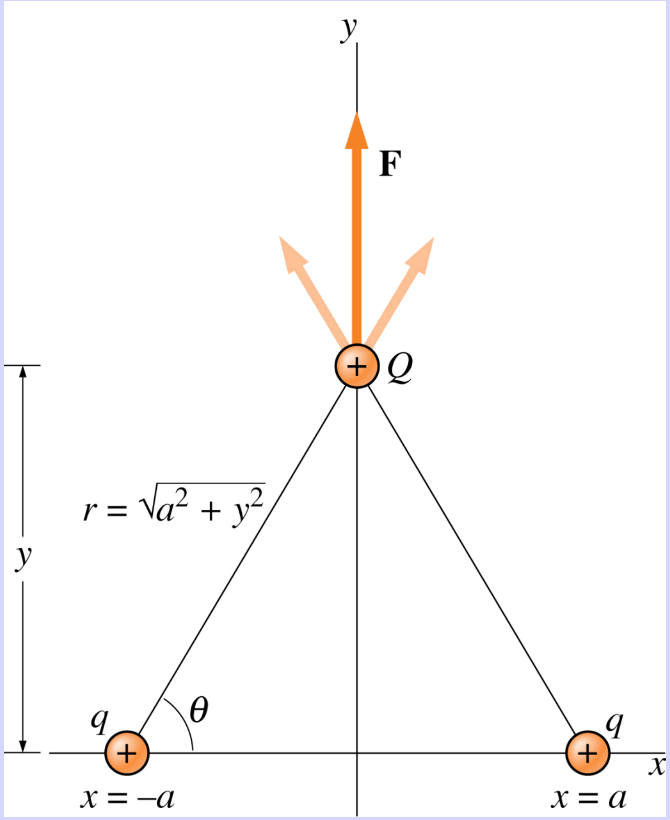
where θ is the angle between the two vectors

Point Charges and the Superposition Principle

Electric forces simply add vectorially.

The TOTAL FORCE on the object is just the VECTOR SUM of the individual forces.

Example: What is the force on q when both q1 and q2 are present??



$$\mathbf{F}_{q>0} = \frac{kQq}{a^2 + y^2}\left[\frac{a}{\sqrt{a^2 + y^2}}\hat{\mathbf{i}} + \frac{y}{\sqrt{a^2 + y^2}}\hat{\mathbf{j}}\right]$$

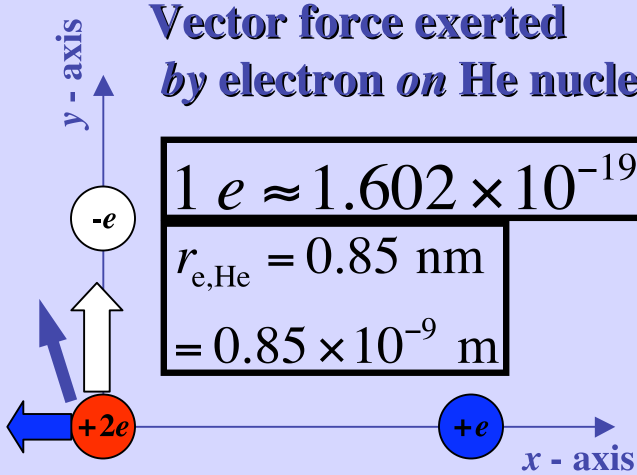
$$\mathbf{F}_{\text{total force on } Q} = \mathbf{F}_{q>0} + \mathbf{F}_{q<0} = \frac{kQq(2a)}{\left(a^2 + y^2\right)^{3/2}}\hat{\mathbf{i}}$$

$$\mathbf{F}_{q<0} = \frac{kQq}{a^2 + y^2}\left[\frac{a}{\sqrt{a^2 + y^2}}\hat{\mathbf{i}} - \frac{y}{\sqrt{a^2 + y^2}}\hat{\mathbf{j}}\right]$$

Example: Problem 11, Chapter 23:

A proton is on the x-axis at x = 1.6 nm. An electron is on the y-axis at y = 0.85 nm. Find the net force the two exert on a helium(He) nucleus (charge +2e) at the origin.

