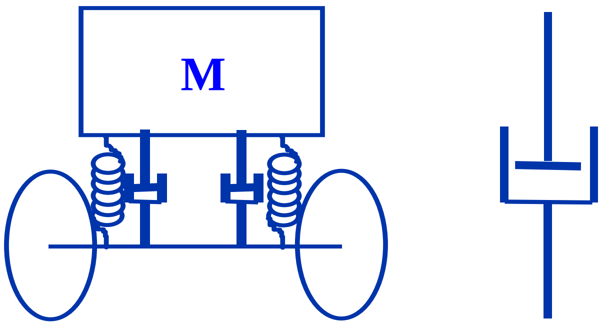
Experiment 3

Construct and test a critical damping system for a spring



- A shock absorber damps oscillations of springs
- If overdamped, recovery is very slow
- If underdamped, the spring will go through many oscillations before returning to equilibrium
- If the damping is just right, we call it critically damped. It reduces shocks and returns to equilibrium as quickly as possible

The Damped Oscillator

$$F = -mg - k\left(y - y_0\right) - bv$$

gravity spring damping

$$ma = F = -k\left(y - \left[y_0 - \frac{mg}{k}\right]\right) - bv$$

gravity changes equilibrium position y_0 but nothing else

$$\ddot{y} = -\frac{k}{m}y - \frac{b}{m}\dot{y}$$

write the differential equation in y and solve it using an exponential function

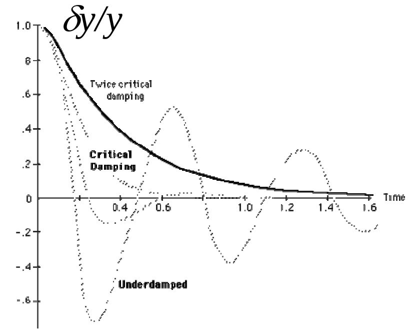
$$y = y_0 \exp\left(-\frac{b}{2m}t\right)\exp\left(\pm i\omega_0 t\right)$$

$$\omega_0 = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

damped oscillator frequency

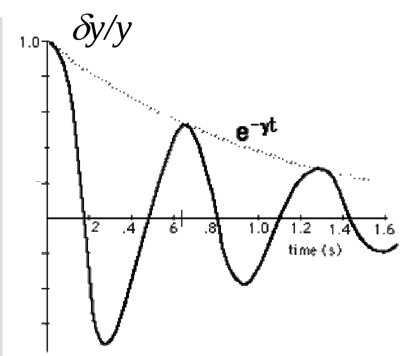
overdamped $\quad \dfrac{b^2}{4m^2} > \dfrac{k}{m}$

$$y = y_0 e^{-\left(\frac{b}{2m} - \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right)t}$$



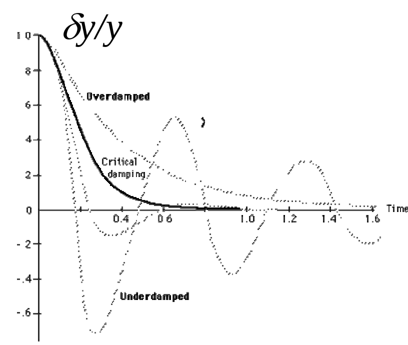
underdamped $\quad \dfrac{b^2}{4m^2} < \dfrac{k}{m}$

$$y = y_0 e^{-\frac{b}{2m}t}\cos\left(\omega_0 t\right)$$



critically damped $\quad \dfrac{b^2}{4m^2} = \dfrac{k}{m}$

$$y = y_0 e^{-\frac{b}{2m}t}$$



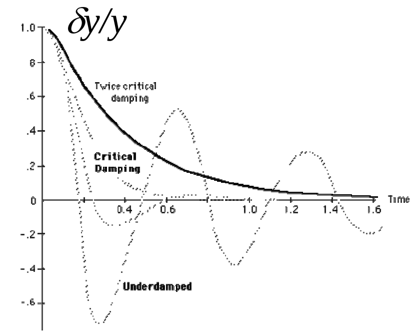
The Damped Oscillator

when the mass is displaced from equilibrium, it will return to its equilibrium position exponentially as a function of time without oscillating

large *b* would result in a stiff suspension for a car and would be equivalent to having no springs

overdamped $\frac{b^2}{4m^2} > \frac{k}{m}$

$$y = y_0 e^{-\left(\frac{b}{2m} - \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right)t}$$

