Homework #7

Ch #9

(Q7) The rich man could have used momentum conservation to his advantage to move across the ice. Had he thrown his bag of coins, it would have been like a totally inelastic collision run in reverse. Since the system (him + coins) has zero momentum before he throws the coins, the momentum of the center of mass remains zero (because there are no other, external forces acting on them). Therefore, he will get the same momentum as the coins but in the opposite direction. If he pushes hard enough, he could get enough momentum so that he would reach the ~~other side~~ shore before he froze to death.

(Q14) I would expect more damage to the occupants when the cars rebound backward. Remember that momentum is a vector - a change in direction is a larger change in momentum then simply coming to rest. Since momentum change is proportional to force ($\Delta p = F_{net} \Delta t$) assuming the collisions take the same amount of time, the larger the change in momentum, the larger the force. Therefore, the ~~for~~ rebound collision would require more force, and hurt the occupants more.

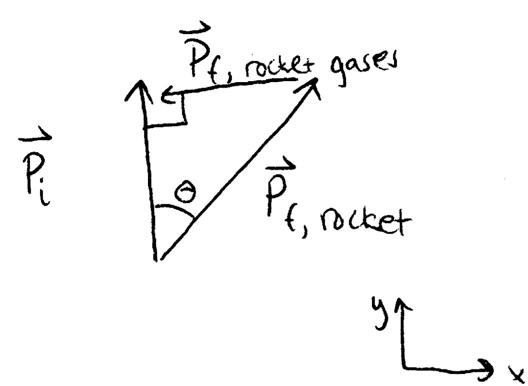
(Q22) The only way a car can accelerate is if there is friction between the road and the tires. (Think about trying to take a turn on an icy road - if there is no friction, you can't change direction/accelerate.) It is not the internal force of the engine that causes acceleration, it is the external force of the road on the tires via friction that causes acceleration.

(Q25) Neither Jim nor Bob wins or loses. Since the ice is frictionless no external forces can act on the Jim-Bob system, so the center of mass of the system remains at rest.

(P6)

$$\Delta \vec{p} = 0$$

$$\vec{P}_i = \vec{P}_{f,rocket} + \vec{P}_{f,\,rocket\ gases}$$



look at $\hat{x}$-direction

$$\vec{P}_{f,\,rocket\ gases} = -\vec{P}_{f,\,rocket} \sin\theta$$

$$(mv)_{rocket\ gases} = +(mv)_{rocket} \sin\theta$$

$$m_{rocket\ gases} = \frac{(mv)_{rocket} \sin\theta}{v_{rocket\ gases}}$$

$$m_{rocket} + m_{rocket\ gases} = 4200\,kg = M_T$$

look at $\hat{y}$ - direction

$$\vec{P}_{f,\,rocket} = \vec{P}_i$$

$$(m_{rocket} + m_{rocket\ gases})V_i = (mv)_{rocket} \cos\theta$$

$$M_T V_i = (M_T - m_{rocket\ gases})\, V_{rocket} \cos\theta$$

$$V_{rocket} = \frac{M_T V_i}{(M_T - m_{rocket\ gases})\cos\theta}$$

Plug back into equation from $\hat{x}$ direction

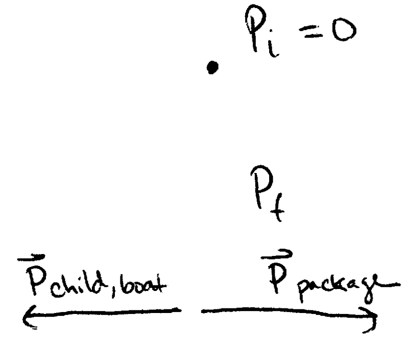
$$m_{rocket\ gases} = \frac{\cancel{(M_T - m_{rocket\ gases})}\, M_T V_i \sin\theta}{\cancel{(M_T - m_{rocket\ gases})} \cos\theta \; V_{rocket\ gases}}$$