Solution: **draw a picture; set up the algebra; plug in the numbers with their units/dimensions; cancel out and re-work units to the final units and make sure they make sense!**



Vector force exerted *by* electron *on* He nucleus:

$$\boxed{1\ e \approx 1.602 \times 10^{-19}\ \mathrm{C}}$$

$$\boxed{r_{\mathrm{e,He}} = 0.85\ \mathrm{nm} = 0.85 \times 10^{-9}\ \mathrm{m}}$$

$$\mathbf{F}_{\mathrm{e,He}} = \frac{k(-e)(+2e)}{r_{\mathrm{e,He}}^2}\left(-\hat{\mathbf{j}}\right) = \frac{k\left(2e^2\right)}{r_{\mathrm{e,He}}^2}\hat{\mathbf{j}}$$

$$\approx \frac{\left(9 \times 10^9\ \mathrm{N} \cdot \cancel{\mathrm{m}}^2/\cancel{\mathrm{C}}^2\right)2\left(1.602 \times 10^{-19}\ \cancel{\mathrm{C}}\right)^2}{\left(0.85 \times 10^{-9}\ \cancel{\mathrm{m}}\right)^2}\hat{\mathbf{j}}$$

$$\approx \left(6.38 \times 10^{-10}\mathrm{N}\right)\hat{\mathbf{j}} = \left(0.638\ \mathrm{nN}\right)\hat{\mathbf{j}}$$

Vector force exerted *by* proton *on* He nucleus:

$$\mathbf{F}_{\mathrm{tot}} = \mathbf{F}_{\mathrm{e,He}} + \mathbf{F}_{\mathrm{p,He}} \approx \left(-0.18\ \mathrm{nN}\right)\hat{\mathbf{i}} + \left(0.638\ \mathrm{nN}\right)\hat{\mathbf{j}}$$

$$\mathbf{F}_{\mathrm{p,He}} = \frac{k(+e)(+2e)}{r_{\mathrm{p,He}}^2}\left(-\hat{\mathbf{i}}\right) = \frac{k\left(2e^2\right)}{r_{\mathrm{p,He}}^2}\left(-\hat{\mathbf{i}}\right) \approx \frac{\left(9 \times 10^9\ \mathrm{N} \cdot \cancel{\mathrm{m}}^2/\cancel{\mathrm{C}}^2\right)2\left(1.602 \times 10^{-19}\ \cancel{\mathrm{C}}\right)^2}{\left(1.6 \times 10^{-9}\ \cancel{\mathrm{m}}\right)^2}\left(-\hat{\mathbf{i}}\right)$$

$$\approx \left(-0.180\ \mathrm{nN}\right)\hat{\mathbf{i}}$$

Vectors in 3 Dimensions

A simple generalization of what we just did in the examples in two dimensions!

At a particular point in space x, y, z, the (vector) force on a point particle with charge *q* is