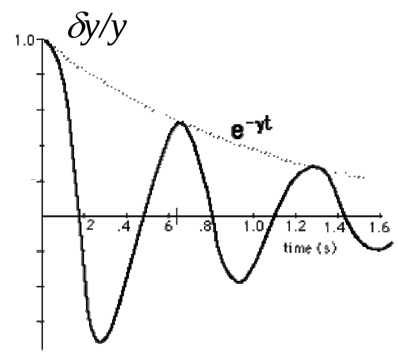


displacement from equilibrium would result in oscillations that would continue for many cycles

car would oscillate after a bump

underdamped $\frac{b^2}{4m^2} < \frac{k}{m}$

$$y = y_0 e^{-\frac{b}{2m}t} \cos(\omega_0 t)$$



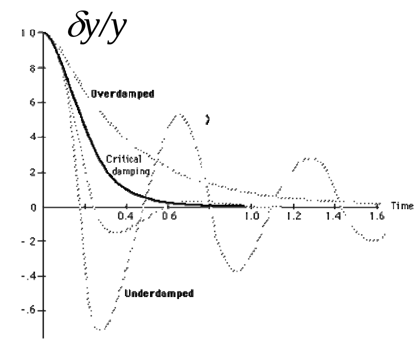
displacement returns to zero exponentially in the shortest time

the car suspension is as soft as possible without a disturbing oscillation after a bump