$$= \frac{M_T V_i}{V_{rocket\ gases}} \tan\theta$$

$$= \frac{(4200\,kg)(120\,m/s)}{(2200\,m/s)} \tan 23^\circ$$

$$m_{rocket\ gases} = 97\,kg$$

(P13) $\Delta\vec{p} = 0$



$$\vec{P}_{child, boat} + \vec{P}_{package} = 0$$

$$(mv)_{child, boat} = (mv)_{package}$$

$$V_{child, boat} = \frac{m_{package}}{m_{child, boat}} V_{package}$$

$$= \frac{5.40\,kg}{(26.0\,kg + 55.0\,kg)} (10.0\,m/s)$$

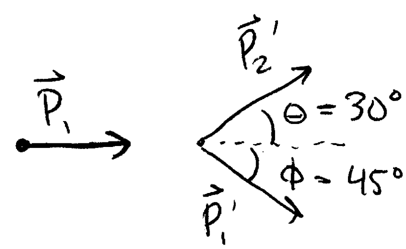
$$= 0.667\ m/s$$

(P16) $\Delta\vec{p} = 0$ in both directions

$\hat{x}$: $\vec{P}_{2',x} + \vec{P}_{1'x} = P_1 \longrightarrow m_2 V_2' \cos\theta + m_1 V_1' \cos\phi = m_1 V_1$

$\hat{y}$: $\vec{P}_{2'y} + \vec{P}_{1'y} = 0 \longrightarrow m_2 V_2' \sin\theta - m_1 V_1' \sin\phi = 0$

$$V_1' = \frac{m_2 V_2' \sin\theta}{m_1 \sin\phi}$$



Substituting

$$m_2 V_2' \cos\theta + m_1 \left(\frac{m_2 V_2'}{m_1} \frac{\sin\theta}{\sin\phi} \right) \cos\phi = m_1 V_1$$

$$V_2' \left[m_2 \left(\cos\Theta + \frac{\sin\Theta}{\sin\phi} \cos\phi \right) \right] = m_1 V_1$$

$$V_2' = \frac{m_1}{\cancel{m_2}} \frac{V_1}{\left[\cos\Theta + \frac{\sin\Theta}{\tan\phi} \right]} \quad ; \; m_1 = m_2$$

$$V_2' = \frac{17\,m/s}{\cos 30^\circ + \frac{\sin 30^\circ}{\tan 45^\circ}} = \boxed{12.4\,m/s = V_2'}$$

$$V_1' = \frac{(12.4\,m/s)\sin 30^\circ}{\sin 45^\circ} = \boxed{8.7\,m/s = V_1'}$$

(P28)

a) Impulse $= \int F\,dt = F_{ave}\,\Delta t = (100\,N)(0.05\,s)$

$= 5\ N\cdot s$

or $\rightarrow$ (# of boxes)(Area per box) $= 10\,(0.01s \times 50N)$

$= 5\,N\cdot s$

b) Impulse $= \Delta p$

$$V_f = \frac{\text{Impulse}}{m} = \frac{5\,Ns}{0.060\,kg} = 80\,m/s$$

(P4b) Plan → a) use Work - Energy to figure out intitial speed of cars just after collision

→ b) use momentum conservation to figure out speed of Toyota before collision

a) $W = \Delta KE$

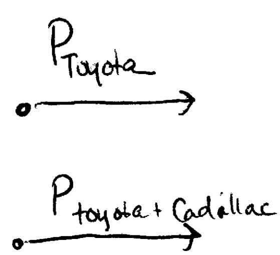
$$\vec{F} \cdot \vec{d} = \Delta KE$$

$$-F_k d = \frac{1}{2} m (\cancel{v_f^2}^{\,0} - v_i^2)$$

$$\mu_k \cancel{m_T} g d = \frac{1}{2} \cancel{m_T} v_i^2$$

$$v_i = \sqrt{2 \mu_k g d}$$

b) $\vec{P}_T = +\vec{P}_f$



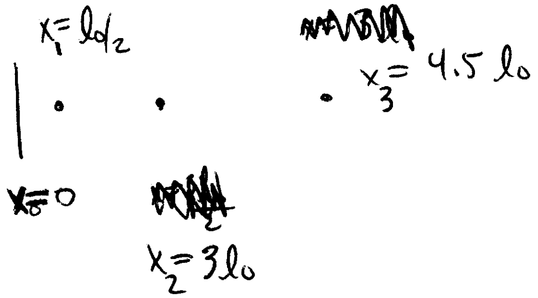
$$m_T v_T = (m_T + m_C) v_i$$

$$v_T = \left(\frac{m_T + m_C}{m_T}\right) \sqrt{2 \mu_k g d}$$

$$= \left(\frac{950\ kg + 2200\ kg}{950\ kg}\right) \sqrt{2(0.40)(9.8\ m/s^2)(4.8\ m)}$$

$$v_T = 20.\ m/s$$

(P62) Consider 3 point masses, each at center of each box.