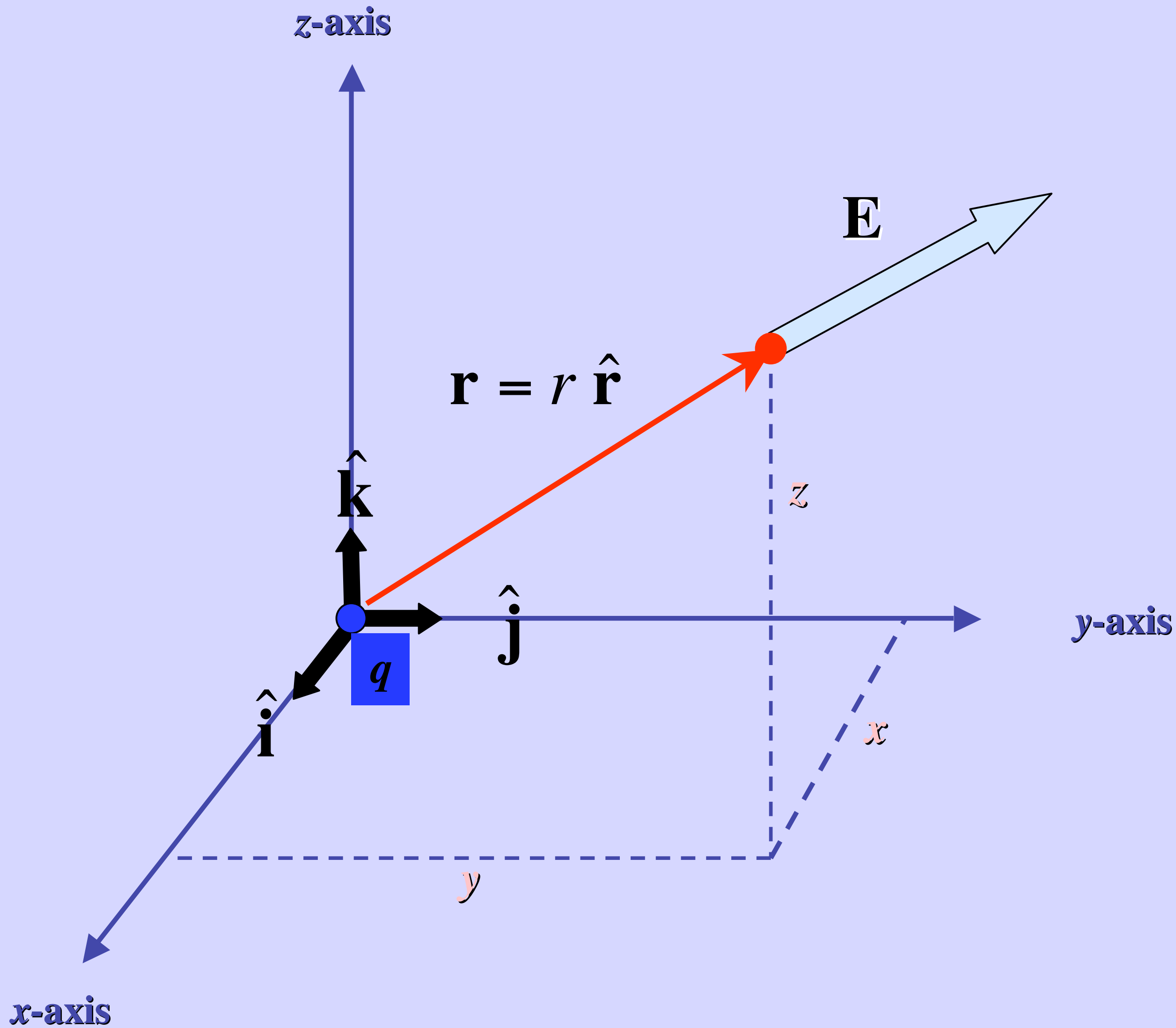
$$\mathbf{F} = q\mathbf{E}(x,y,z)$$

So, you can think of a vector residing at each point in space.
Of course, the vector will have three components:
a scalar function $E_x(x, y, z)$, the component along the *x*-direction;
a scalar function $E_y(x, y, z)$, the component along the *y*-direction;
a scalar function $E_z(x, y, z)$, the component along the *z*-direction.

$$\mathbf{E}(x,y,z) = E_x(x,y,z)\hat{\mathbf{i}} + E_y(x,y,z)\hat{\mathbf{j}} + E_z(x,y,z)\hat{\mathbf{k}}$$

$$\mathbf{E}(x,y,z) = \frac{kq}{x^2+y^2+z^2}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{i}} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{j}} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{k}}\right)$$

$$= \frac{kq}{r^2}\hat{\mathbf{r}}$$



The radius vector from the origin to point (x, y, z) is

$$\mathbf{r} = r\,\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

$$\hat{\mathbf{r}} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{i}} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{j}} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{k}}$$

$$r = |\mathbf{r}| = \sqrt{x^2+y^2+z^2}$$

Remember your vector algebra . . .

$$\mathbf{r}\cdot\mathbf{r} = r^2\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = r^2 = x^2 + y^2 + z^2$$

$$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = 1$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$$

inner (scalar) product of two vectors

$$\mathbf{A} = (A_x)\hat{\mathbf{i}} + (A_y)\hat{\mathbf{j}} + (A_z)\hat{\mathbf{k}}$$

x - component A_x = projection along x - axis $= \mathbf{A} \cdot \hat{\mathbf{i}}$

y - component A_y = projection along y - axis $= \mathbf{A} \cdot \hat{\mathbf{j}}$

z - component A_z = projection along z - axis $= \mathbf{A} \cdot \hat{\mathbf{k}}$

$$\mathbf{B} = (B_x)\hat{\mathbf{i}} + (B_y)\hat{\mathbf{j}} + (B_z)\hat{\mathbf{k}}$$

x - component B_x = projection along x - axis $= \mathbf{B} \cdot \hat{\mathbf{i}}$

y - component B_y = projection along y - axis $= \mathbf{B} \cdot \hat{\mathbf{j}}$