$$b = 2\sqrt{mk}$$

critically damped $\frac{b^2}{4m^2} = \frac{k}{m}$

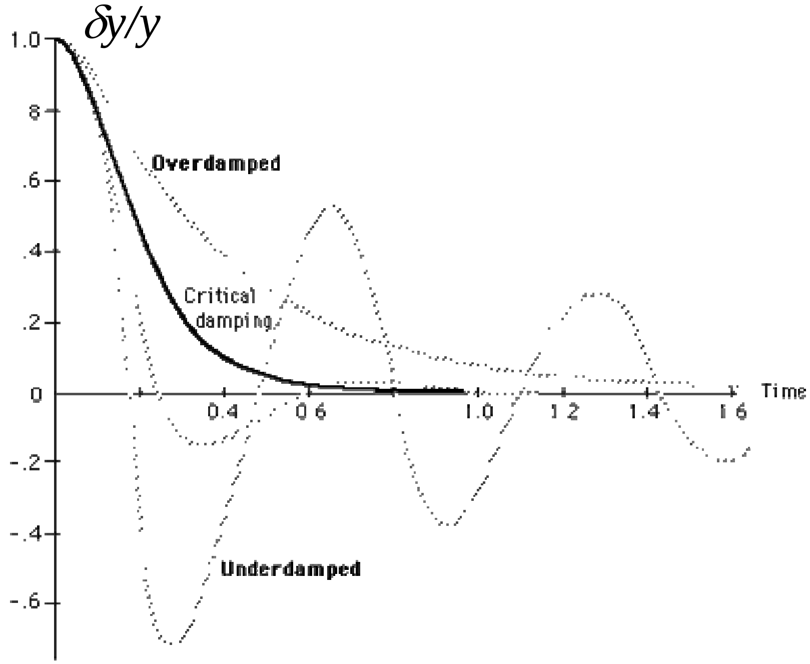
$$y = y_0 e^{-\frac{b}{2m}t}$$

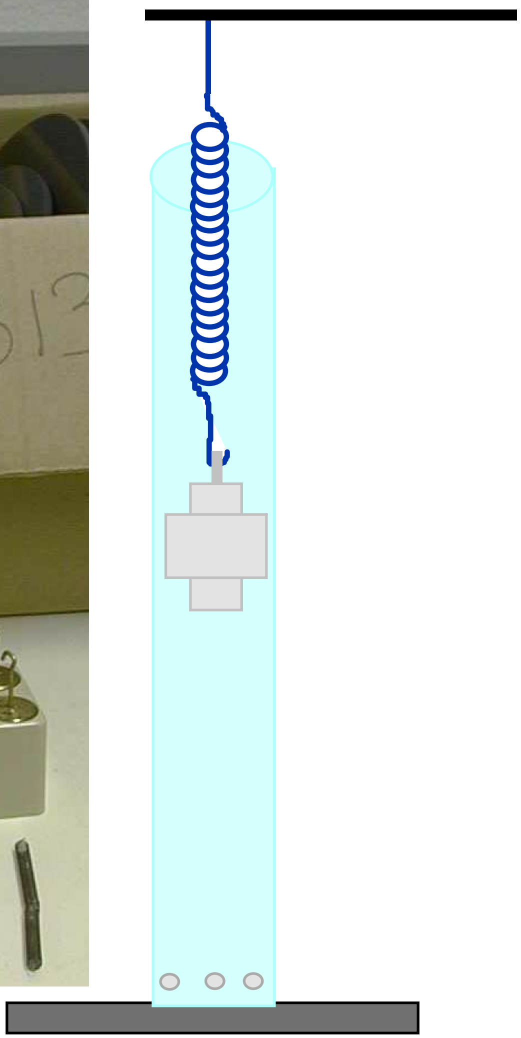


The Equipment

$$b = 2\sqrt{mk}$$

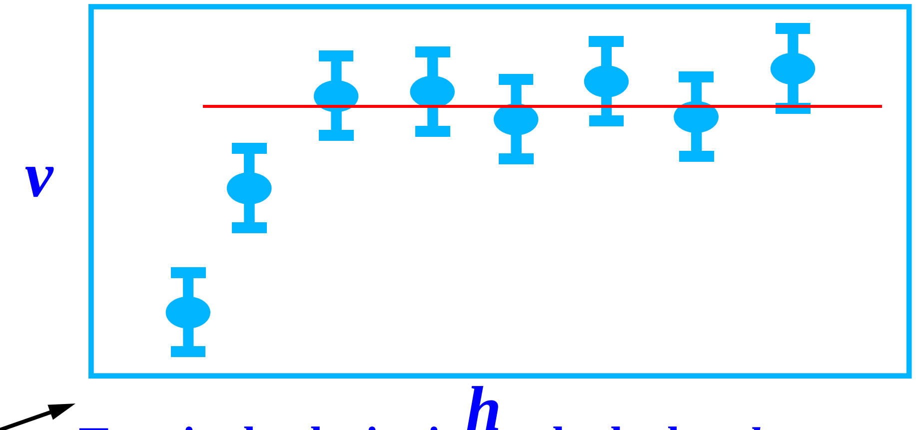
Valve plus holes adjustment of air flow out of cylinder → adjustment of *b*



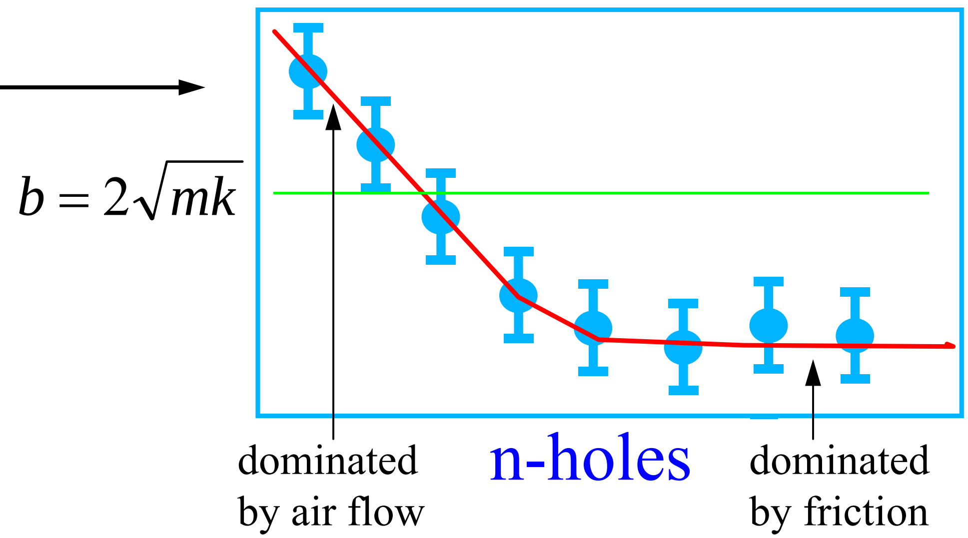


Construct and Test a Critical Damping System

- Measure the spring constant *k*
- Measure mass *m*
- Compute the damping coefficient *b* needed for critical damping $b = 2\sqrt{mk}$
- Use terminal velocity measurements to determine *b* as a function of the number of holes covered and valve position
- Test spring plus shock absorber and optimize



Terminal velocity is reached when *bv=mg*.



