PHYSICS 1B – Fall 2007

Electricity
&
Magnetism

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UCSD

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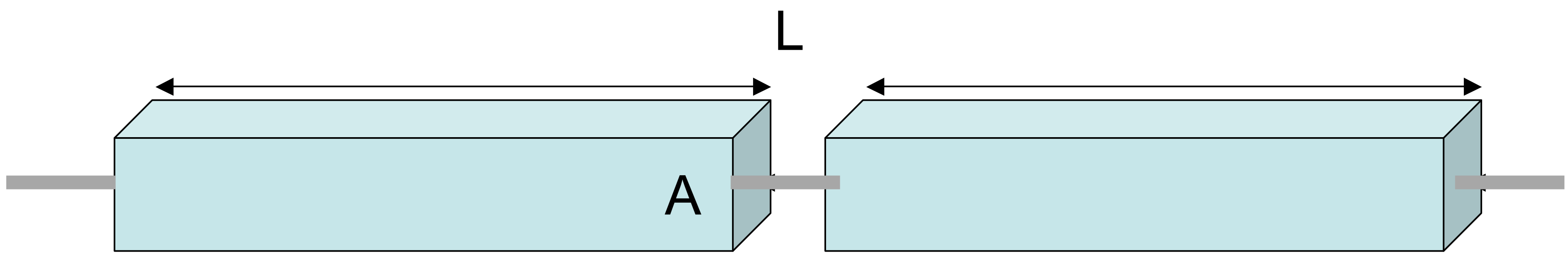
Why is the series law easy to understand?

- Recall that the resistance of a resistor is

$$R = \rho \frac{L}{A}$$

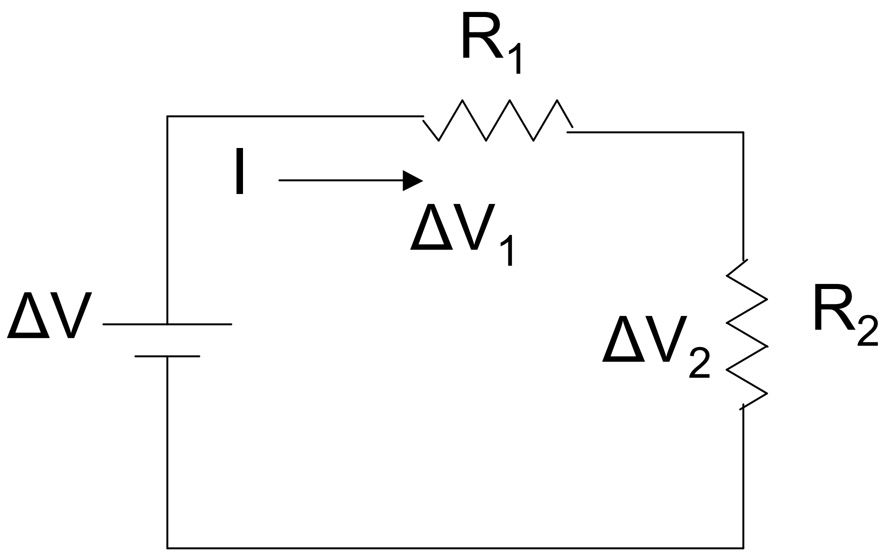


R~L

$R_{tot} \sim L_1 + L_2$

Resistors in Series   I same, ΔV different

What is the equivalent resistance R_{eq} ?



$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = IR_{eq} = IR_1 + IR_2$$

$$R_{eq} = R_1 + R_2$$

For N resistors in series

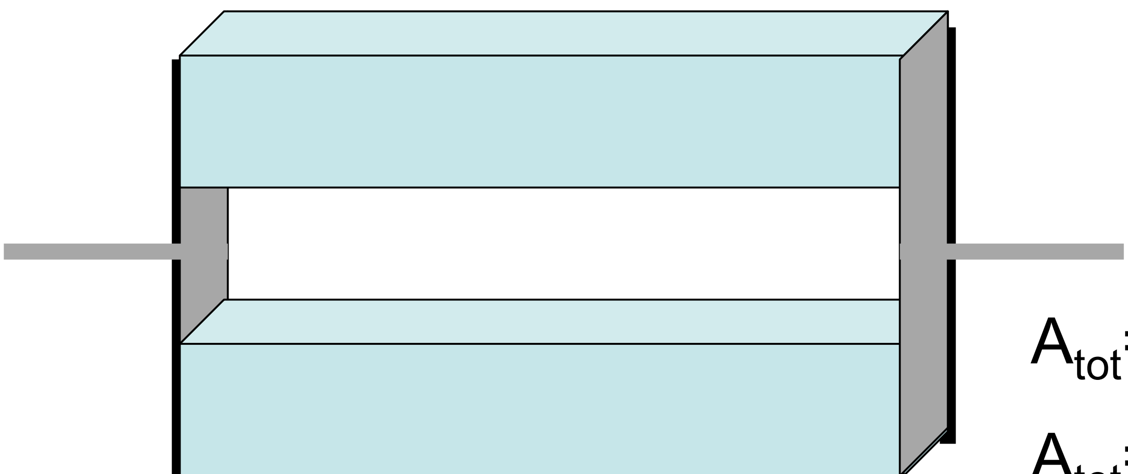
$$R_{eq} = R_1 + R_2 + \ldots\ldots. R_N$$

R_{eq} is larger than any R

Why is the parallel law easy to understand?

- Recall that the resistance of a resistor is

$$R = \rho \frac{L}{A}$$



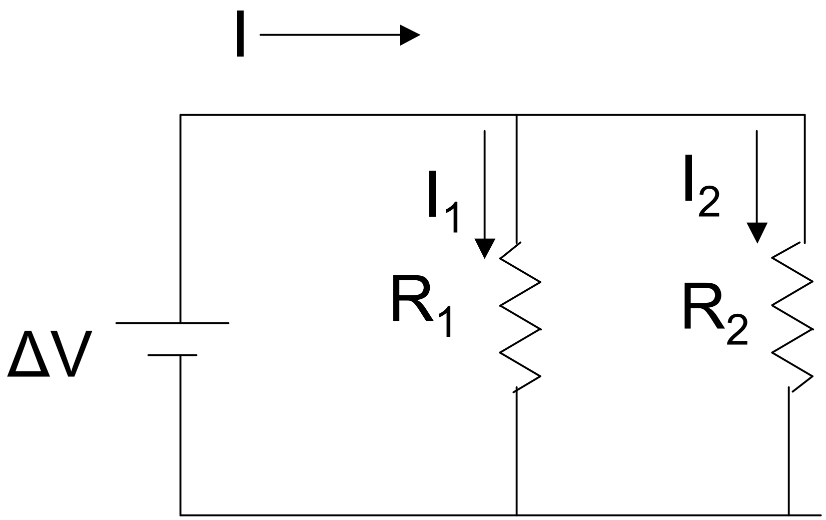
R ~ 1/Area

$A_{tot}=A_1 + A_2$

$A_{tot}=1/R_1 + 1/R_2$

R_{tot}~$1/A_{tot}$ ~ $1/(1/R_1 + 1/R_2)$

Resistors in parallel, ΔV same, I different



$I = I_1 + I_2$

$$\frac{\Delta V}{R_{eq}} = I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

For N resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots\dots\dots + \frac{1}{R_N}$$