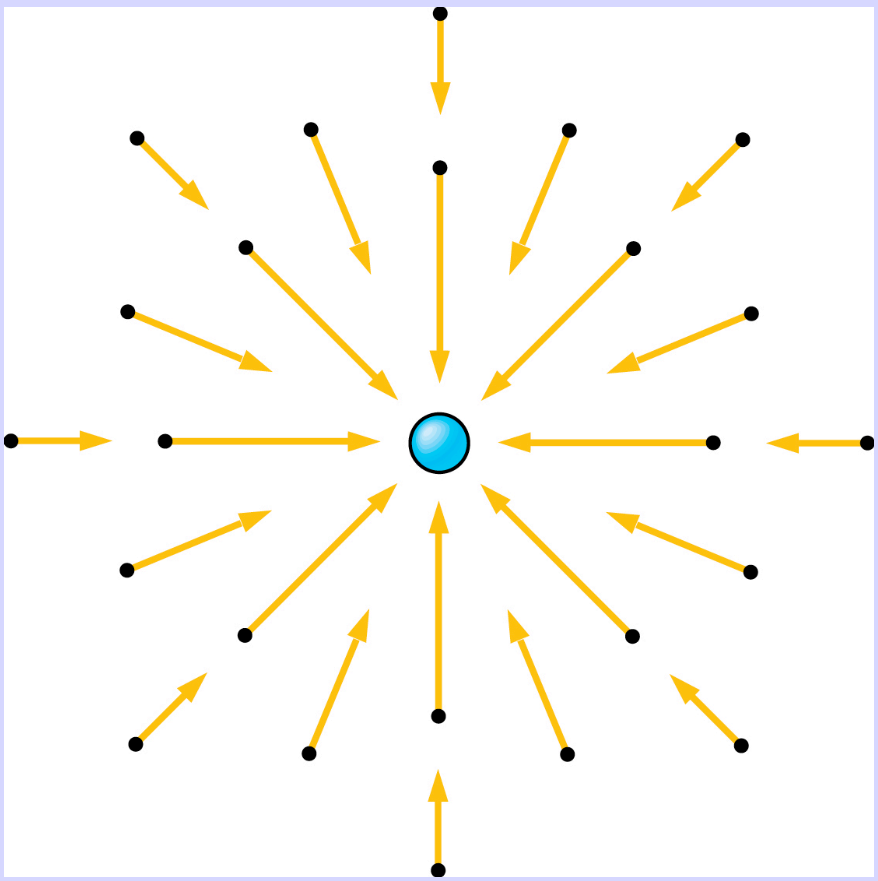
z - component B_z = projection along z - axis $= \mathbf{B} \cdot \hat{\mathbf{k}}$

inner product

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= |\mathbf{A}||\mathbf{B}|\cos\theta$$

where θ is the angle between the two vectors

The Electric Field of a Point Charge $q < 0$



$$\mathbf{E} = \frac{kq}{r^2}\hat{\mathbf{r}}$$

A positive charge causes E field in opposite direction, ie. outward.

The Electric Field from a Distribution of Point Charges

Again, simply use the superposition principle.

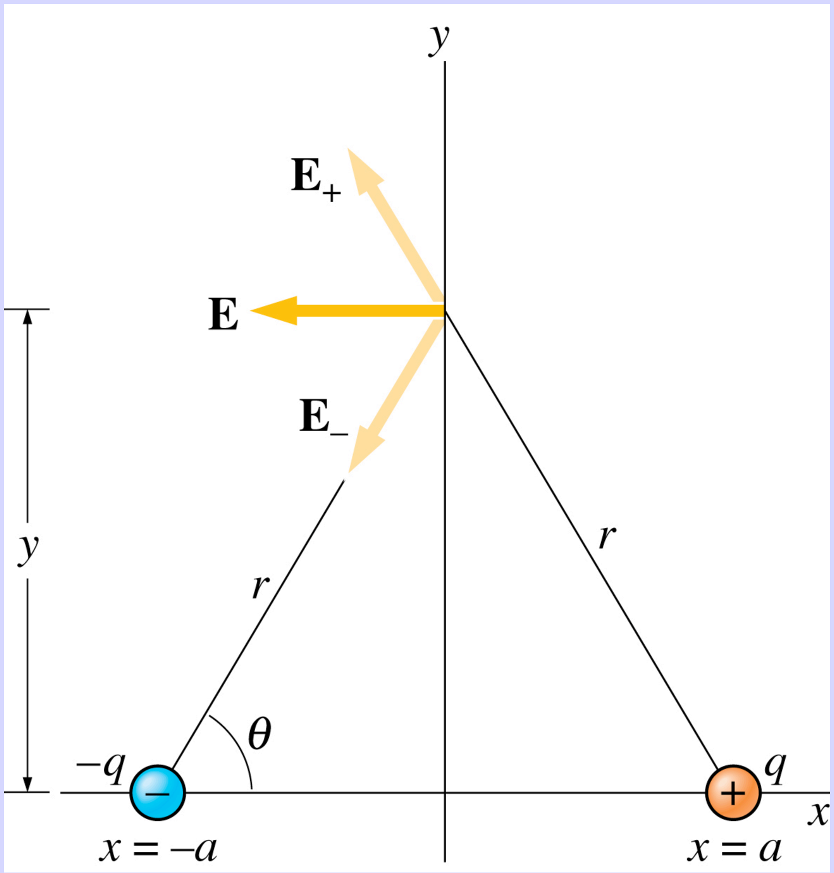
To find the Electric Field at a given point in space, simply add up the contributions from each individual charge.