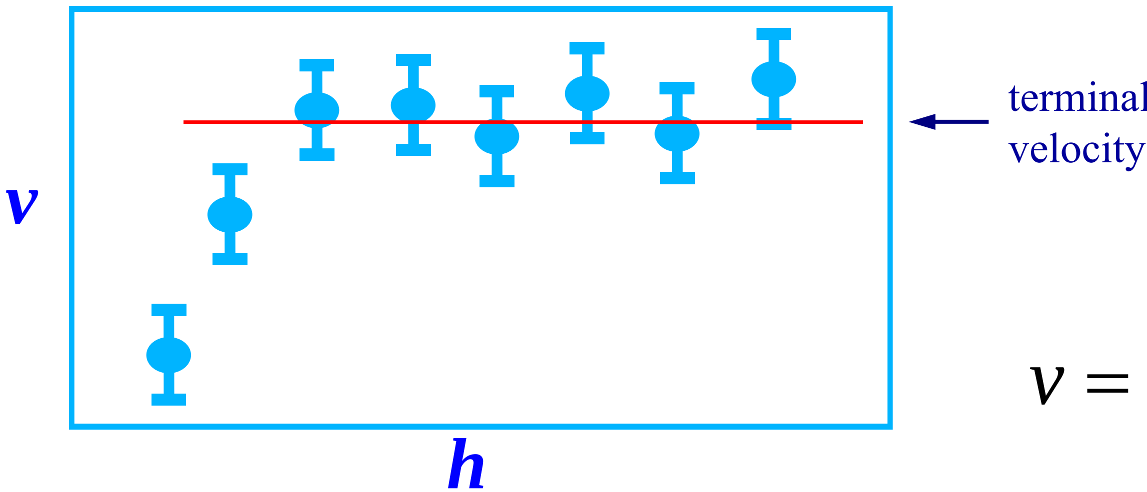
the valve is good for fine adjustment

Terminal Velocity

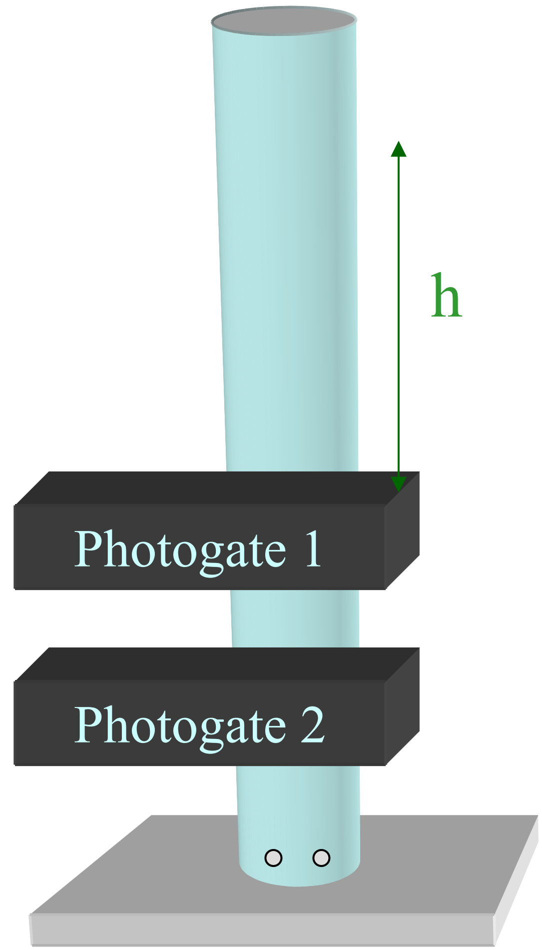
- Terminal velocity is reached when drag force equals force of gravity
- We use this to measure *b*

$$F = -mg - bv = 0$$

$$b = -\frac{mg}{v}$$



$$v = \frac{\Delta h}{\Delta t}$$



Checking the Oscillator

- Good taping and a clean cylinder is crucial to good operation for terminal velocity and oscillation
- Critical damping means no real oscillation but so does overdamping
- Critical damping will have the smallest *b* with no oscillation