R_{eq} is smaller than any R

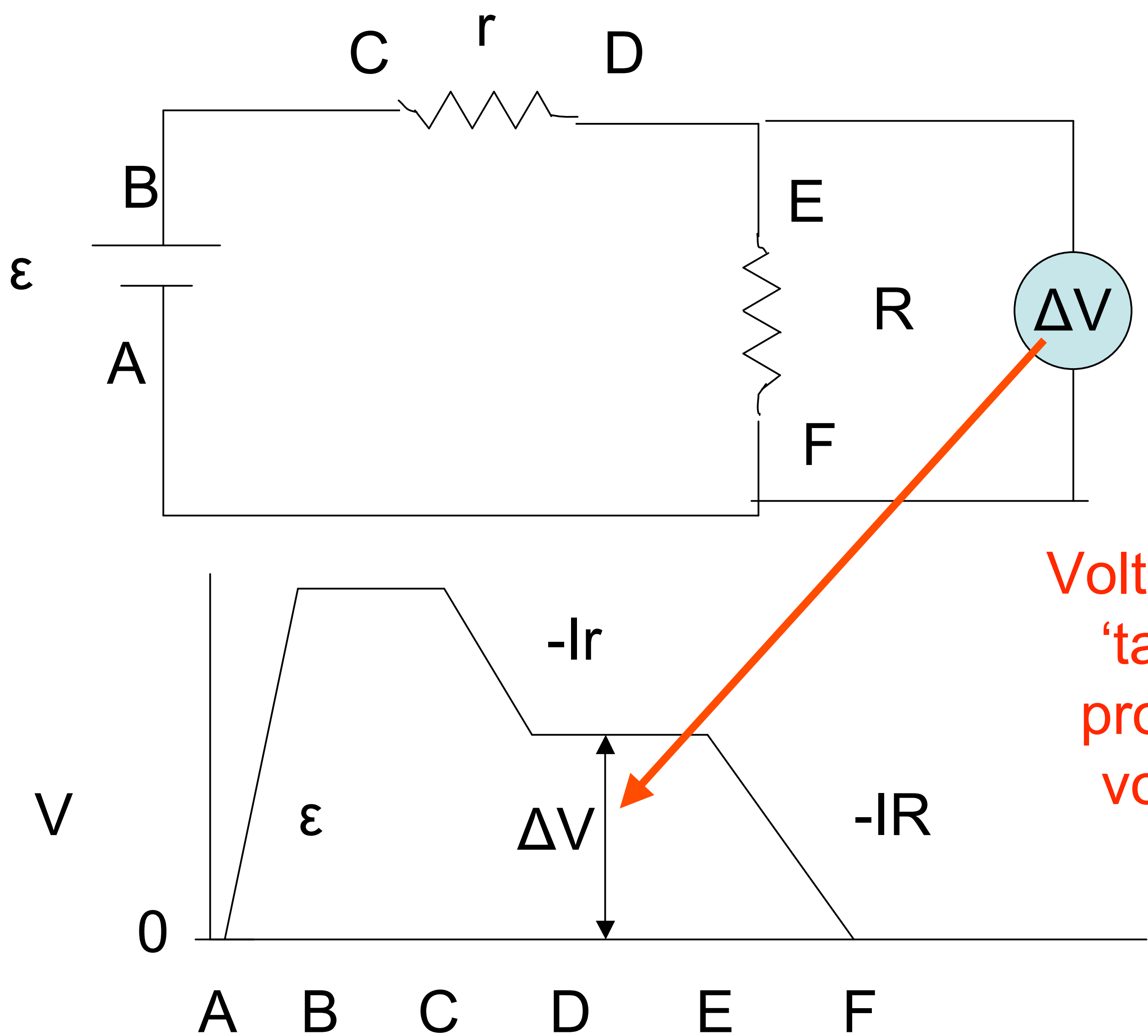
Comparisons: Resistors & Capacitors

- Resistors in series are like capacitors in parallel.

- Resistors in parallel are like capacitors in series.
- This is because R ~ L and C~1/L
- And because R~1/A and C~A

Why do we care?

Consider Simple Circuit: Two resistors in Series



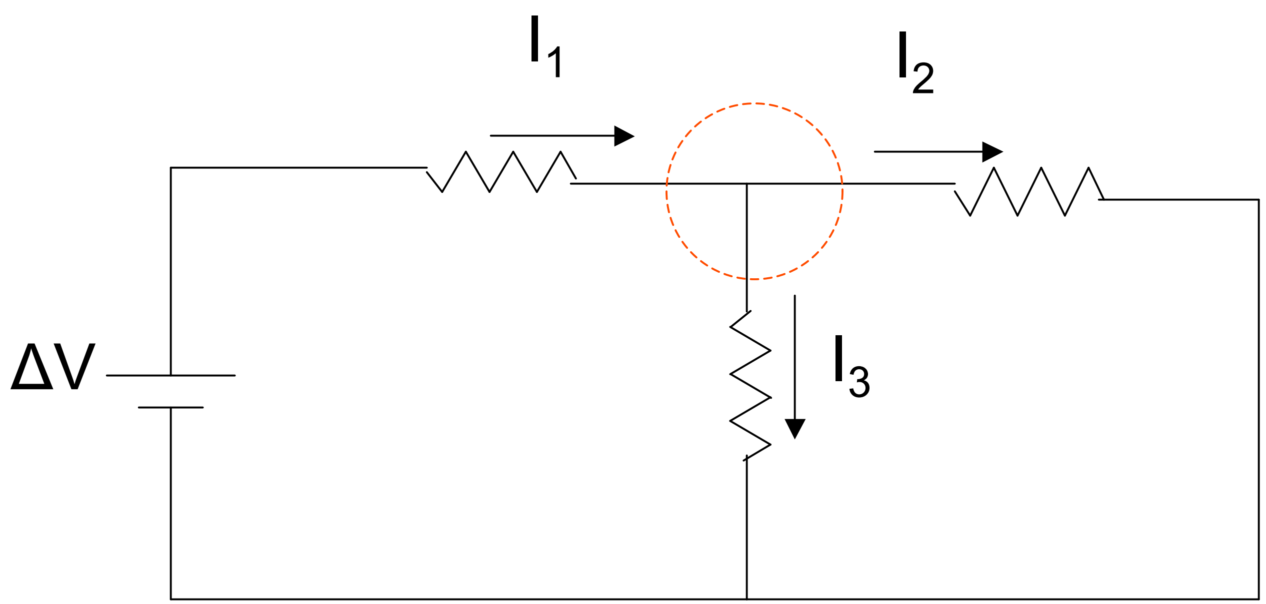
Voltage can be 'tailored' to produce any voltage we desire!

Ch 18 Kirchoff’s 2 Rules

1. Junction rule
2. Loop rule

Rule #1. “*Junction rule*”

The current flowing into a junction is equal to the current flowing out.