Calculate the Electric Field contribution from each one of these charges as if the other ones didn't exist.

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdot \cdot \cdot = \sum_i \mathbf{E}_i = \sum_i \frac{kq_i}{r_i^2}\hat{\mathbf{r}}_{\mathbf{i}}$$

Example: Electric Dipole

Two (point) charges of equal magnitude but opposite sign at a fixed separation.



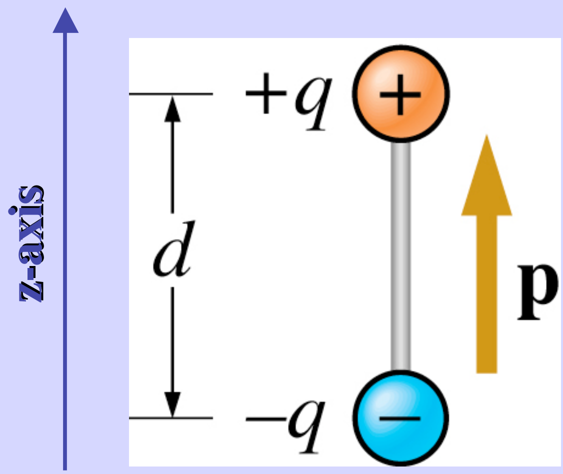
$$\mathbf{E}_{q>0} = \frac{kq}{a^2+y^2}\left[\frac{a}{\sqrt{a^2+y^2}}\hat{\mathbf{i}} + \frac{y}{\sqrt{a^2+y^2}}\hat{\mathbf{j}}\right]$$

$$\mathbf{E}_{\text{total}} = \mathbf{E}_{q>0} + \mathbf{E}_{q<0} = \frac{kq(2a)}{\left(a^2+y^2\right)^{3/2}}\hat{\mathbf{i}}$$

$$\approx \frac{kq(2a)}{y^3}\hat{\mathbf{i}} \text{ for } y >> a$$

$$\mathbf{E}_{q<0} = \frac{kq}{a^2+y^2}\left[\frac{a}{\sqrt{a^2+y^2}}\hat{\mathbf{i}} - \frac{y}{\sqrt{a^2+y^2}}\hat{\mathbf{j}}\right]$$

The Dipole Moment is defined to be the vector with magnitude $p = q\, d$ that points from the negative toward the positive charge.