With uniform density, mass scales with volume.

$$V_1 = l_0^3$$

$$V_2 = (2 l_0)^3 = 8 V_1$$

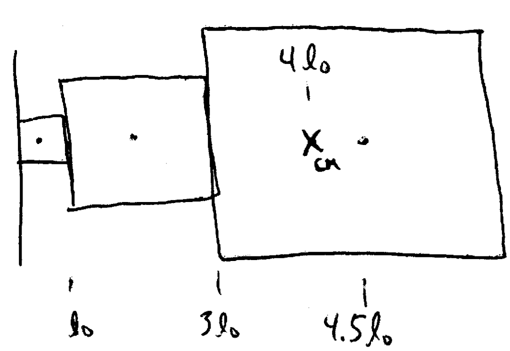
$$V_3 = (3 l_0)^3 = 27 V_1$$

$$X_{cm} = \frac{\sum_{i=1}^{3} m_i x_i}{\sum_{i=1}^{3} m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{V_1 x_1 + 8 V_1 x_2 + 27 V_1 x_3}{V_1 + 8 V_1 + 27 V_1} = \frac{\cancel{V_1}(x_1 + 8x_2 + 27x_3)}{36 \cancel{V_1}}$$

$$= \frac{0.5\, l_0 + 8(3 l_0) + 27(4.5\, l_0)}{36}$$

$$X_{cm} = 4\, l_0$$



(P64) Think about the disk having a density equal to the negative of the density of the disk

$$\rho_{plate} = -\rho_{disk}$$

Let $x = 0$ be at C (r_{disk})

$$M_{plate} = \rho_{plate} V = \rho_{plate} \pi R^2 = -\rho_{disk} \pi R^2$$

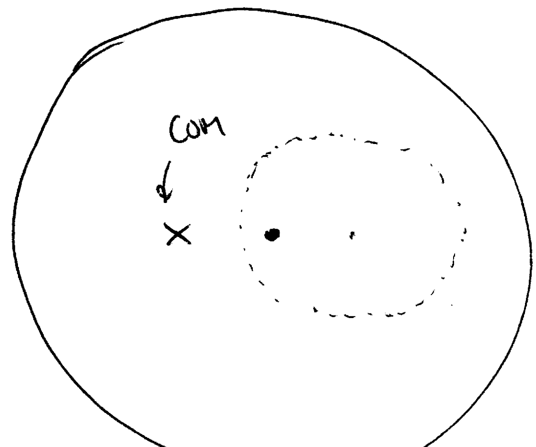
$$M_{disk} = \rho_{disk} V = \rho_{disk} \pi (2R)^2 = \rho_{disk} \pi 4 R^2$$

$$X_{cm} = \frac{\sum_{i=1}^{2} m_i r_i}{\sum_{i=1}^{2} m_i} = \frac{(M_{plate})(r_{plate}) + (M_{disk})(\cancel{r_{disk}})^{\,0}}{M_{plate} + M_{disk}}$$

$$= \frac{-\cancel{\rho_{disk}} \cancel{\pi R^2} (0.80R)}{\cancel{\pi R^2} \cancel{\rho_{disk}} (-1 + 4)}$$

$$= -\frac{(0.80)R}{3}$$

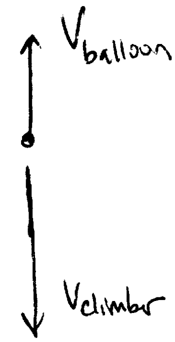
$$= -0.27 R$$



(P73) Momentum must be conserved, so when the passenger climbs downward, the balloon must move upward

$$\vec{P}_{climber} + \vec{P}_{balloon} = 0$$

$$(mv)_{climber} = (mv)_{balloon}$$



$$\vec{V}_{climber} - \vec{V}_{balloon} = V$$

$$V_{climber} + V_{balloon} = V \quad \text{(direction is downward)}$$

$$V_{climber} = V - V_{balloon}$$

$$m_{climber}(v - V_{balloon}) = M_{balloon} V_{balloon}$$

$$mV = V_{balloon}(M + m)$$

$$\vec{V}_{balloon} = \frac{mV}{M+m} \quad \text{upward}$$

(P76) $M \frac{d\vec{v}}{dt} = \sum F_{ext} + \vec{V}_{rel} \frac{dM}{dt}$

$$M \frac{dv}{dt} = V_{rel} \frac{dM}{dt} - Mg$$

$$V_{rel} = \frac{M\left(\frac{dv}{dt} + g\right)}{dM/dt} = \frac{(2500\,kg)(3.0g + g)}{30\,kg/s}$$