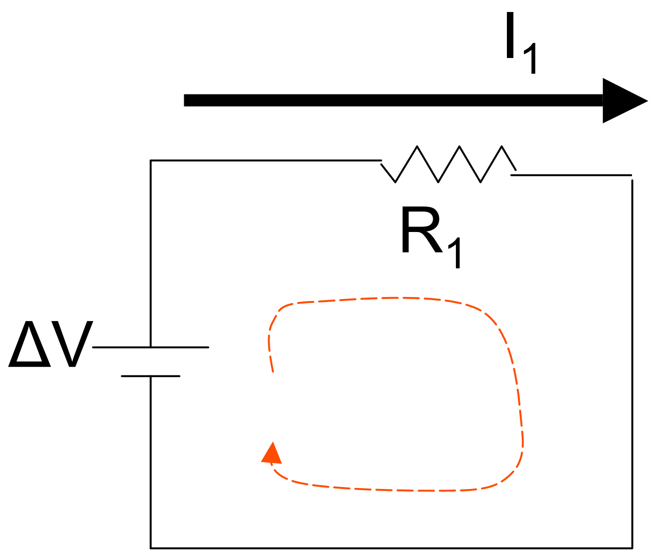


$$I_1 = I_2 + I_3$$

This comes from ‘conservation of charge’

#2. Loop rule

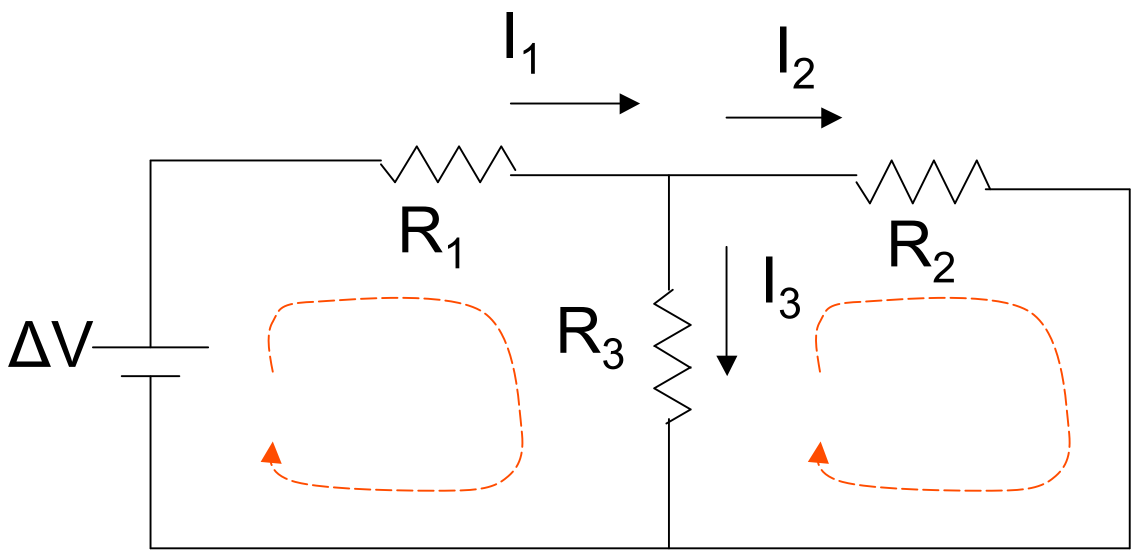
"The sum of voltage differences in going around a closed current loop is equal to zero"



$$\sum_{loop} \Delta V_i = 0$$

#2. Loop rule

The sum of voltage differences in going around a closed current loop is equal to zero

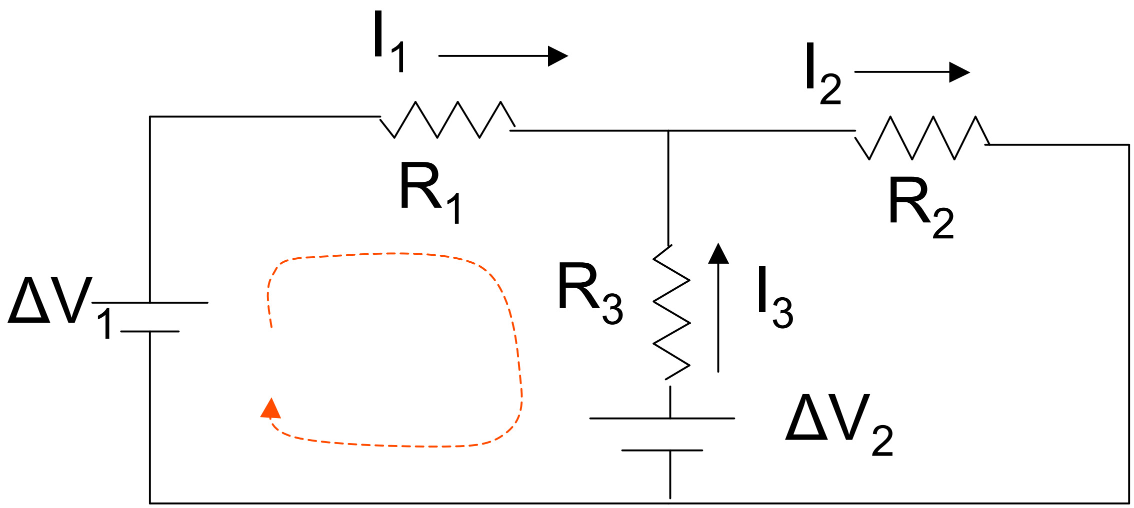


$$\sum_{loop} \Delta V_i = 0$$

Voltage changes in traversing the loop

Choose a current direction

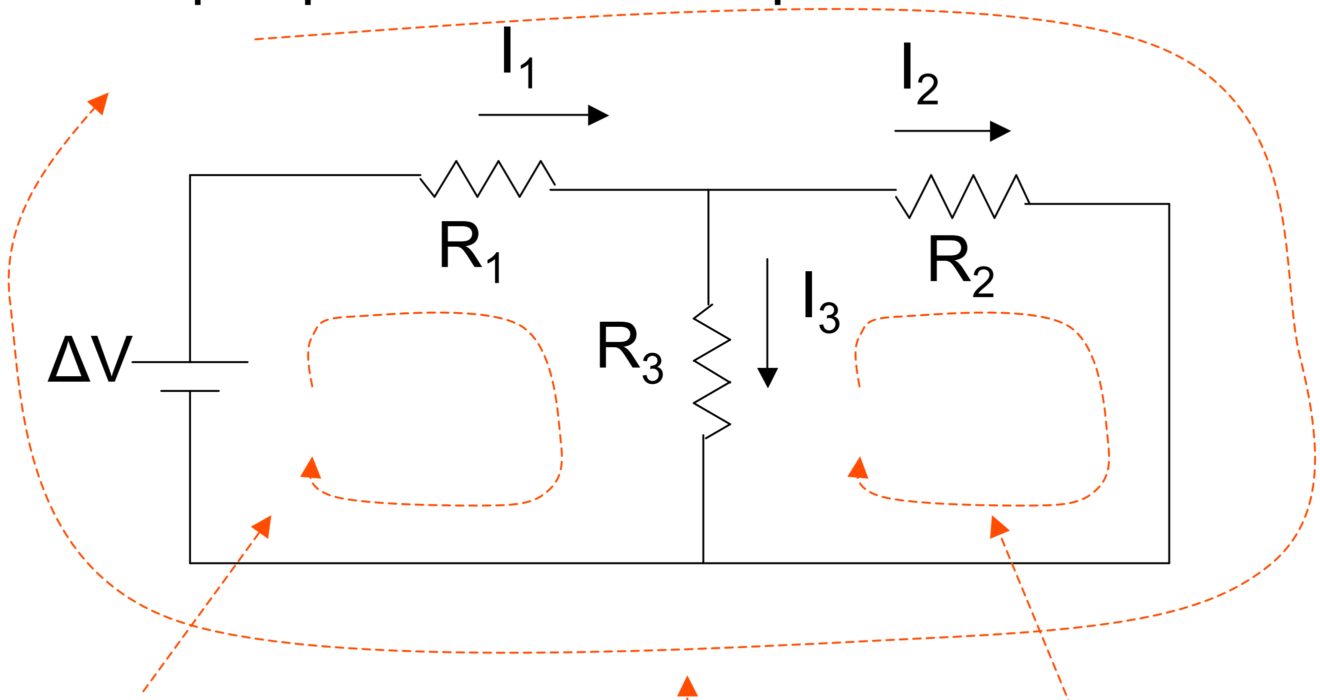
-IR, current in traversal direction
+IR current in opposite direction
+ΔV voltage increases along traversal direction
-ΔV voltage decreases along traversal direction



$$\Delta V_1 - I_1R_1 + I_3R_3 - \Delta V_2 = 0$$

If I is negative when you solve the equations, the current flows in the opposite direction than you chose.