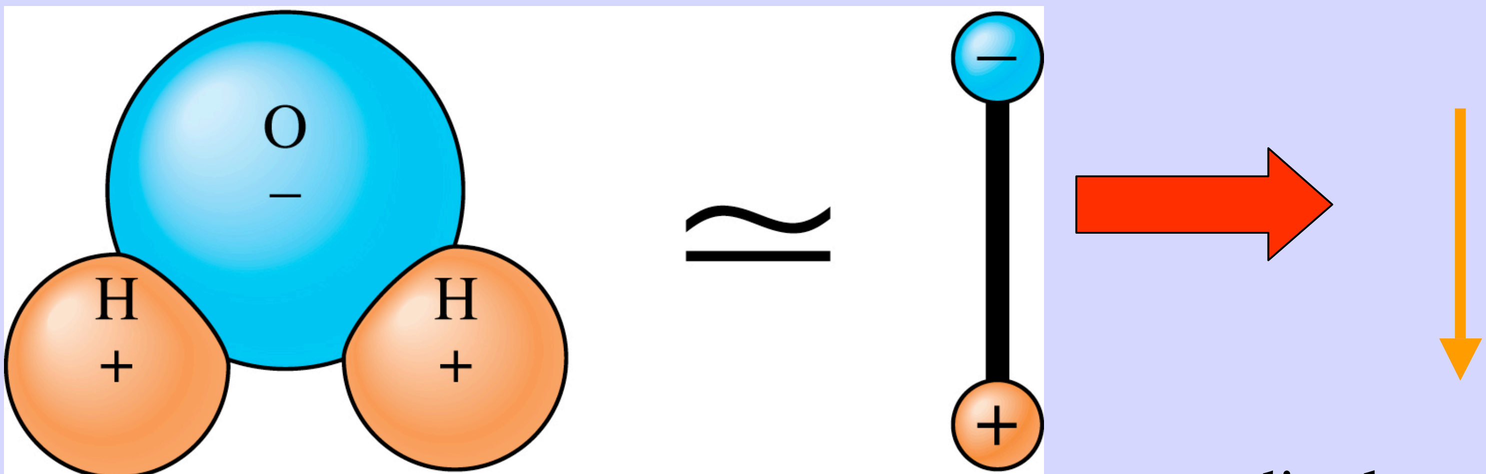


$$\mathbf{p} = qd\,\hat{\mathbf{k}}$$

Example:

Molecules can be overall charge neutral but may nevertheless possess an electric dipole moment.

The classic example is the water molecule.



dipole moment **p**

Continuous Distributions of Charge

Of course, real materials are composed of point-like electrons, protons, and neutrons. The protons and electrons can be regarded, for our purposes, as positive and negative point charges, respectively.

Any macroscopic object will contain ~ 10^{24} particles, however. In this case we can approximate the distribution of charge as continuous across the volume of the object, given locally by a *volume charge density* r (with dimensions *Coulombs per unit volume*, or C/m^3).

Likewise, for charge spread out over a surface, we can define a *surface charge density* s (with dimensions Coulombs per unit area, or C/m^2). For charge spread out along a line (e.g., an extremely thin wire) we can define a *line charge density* l (with dimensions Coulombs per unit length, or C/m).

Then the increment in charge associated with an increment in volume, area, or length is:

$$dq = \rho dV , \quad dq = \sigma dA , \quad dq = \lambda dx$$

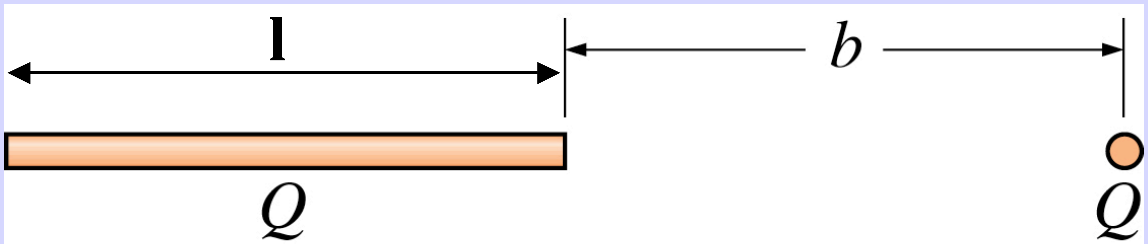
With the charge in an object broken down into particles or small increments *dq*, we can employ the superposition principle to find the Electric Field at any position.

$$d\mathbf{E} = \frac{k\,dq}{r^2}\hat{\mathbf{r}}$$

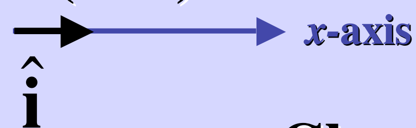
$$\mathbf{E} = \int d\mathbf{E} = \int \frac{k\,dq}{r^2}\hat{\mathbf{r}}$$

Example: a uniformly charged rod of length *l* and charge *Q*.
Find the electric field on the rod axis at distance b.

$$dq = \lambda\, dx = \frac{Q}{l}\, dx$$



E(x=0)?



