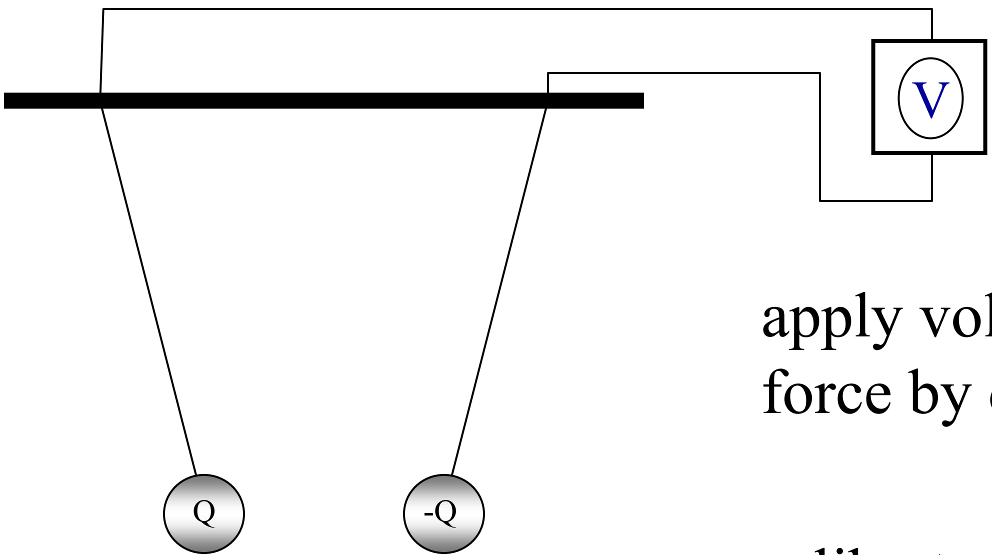
Four Experiments

- **Measure the density of the earth**
  - Learn to estimate and propagate errors
- **Measure moments of inertia**
  - Use repeated measurements to reduce random errors
- **Design, build, and test shock absorber**
  - Design something using mechanical systems to solve a problem
- **Measure coulomb force and calibrate voltmeter**
  - Reduce systematic errors in a precise measurement

Experiment 4

Construct a device to measure the absolute value of a voltage through the measurement of a force

The actual measurements you will make will be of mass, distance, and time but the result will be a measurement of an electric potential in Volts

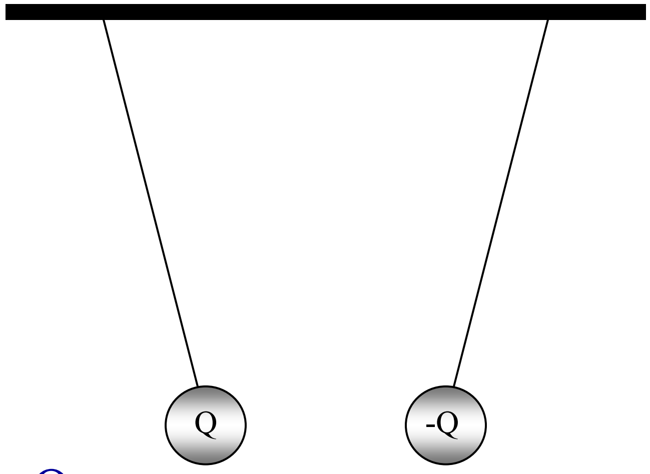


apply voltage and measure force by deflection

↓

calibrate the voltmeter

Review the Basic Equations



$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

Force between point charges
Coulomb's low

$$\varepsilon_0 = 8.85 \times 10^{-12} \ \frac{\text{F}}{\text{m}}$$

Permittivity constant

$$E = \frac{F}{Q_2} = \frac{Q_1}{4\pi\varepsilon_0 r^2}$$

Electric field from a point charge Q_1
Coulomb force acting on a unit charge
Coulomb force acting on a charge Q_2 $\quad F = EQ_2$

$$V = \frac{1}{Q_2}\int_r^{\infty} F dr = \frac{Q_1}{4\pi\varepsilon_0 r}$$

Voltage - potential energy per unit charge

$$\Delta V = \int_{r_1}^{r_2} E dr = -\frac{Q_1}{4\pi\varepsilon_0 r_1} + \frac{Q_1}{4\pi\varepsilon_0 r_2}$$