Not all loop equations are independent



$$\Delta V - I_1 R_1 - I_3 R_3 = 0$$

$$I_3 R_3 - I_2 R_2 = 0$$

$$\Delta V - I_1 R_1 - I_2 R_2 = 0$$

only 2 of these equations are independent

Using Kirchoff's rules

(1) Write the equations for the junction rule.

(2) Write the equations for the loop rule. Choose a direction for currents. If the current is negative then it flows in the opposite direction. Use as many equations as necessary to solve for all unknown quantities. (for n unknowns need n equations).

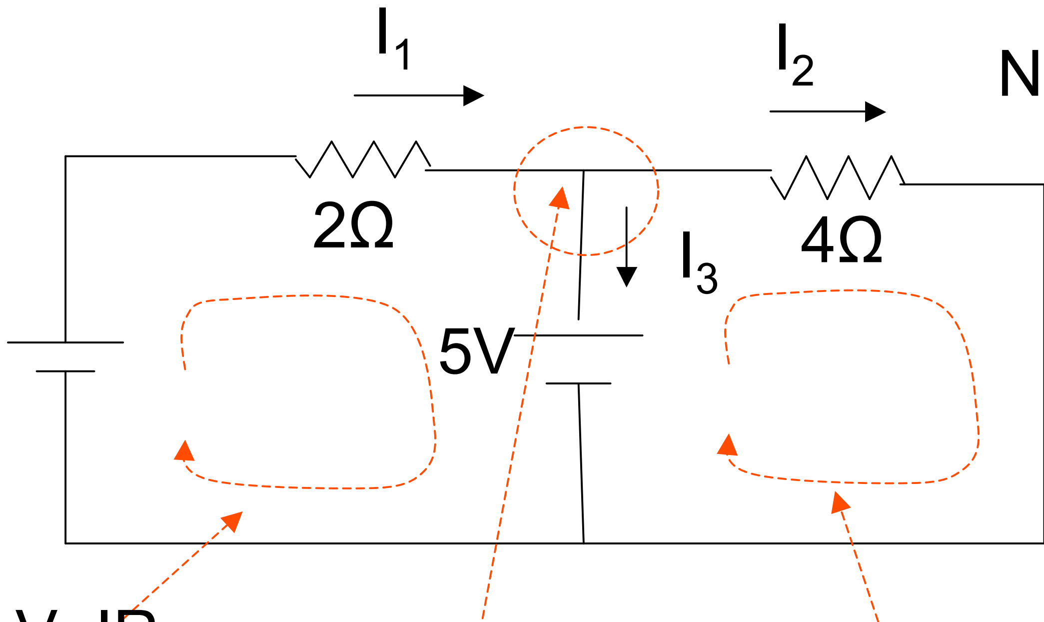
(3) Solve the set of equations for n unknown quantities.

Find I_1, I_2, I_3

No. equations needed= 3

no. Junction= 1

No. loop = 2



V=IR

$$10 - 2I_1 - 5 = 0$$

$$I_1 = \frac{10-5}{2} = 2.5A$$

$$I_1 = I_2 + I_3$$

$$5 - 4I_2 = 0$$

$$I_2 = \frac{5}{4} = 1.25A$$

$$I_3 = I_1 - I_2$$

$$I_3 = 2.5 - 1.25 = 1.25A$$

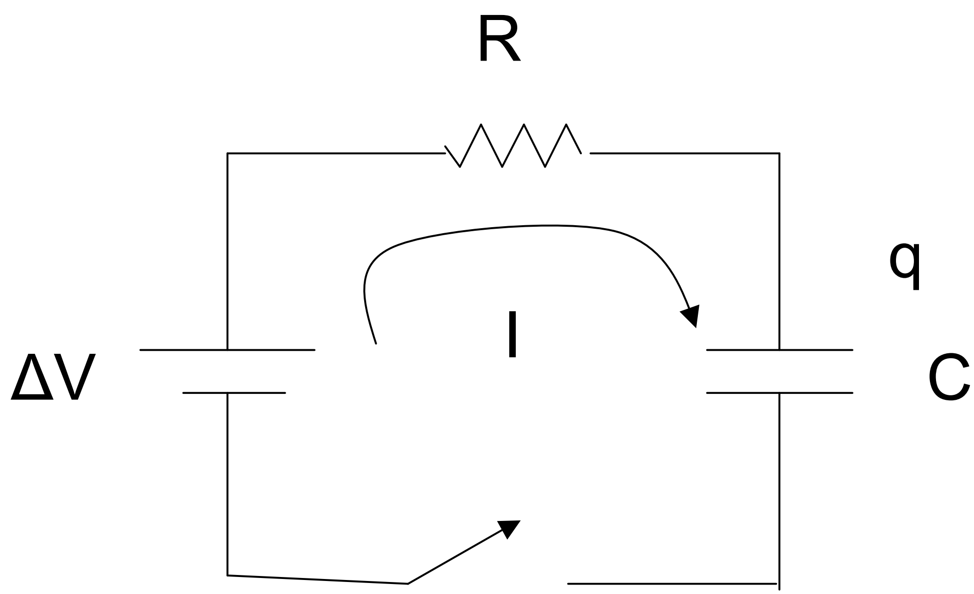
Chapter 18.5 RC circuit

Time dependent currents and voltages.

Applications. clocks, timing circuits, computers.

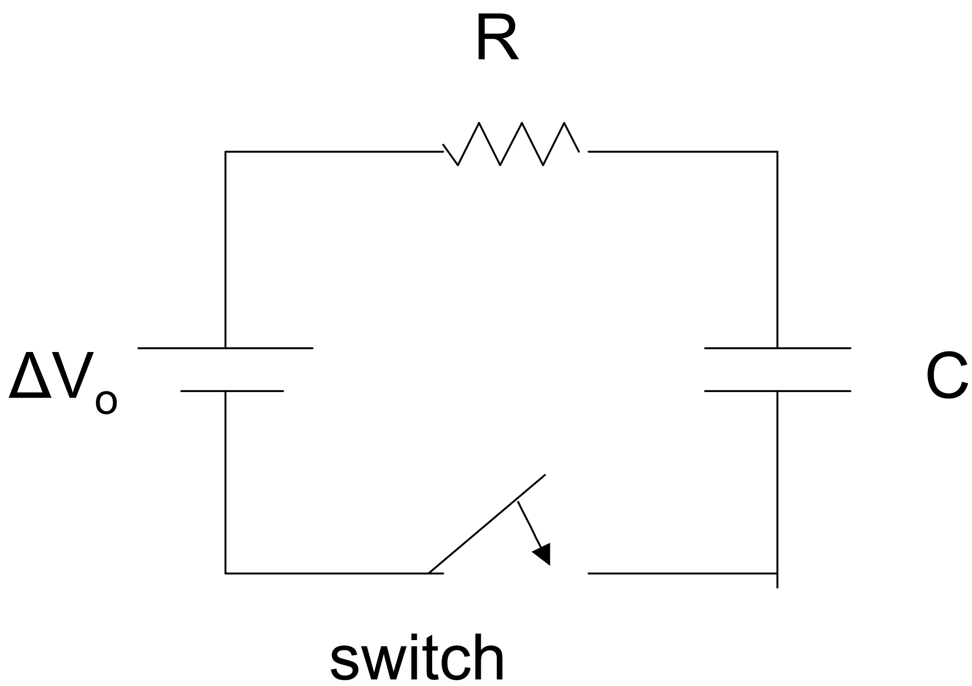
Time to charge and discharge of a capacitor

RC circuit



When the switch is closed how does the current and voltage change with time?