$$d\mathbf{E} = \frac{k\,dq}{x^2}\,\hat{\mathbf{i}}$$

$$\vec{E} = \hat{i}\int_{-2b}^{-b} \frac{k\,dq}{x^2}$$

$$\vec{E} = \hat{i}\int_{-(b+l)}^{-b} \frac{kQ}{lx^2}dx = \hat{i}\,\frac{kQ}{l}\left(\frac{1}{b} - \frac{1}{(b+l)}\right)$$

$$= \hat{i}\,\frac{kQ}{b(b+l)}$$

Check if makes sense:

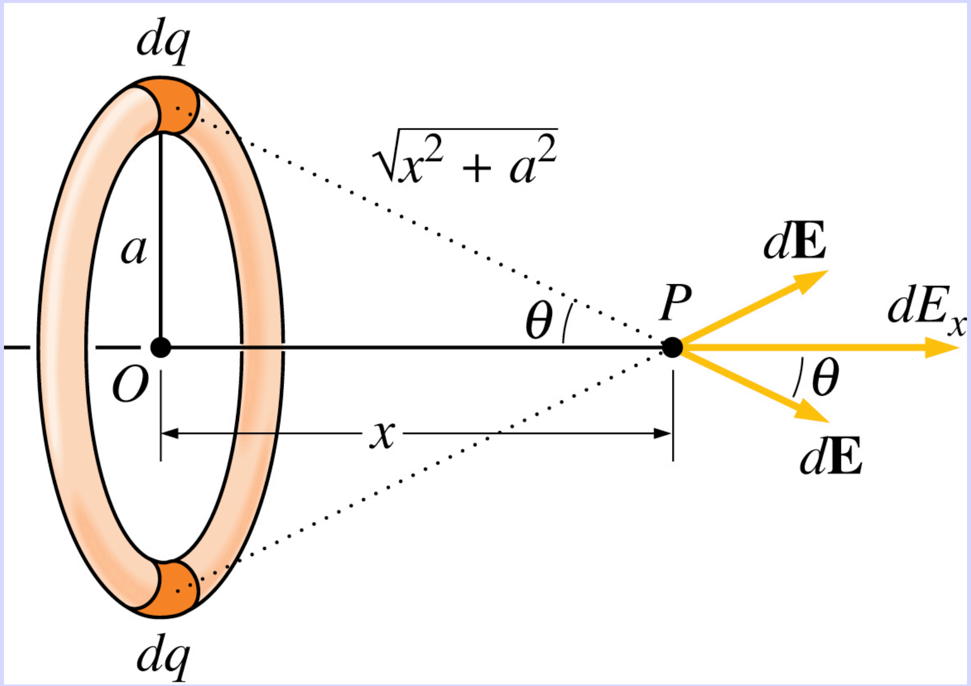
If b>>l

$$\vec{\mathbf{E}} \approx \frac{k\,Q}{b^2}\,\hat{\mathbf{i}}$$

Point charge

Example: a uniformly charged ring of radius *a* and charge *Q*. Find the electric field on the axis through the ring.

Assume that the ring is very thin so that we can regard the charge distribution on it as being a "line charge." Then the "charge density" (charge per unit length) is $\lambda = Q/(2\pi a)$.



$$dE = \frac{k\,dq}{r^2}\hat{\mathbf{r}}$$

$$dE_x = \frac{k\,dq}{r^2}\cos\theta$$

$$= \frac{k\,\mathrm{dq}}{x^2 + a^2}\,\frac{x}{\sqrt{x^2 + a^2}}$$

$$dq = \lambda\,ds = \frac{Q}{2\pi a}\,ds$$

$$E = |\mathbf{E}| = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{kx\,dq}{\left(x^2 + a^2\right)^{3/2}} = \frac{kx}{\left(x^2 + a^2\right)^{3/2}}\int_{\text{ring}} dq$$

$$= \frac{kx}{\left(x^2 + a^2\right)^{3/2}}\int_0^{2\pi a} \frac{Q}{2\pi a}\,ds = \frac{kx\,Q}{\left(x^2 + a^2\right)^{3/2}}$$

Example: continued . . .

