PHYSICS 110A : CLASSICAL MECHANICS
PROBLEM SET #2

[1] Show explicitly that $x(t) = (C + Dt)\exp(-\beta t)$ is a solution to the damped harmonic oscillator equation $\ddot{x} + 2\beta\,\dot{x} + \omega_0^2\,x = 0$ when $\omega_0 = \beta$.

[2] Consider the third order equation

$$\dddot{x} + (2\beta + \gamma)\,\ddot{x} + (\omega_0^2 + 2\beta\gamma)\,\dot{x} + \gamma\omega_0^2\,x = f_0\cos(\Omega t) \tag{1}$$

$$= \left(\frac{d}{dt} + \gamma\right)\left(\frac{d^2}{dt^2} + 2\beta\frac{d}{dt} + \omega_0^2\right) x\ . \tag{2}$$

(a) Find the most general solution.

(b) Show that for long times the solution takes the form

$$x(t) = A(\Omega)\,f_0\cos\big(\Omega t - \delta(\Omega)\big)\ , \tag{3}$$

and find $A(\Omega)$ and $\delta(\Omega)$.

(c) Under what conditions does $A(\Omega)$ have a maximum at a nonzero value of Ω?

[3] Use the Green's function method to determine the response of a damped oscillator to a forcing function of the form

$$f(t) = f_0\,e^{-\gamma t}\,\sin(\Omega t)\,\Theta(t)\ , \tag{4}$$

where $\Theta(t)$ is the step function.

[4] A grandfather clock has a pendulum length of 0.7 m and a bob of mass $m = 0.4$ kg. A mass of 2 kg falls 0.8 m in seven days to keep the amplitude (from equilibrium) of the pendulum oscillations steady at 0.03 rad. What is the Q of the system?

[5] Consider the equation

$$\dot{x} + \gamma\,x = f(t)\ . \tag{5}$$

Show that the general solution to this problem may be written

$$x(t) = x_{\rm h}(t) + \int\limits_{-\infty}^{t} dt'\,G(t-t')\,f(t')\ , \tag{6}$$

and find the general form of the homogeneous solution $x_{\rm h}(t)$ as well as the Green's function $G(t-t')$. *Hint: $G(t-t')$ may be evaluated either by contour integration, or by deriving and using the result*

$$\left(\frac{d}{dt} + \gamma\right) G(t-t') = \delta(t-t')\ . \tag{7}$$