RC circuit

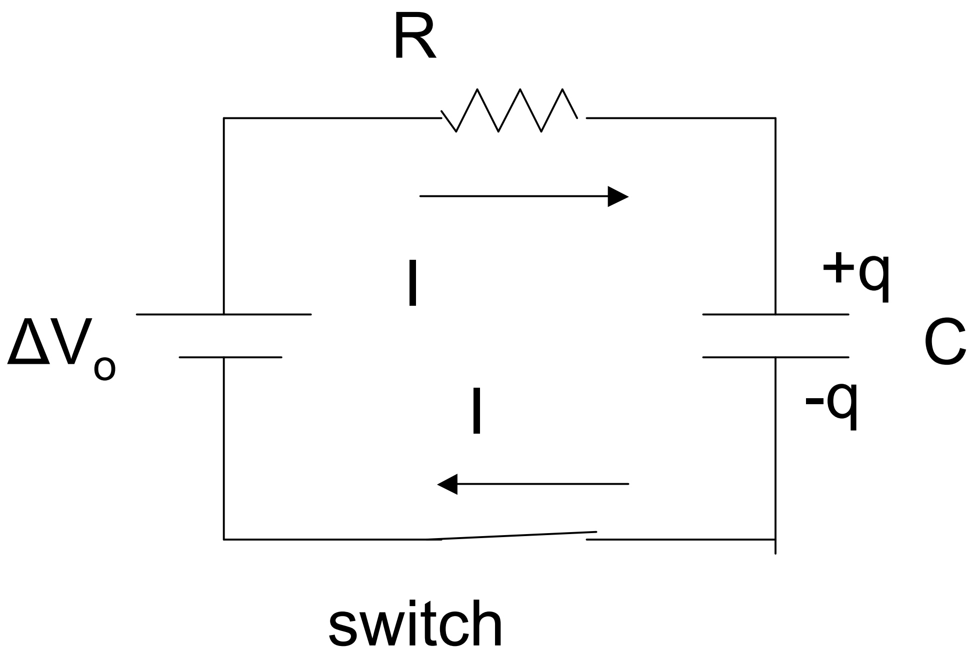


Switch off

Capacitor uncharged

$\Delta V_c = 0$

Charging



Switch on

$$\Delta V_c = \frac{q}{C}$$

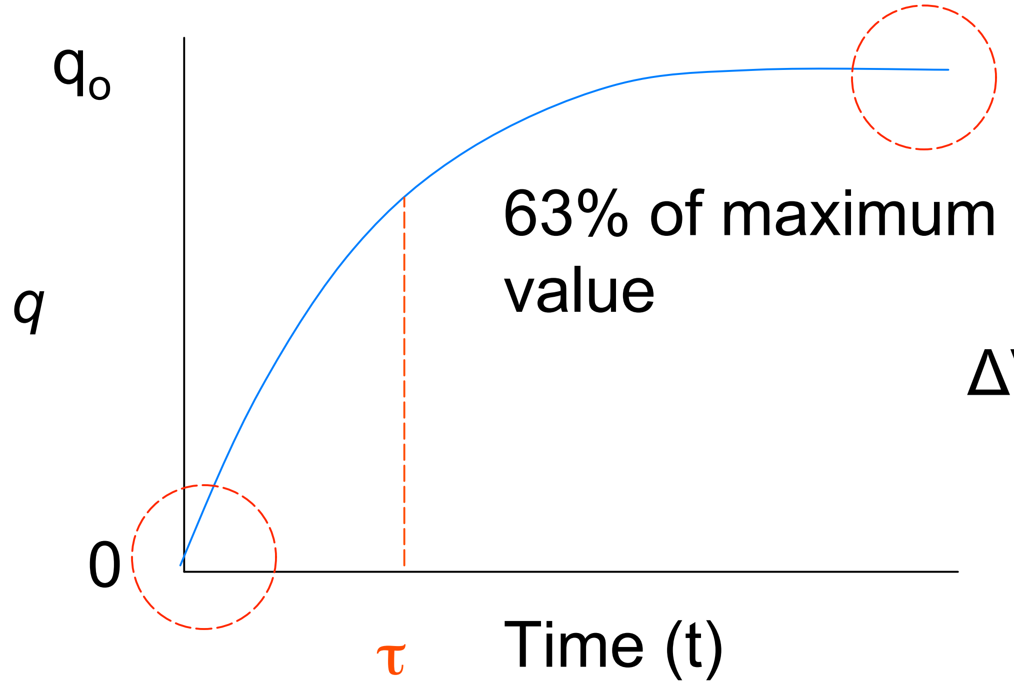
$$\Delta V_o - IR - \Delta V_c = 0$$

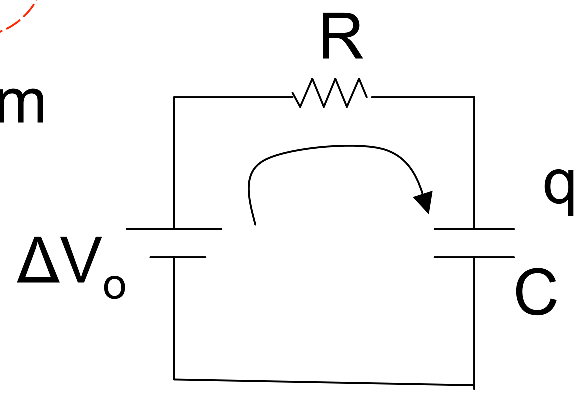
When the switch is initially closed the voltage on the capacitor is zero.

Charge is transferred to the capacitor at a rate I=dq/dt.

As the capacitor is charging the charge and voltage on the capacitor increases with time and the current decreases.

Charging Capacitor





$$\Delta V_C = \frac{q}{C}$$

$$\Delta V_o = IR + \frac{q}{C}$$

short times	intermediate times	long times
$q = \sim 0$	$q = q_o(1 - e^{-\left(\frac{t}{\tau}\right)})$	$q = q_o$
$\Delta V_C = \approx 0$	$\Delta V_c = V_o(1 - e^{-\left(\frac{t}{\tau}\right)})$	$\Delta V_C = \Delta V_o$
$I = \approx \frac{\Delta V_o}{R}$	$I = \frac{\Delta V_o}{R} e^{-\left(\frac{t}{\tau}\right)}$	$I = 0$

$$\tau = RC$$

Time Constant

$$\tau = RC$$

Dimensional analysis

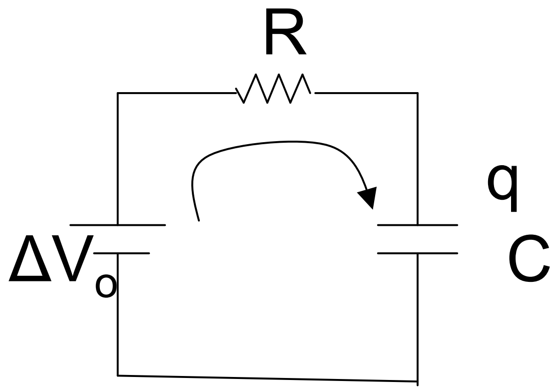
$$RC = \frac{V}{I}\frac{q}{V} = \frac{q}{I} = \frac{q}{q/t} = t$$