Suppose the ring of radius 1 m carries a uniformly distributed charge of 0.01 C. What is the electric field a distance 8 m from the center of the ring and along the center line?
Well this meets the conditions of what we had just derived. We saw that the electric field points along the center line and has magnitude:

$$E = \left|\mathbf{E}\right| = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{kx\, dq}{\left(x^2 + a^2\right)^{3/2}} = \frac{kx}{\left(x^2 + a^2\right)^{3/2}} \int_{\text{ring}} dq$$

$$= \frac{kx}{\left(x^2 + a^2\right)^{3/2}} \int_0^{2\pi a} \frac{Q}{2\pi a}\, ds = \frac{kx\, Q}{\left(x^2 + a^2\right)^{3/2}}$$

$$E \approx \frac{\left(9 \times 10^9 \text{ N m}^2/\text{C}^2\right)\left(8 \text{ m}\right)\left(0.01 \text{ C}\right)}{\left[\left(8 \text{ m}\right)^2 + \left(1 \text{ m}\right)^2\right]^{3/2}} \approx 1.37 \times 10^6 \; \frac{\text{N}\left(\cancel{\text{m}^2}/\cancel{\text{C}^2}\right)\cancel{\text{m}}\,\cancel{\text{C}}}{\cancel{\text{m}^3}\;\;\text{C}}$$

$$\approx 1.37 \times 10^6 \; \frac{\text{N}}{\text{C}} \qquad \text{note}: \; \frac{\text{N}}{\text{C}} = \frac{\text{Joule}}{\text{m C}} = \frac{\text{Volt}}{\text{m}}$$

Kinematics and Dynamics of Matter in Electric Fields

For point particles of mass *m* and charge *q* in an Electric Field E we have

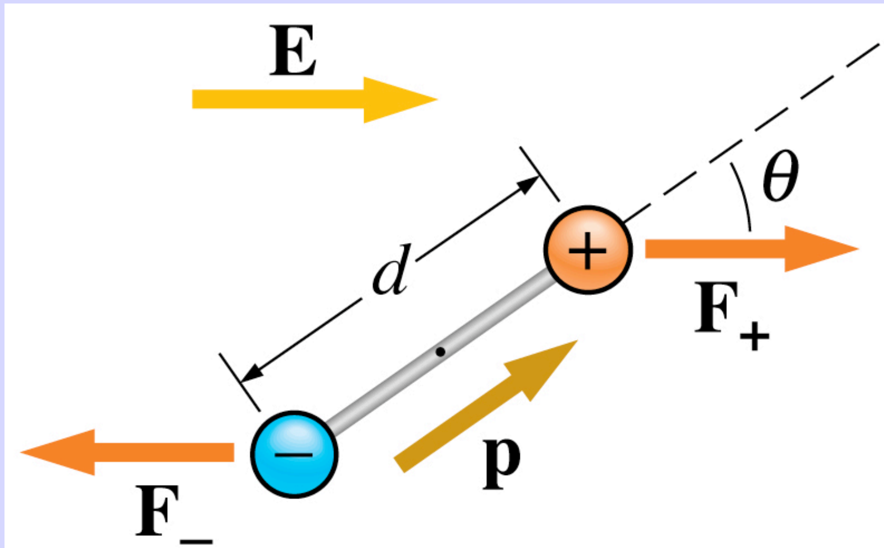
force $\mathbf{F} = q\,\mathbf{E}$

and Newton's law relating force and acceleration $\mathbf{F} = m\mathbf{a}$,

implying that $\mathbf{a} = \dfrac{q}{m}\mathbf{E}$

For example, a uniform(constant in magnitude and direction in some region of space) electric field produces a constant acceleration of a particle.

The Interaction of a Dipole with a uniform Electric Field



$$\tau_- = \tfrac{1}{2}\, d\, q\, E \sin\theta \qquad \tau_+ = \tfrac{1}{2}\, d\, q\, E \sin\theta$$

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

The torque acts to try to line up the dipole along the field.

Calculate the work required to rotate a dipole from a position orthogonal to the Electric Field direction to a new angle q with respect to the field orientation.

$$\mathrm{W} = \int_{\pi/2}^{\theta} \boldsymbol{\tau}\, d\theta = \int_{\pi/2}^{\theta} pE \sin\theta \, d\theta = -pE\cos\theta$$

Associate this work with a change in a potential energy which we can define as

$$U = -\mathbf{p}\cdot\mathbf{E} = -p\,E\cos\theta$$

Conductors, Insulators, and Dielectrics

Materials in which (some) electrons are free to move in response to an applied electric field we term conductors.

Insulators are materials where charges are not free to flow as large scale electric currents.

However, the molecules in insulators may be able to respond to an applied electric field, for example, lining up the intrinsic dipole moments of these molecules. Molecules without intrinsic dipole moments may acquire *induced* dipole moments in response to the electric field. In either case, we call these Substances dielectrics.