PHYSICS 160: Stellar Structure

Instructor: Dr. A. M. Wolfe (phone: 47435)
Homework no. 2
Due: Thurs. Nov. 8

1

In class we found an expression for pressure given by

$$P = \frac{1}{3} n < vp >$$

where n is particle density, $< vp >$ is the average dot product of velocity and momentum. In the case of isotropic radiation we found that $< vp >=< E >$ where $< E >$ is the average photon energy.

(a) Repeat the calculation for non-relativistic particles of mass m. Express P in terms of average energy density $U = n < E >$.

(b) Assuming a Maxwellian speed distribution given by

$$f(v) = (\frac{m}{2\pi kT})^{3/2} \exp(-mv^2/2kT) 4\pi v^2$$

find expressions for U and P as functions of n and T.
Hint: $\int_0^\infty x^4 \exp(-x^2)dx = \frac{3\sqrt{\pi}}{8}$

2

To explain the phenomenon of limb darkening we showed that at each polar angle θ we observe photons that propagate from a surface defined by a fixed optical depth. This surface is defined to be one mean-free-path l_λ below the outer solar radius $R_\odot$ along the *line of sight*. Thus, at the middle position of the star where θ=$\pi/2$, the radial distance from the center of the star to this surface is given by $d(\pi/2)$=$R_\odot - l_\lambda$.

(a) Compute an expression for d as a function of θ (Hint: assume $l_\lambda << R_\odot$).

(b) Suppose the temperature of the photosphere at radius $R > d(\pi/2)$ declines with radius as

$$T(R) = T_e \exp\left[- \frac{\left(R - d(\pi/2)\right)}{H} \right],$$

where the scale-height H=l_λ and T_e is the effective temperature. Use the expression computed in (a) to compare T at the middle position ($\theta = \pi/2$) to T at the limb (θ=0).

(c) Use the result in (b) to compare the black body intensities at the two positions.

3

Estimate an order-of-magnitude expression for the luminosity of a star as follows:

(a) In class we worked out the time it takes for randomly walking photons to escape (or diffuse) from a star. We also worked out an expression for the energy density of radiation.

(b) Argue that the luminosity of star is given by

$$\frac{L_\odot = (\text{volume}) * (\text{radiation energy per unit volume})}{(\text{escape time})}$$

$$\frac{L = (4\pi R_\odot)(caT^4)}{(3\sigma_\nu n)}$$

where σ_ν is the photon scattering cross-section at frequency ν and n is density.

(c) Compare your result with the expression derived in class for $L_\odot$