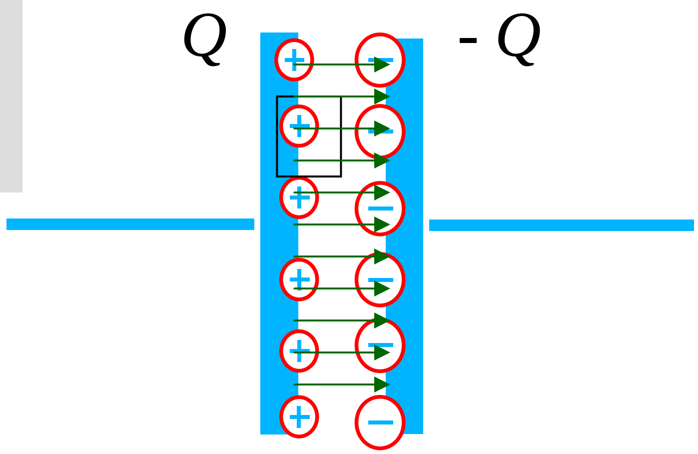
Voltage difference

The Parallel Plate Capacitor

We suggest the use of a parallel plate capacitor rather than charged spheres

The area integral of the electric field over any closed surface is equal to the net charge enclosed in the surface divided by the permittivity of space.

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$



$$E = \frac{Q}{A\varepsilon_0}$$ from Gauss's Law

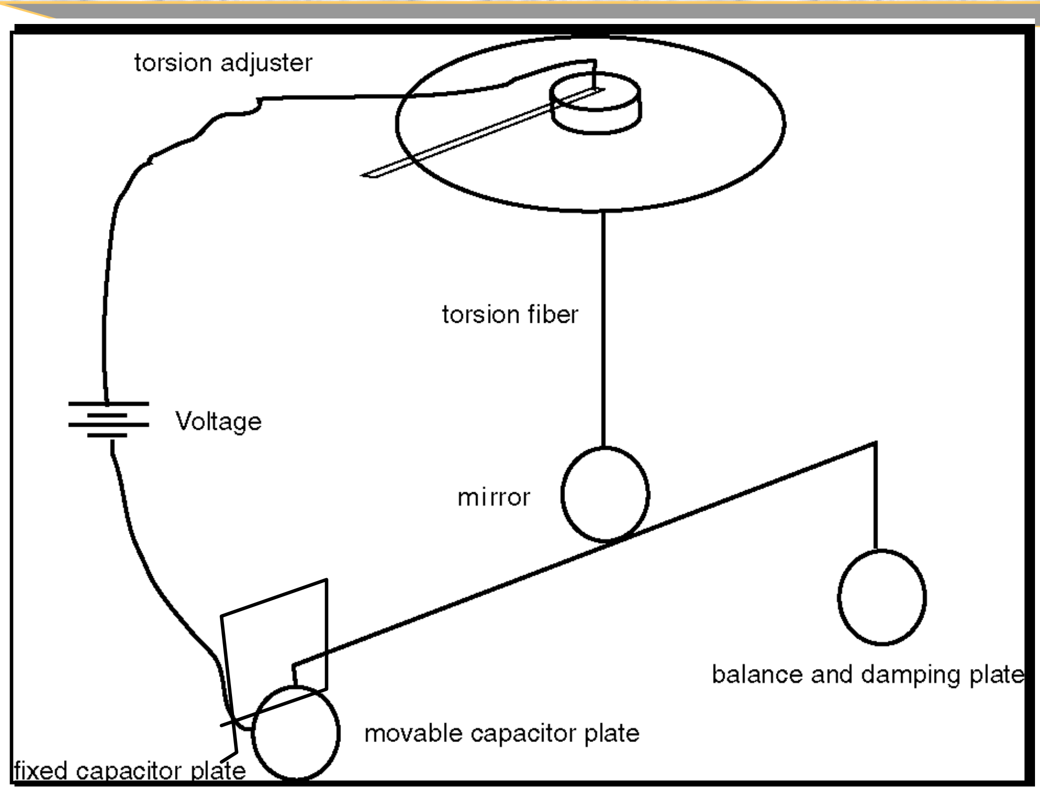
$$V = Ed = \frac{Qd}{A\varepsilon_0}$$ voltage difference