RC has units of time

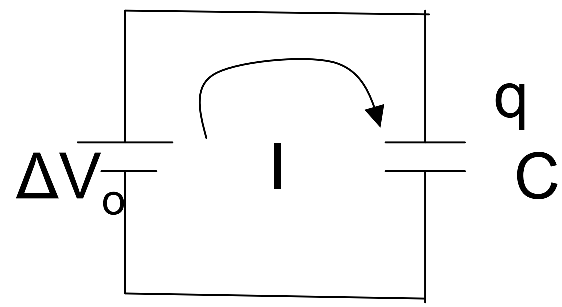
Time required to charge the capacitor

- increases with R – lower current flow
- Increases with C - more charge on capacitor

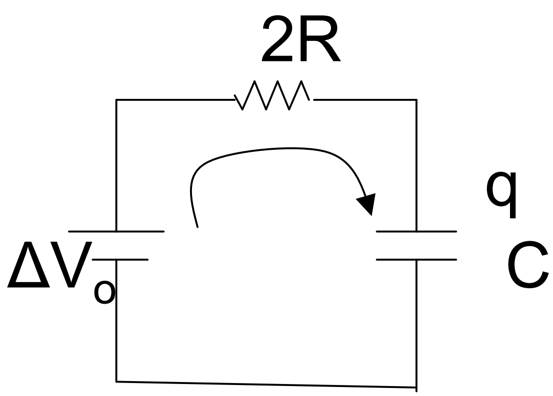
How does the time to charge the capacitor depend on R and C



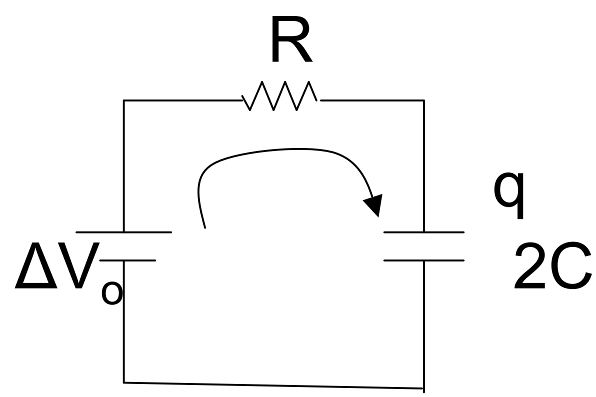
Charging time τ_o



shorter than τ_o because the current is larger

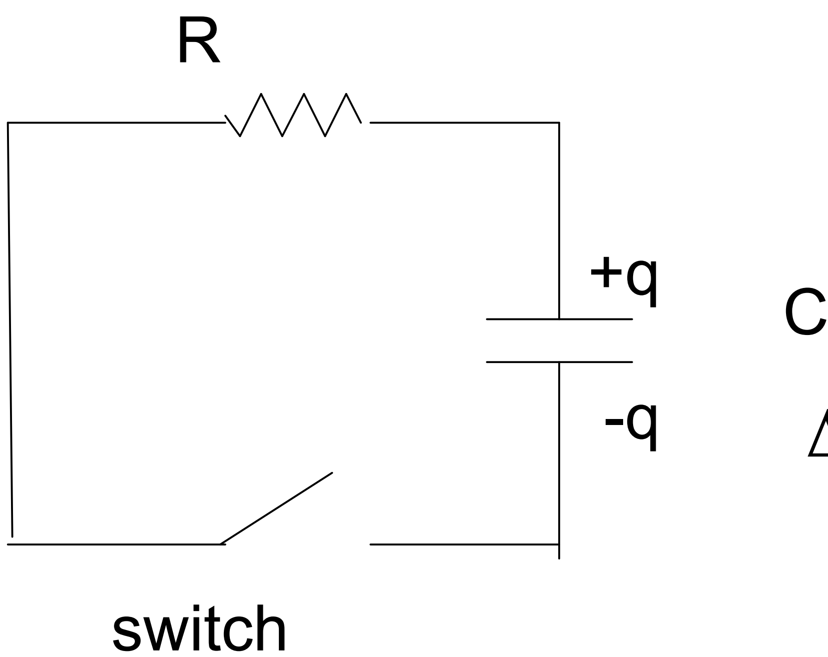


longer than τ_o
the current is smaller



longer than τ_o
more charge is transferred

Discharging



Switch off

Capacitor charged

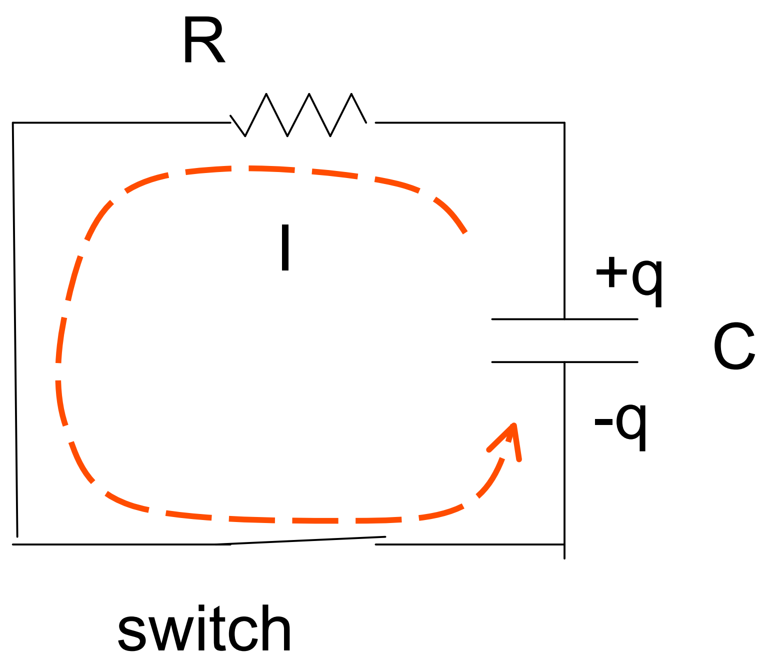
$$\Delta V_c = \frac{q}{C}$$

When the switch is closed to discharge the capacitor the capacitor has a maximum charge of q_o and maximum voltage V_o.

As the capacitor discharges the charge and voltage decrease with time.

The current will also decrease with time.