$$= \frac{4(2500\,kg)(9.8\,m/s^2)}{30\,kg/s}$$

$$V_{rel} = 3300\,m/s$$

(P78) $M \frac{d\vec{v}}{dt} = \sum F_{net} + \vec{V}_{rel} \frac{dM}{dt}$

$$F_{ext} = \frac{G M_E M_{rocket}}{r^2} = \frac{(6.67\times10^{-11})(5.97\times10^{24}\,kg)(25{,}000\,kg)}{(6.38\times10^{6}\,m + 6400\times10^{3}\,m)^2}$$

$$F_{ext} = 6.11\times10^{4}\,N$$

$$M \frac{dv}{dt} = V_{rel} \frac{dM}{dt} - F_{ext}$$

$$\frac{dM}{dt} = \frac{M\frac{dv}{dt} + F_{ext}}{V_{rel}} = \frac{(25000\,kg)(1.7\,m/s^2) + (6.11\times10^{4}\,N)}{1200\,m/s}$$

$$\boxed{\frac{dM}{dt} = 86\,kg/s}$$

(P96) Plan → (i) Use energy conservation to find speed of m just before collision

→ (ii) Use momentum conservation to determine the speed of both blocks

→ Use energy conservation to find final height of the smaller mass

a) (i) $\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$v = \sqrt{2gh}$

(ii) Collision is elastic, so we have to use both energy and momentum conservation

$\vec{P}_i = \vec{P}_f$

$mv = mv_1' + Mv_2'$ (Momentum cons.)

$\frac{1}{2}mv^2 = \frac{1}{2}m(v_1')^2 + \frac{1}{2}M(v_2')^2$ (energy cons.)

$v_1' = \dfrac{mv - Mv_2'}{m} = v - \dfrac{M}{m}v_2'$

$(v_1')^2 = v^2 + \dfrac{M^2}{m^2}(v_2')^2 - 2\dfrac{M}{m}v_2'v$ (plug into energy conservation)

$\frac{1}{2}\cancel{m}\cancel{v^2} = \frac{1}{2}\cancel{m}\left[\cancel{v^2} + \dfrac{M^2}{m^{\cancel{2}}}(v_2')^2 - 2\dfrac{M}{\cancel{m}}v_2'v\right] + \frac{1}{2}M(v_2')^2$

$0 = \dfrac{M^2}{2m}(v_2')^2 + \dfrac{M}{2}(v_2')^2 - Mv_2'v$

$$0 = v_2' \left[\frac{M}{2}\left(\frac{M}{m} v_2' + v_2' \right) - Mv \right]$$

For this equation to be zero either $v_2' = 0$ or the term in brackets is zero

$$0 = \frac{v_2'}{2}\left(\frac{M}{m} + 1 \right) - v$$

$$\frac{2v}{\left(\frac{M}{m} + 1 \right)} = v_2' \longrightarrow v_2' = \frac{2mv}{M+m} = \frac{2m\sqrt{2gh}}{M+m}$$

$$v_2' = \frac{2(2.20\,\text{kg})\left(\sqrt{2(9.8\,\text{m/s}^2)(3.6\,\text{m})}\right)}{(2.20\,\text{kg} + 7.00\,\text{kg})}$$

$$\boxed{v_2' = 4.0\ \text{m/s}}$$

Remember $v_1' = v - \frac{M}{m} v_2' = \sqrt{2gh} - \frac{M}{m} v_2'$

$$v_1' = \sqrt{2(9.8\,\text{m/s}^2)(3.6\,\text{m})} - \left(\frac{7.00\,\text{kg}}{2.20\,\text{kg}} \right)(4.0\,\text{m/s})$$

$\boxed{v_1' = 4.3\ \text{m/s}}$ up the incline

b) $\frac{1}{2}\cancel{m}(v_1')^2 = \cancel{m}gh'$

$$h' = \frac{(v_1')^2}{2g} = \frac{(4.3\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 0.94\ \text{m}$$