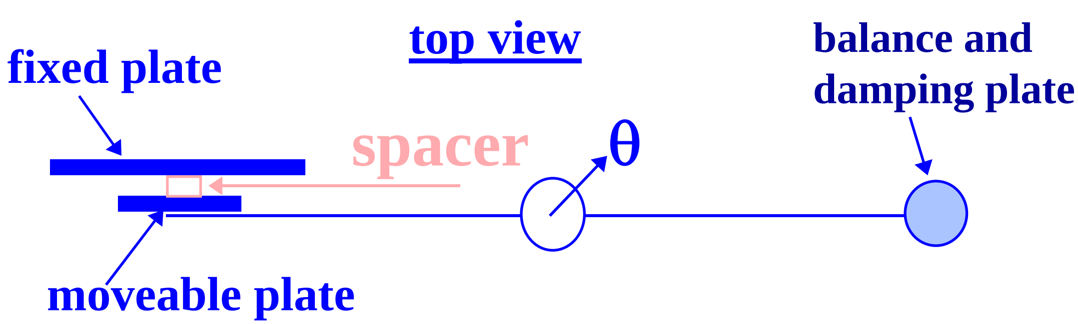
$$F = \frac{1}{2}EQ = \frac{1}{2}\frac{Q^2}{A\varepsilon_0} = \frac{1}{2}\frac{A\varepsilon_0}{d^2}V^2$$ the force

$$F = \frac{1}{2}\frac{\left(A = 3\times10^{-4}\ \mathrm{m}^2\right)\left(\varepsilon_0 = 8.8\times10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}}\right)}{\left(d = 10^{-3}\ \mathrm{m}\right)^2}\left(V = 1000\ \mathrm{V}\right)^2 = 1.3\times10^{-3}\ \mathrm{N}$$

The weight of 0.1 g.

Calibrate a Voltmeter





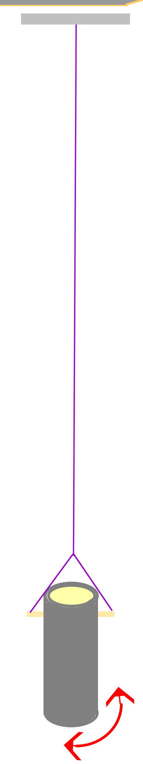
- Set up the apparatus.
- Keep table dry.
- Make the plates parallel for spacer in contact. ·
- Measure the spacer.
- Measure κ .
- Find twisting angle that just causes plates to move apart at applied voltage. ·
- Try calibration at about 1000 Volts. ·
- Now get several measurements at lower voltage.

Measuring *k* by the Torsion Pendulum

$N = -\kappa\theta = I\ddot{\theta}$ torque equation gives diff. eq. in θ

$T = 2\pi\sqrt{\dfrac{I}{\kappa}}$ the period

- measure the restoring torque constant κ of the wire using a solid cylinder for which I can be computed



minimize the wobble of the pendulum since this couples it to other modes and changes the period

Experimental Technique

1. Adjust the fiber so that plates just touch each other (with spacer) at zero voltage
2. Apply voltage between the plates
3. Increase torque from the fiber by twisting the top end of the fiber
and determine the twisting angle that just causes plates to move apart

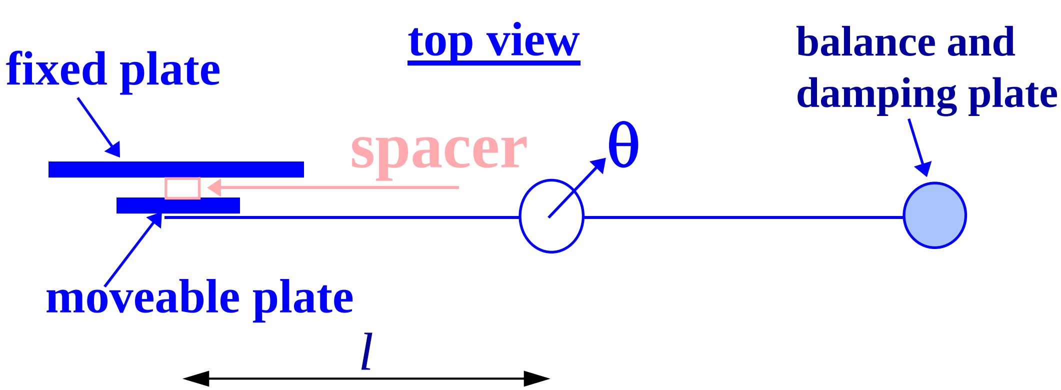
torque resulting from the electrostatic force — torque resulting from the fiber