Discharge



Switch on

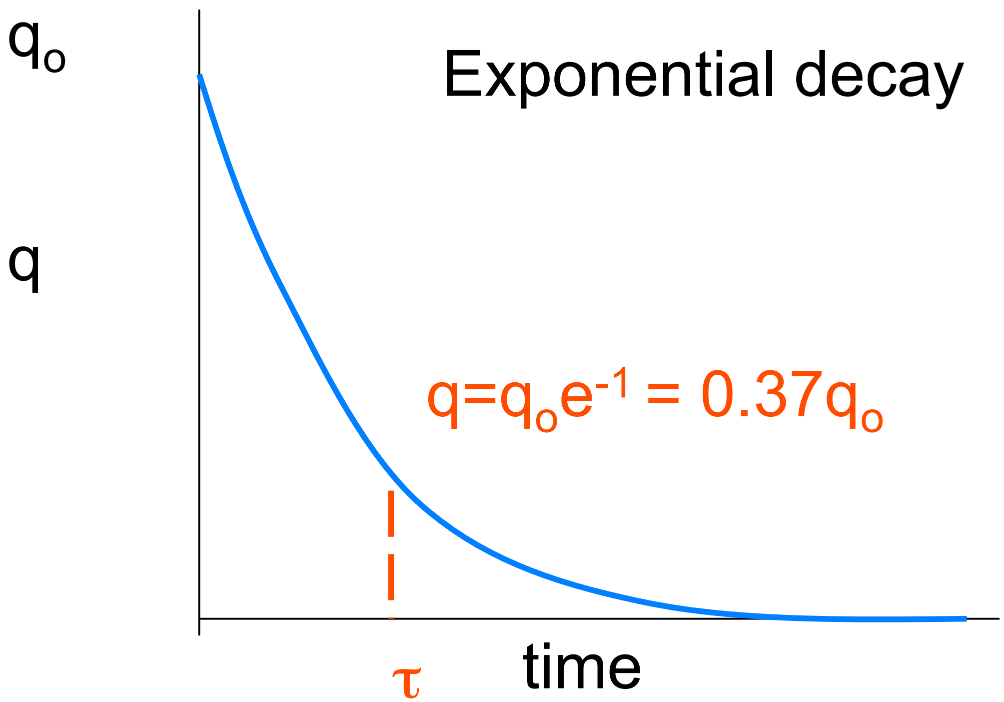
Current flows

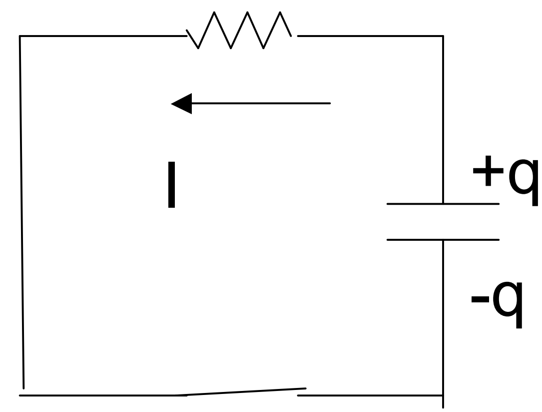
$$\Delta V_R = IR = \Delta V_c = \frac{q}{C}$$

$$I = -\frac{\Delta q}{\Delta t} = \frac{q}{RC}$$

$$q = q_o e^{-\left(\frac{t}{\tau}\right)}$$

The charge decays exponentially with time





$$\Delta V_c - \Delta V_R = 0$$

short times	intermediate times	long times
$q = q_o$	$q = q_o e^{-\left(\frac{t}{\tau}\right)}$	0
$\Delta V_C = \Delta V_o$	$\Delta V_c = V_o e^{-\left(\frac{t}{\tau}\right)}$	0
$I = \frac{\Delta V_o}{R}$	$I = \frac{\Delta V_o}{R} e^{-\left(\frac{t}{\tau}\right)}$	0

$$\tau = RC$$

Exponential decay

Found in many other systems-
Chemical reaction, nuclear decay

$$A \rightarrow B$$

When the rate of decay of a species is proportional to the amount of the species

$$\frac{\Delta A}{\Delta t} = -\frac{A}{\tau}$$

The result is exponential decay

$$A = A_o e^{-\left(\frac{t}{\tau}\right)}$$

τ is a constant

A 12 μfarad capacitor is discharged through a 2 kΩ resistor. How long does it take for the voltage to decay to 5% of the initial voltage.

$$\tau = RC = 2x10^{3}(12x10^{-6}) = 24x10^{-3}s = 24ms$$

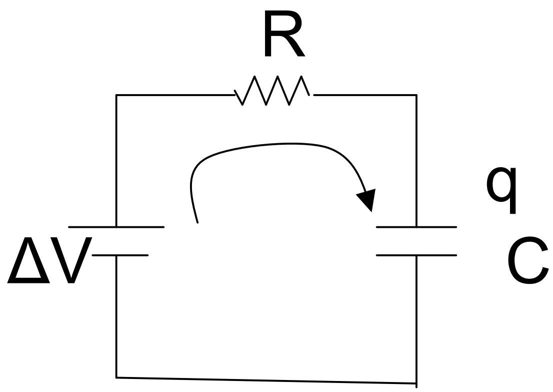
$$V = V_o e^{-\left(\frac{t}{\tau}\right)}$$

$$\frac{V}{V_o} = e^{-\left(\frac{t}{\tau}\right)}$$

$$\ln\left(\frac{V}{V_o}\right) = -\frac{t}{\tau}$$

$$t = -\tau \ln\frac{V}{V_o} = -24x10^{-3}(\ln(0.05)) = 7.2x10^{-2}s$$

33. Consider a series RC circuit for which R=1.0 MΩ, C=5.0 μF and ε=30 V. The capacitor is initially uncharged when the switch is open. (a) Find the charge on the capacitor 10 s after the switch is closed.



$$\tau = RC = 1x10^{6}(5x10^{-6}) = 5.0s$$

$$q = q_o(1 - e^{-\frac{t}{RC}}) = q_o(1 - e^{-\frac{t}{\tau}})$$

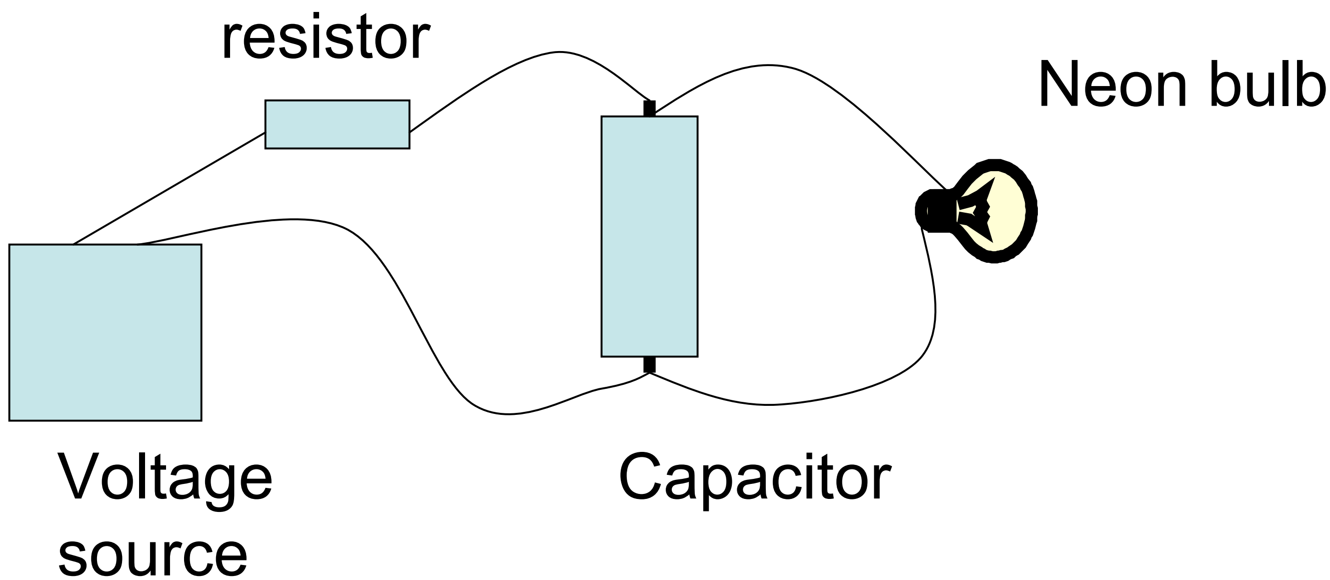
$$C = \frac{q}{\Delta V}$$

$$q_o = \Delta VC = 30(5x10^{-6}) = 1.5x10^{-4}C$$

$$q = q_o(1 - e^{-\frac{t}{RC}}) = 1.5x10^{-4}(1 - e^{-\frac{10}{5}})$$

$$q = 1.3x10^{-4}C$$

You plan to make a flasher circuit that charges a capacitor through a resistor up to a voltage at which a neon bulb discharges (about 100V) about once every 5 sec. If you have a 10 microfarad capacitor what resistor do you need?