At twisting angle that just causes plates to move apart $$F\,l = k\,\theta$$

electrostatic attraction $$F = \frac{1}{2}\frac{A\varepsilon_0}{d^2}V^2$$

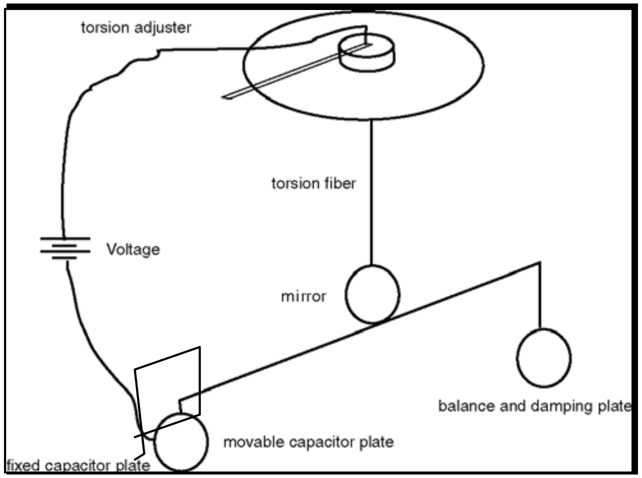
$$\frac{1}{2}\frac{A\varepsilon_0}{d^2}V^2 l = k\theta$$

$$V = d\sqrt{\frac{2k\theta}{lA\varepsilon_0}}$$



Experimental Technique

- Because of the small forces involved, the apparatus is very sensitive to
  - flow in the water
  - air currents
  - vibrations



- We can get these to a minimum but we can't eliminate them

Water must be stable.
Move slowly.
Protect your apparatus from air currents.

Error estimation is optional

Example Problem

Two students measure the radius of a planet and get final answers R_A=25,000±3,000 km and R_B=19,000±2,500 km.
(a) Assuming all errors are independent and random, what is the discrepancy and what is its uncertainty?
(b) Assuming all quantities are normally distributed as expected, what would be the probability that the two measurements would disagree by more than this?
Do you consider the discrepancy in the measurements significant (at the 5% level)?

(a) $$R_A - R_B = 25,000 - 19,000 = \underline{6,000 km}$$

$$\sigma_{R_A - R_B} = \sqrt{{\sigma_{R_A}}^2 + {\sigma_{R_B}}^2} = \sqrt{3,000^2 + 2,500^2} = 3,905 km \rightarrow \underline{4,000 km}$$

$$R_A - R_B = 6,000 \pm 4,000 km$$

(b) $$t = \frac{6,000}{4,000} = 1.5$$

Table A: Probability to be within 1.5σ is 86.64 % ≈ 87 %. Therefore, the probability that the two measurements would disagree by more than this is 100 – 87 = 13 %.
The discrepancy in the measurements is not significant (at the 5% level).

Example Problem

Two students measure the radius of a planet and get final answers R_A=25,000±3,000 km and R_B=19,000±2,500 km.
The best estimate of the true radius of a planet is the weighted average. Find the best estimate of the true radius of a planet and the error in that estimate.

$$x_{wav} = \frac{w_A x_A + w_B x_B}{w_A + w_B} \qquad w_A = \frac{1}{\sigma_A^2} \qquad w_B = \frac{1}{\sigma_B^2} \qquad \sigma_{wav} = \frac{1}{\sqrt{w_A + w_B}}$$

$$R_{wav} = \frac{\frac{R_A}{\sigma_A{}^2} + \frac{R_B}{\sigma_B{}^2}}{\frac{1}{\sigma_A{}^2} + \frac{1}{\sigma_B{}^2}} = \frac{\frac{25,000}{3,000^2} + \frac{19,000}{2,500^2}}{\frac{1}{3,000^2} + \frac{1}{2,500^2}} = 21,459km \rightarrow \underline{21,500km}$$

$$\sigma_{wav} = \frac{1}{\sqrt{\frac{1}{\sigma_A{}^2} + \frac{1}{\sigma_B{}^2}}} = \frac{1}{\sqrt{\frac{1}{3,000^2} + \frac{1}{2,500^2}}} = 1,921km \rightarrow \underline{1,900km}$$

$$\underline{R_{wav} = 21,500 \pm 1,900km}$$