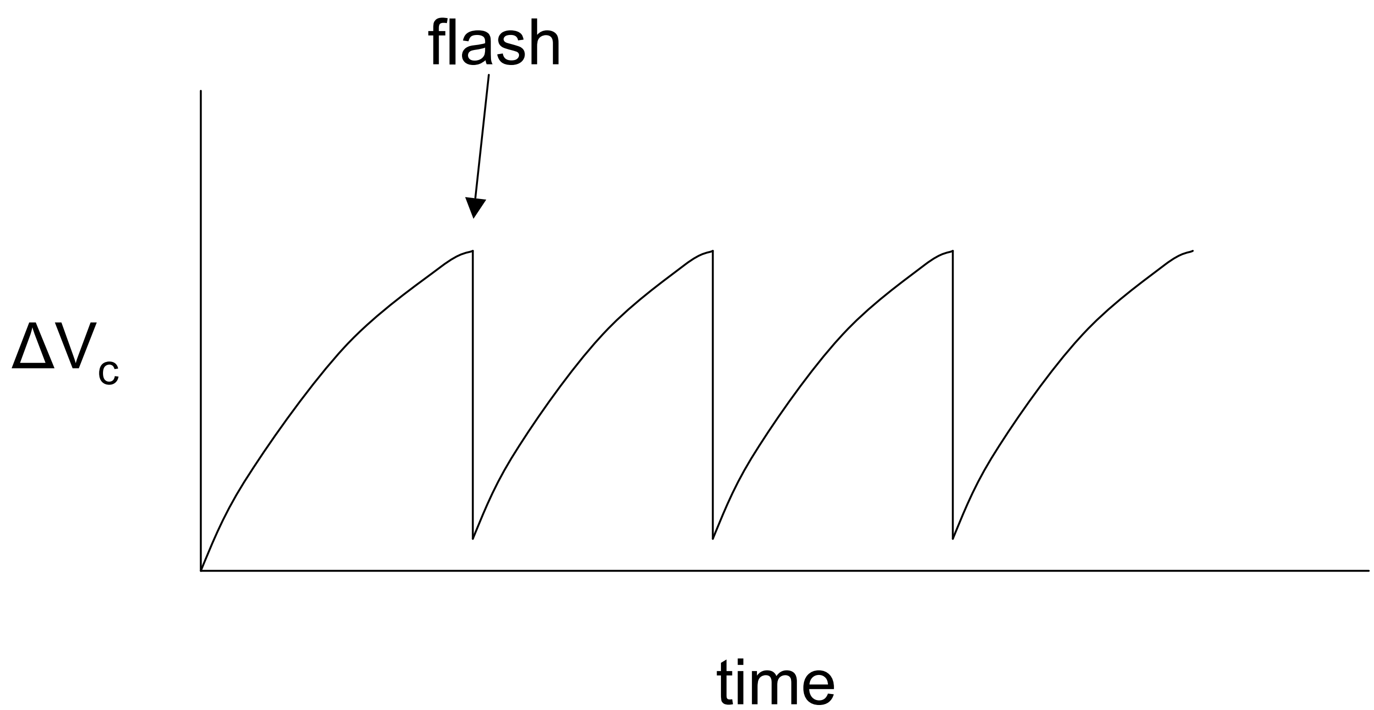


$$\tau = RC$$

$$R = \frac{\tau}{C} = \frac{5}{10x10^{-6}} = 0.5x10^{6}\,\Omega$$

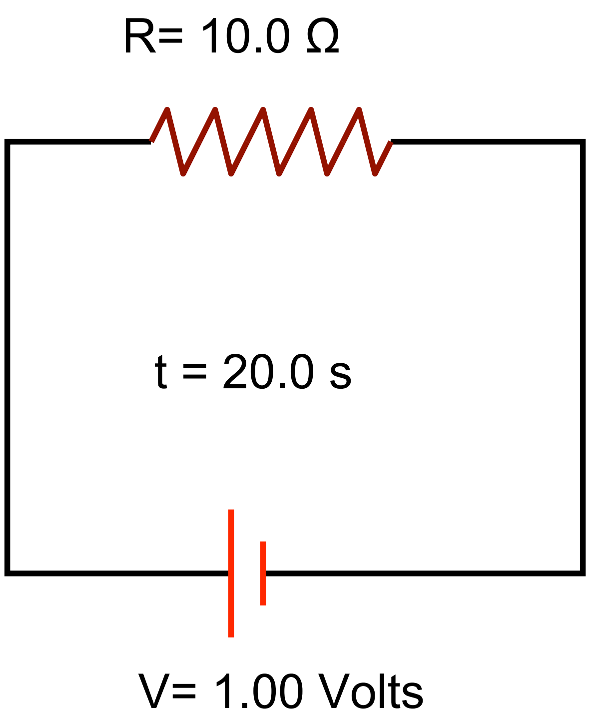
About 0.5MΩ

Charging



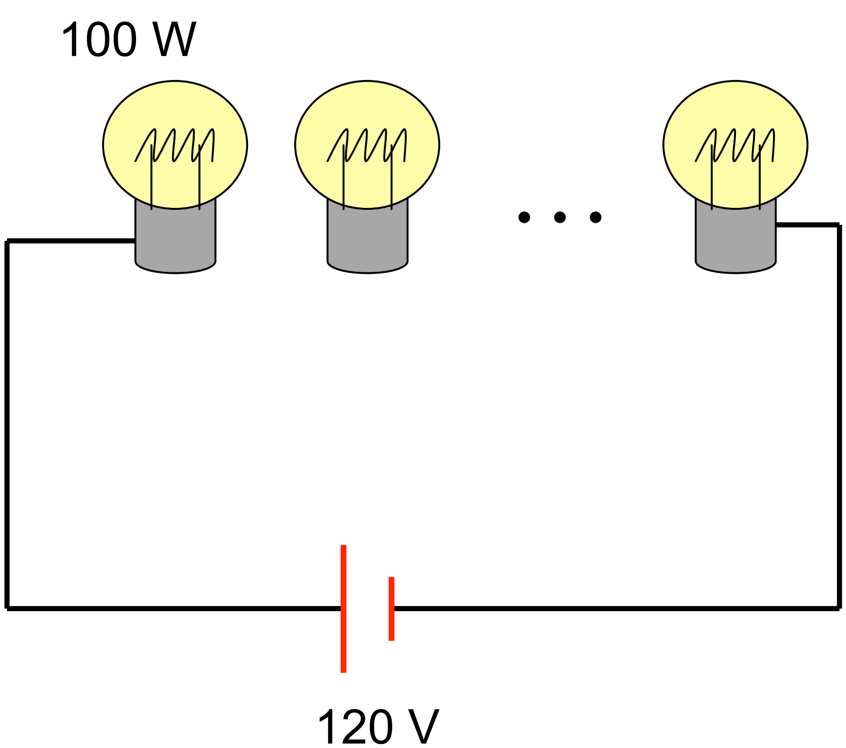
HW – Clickers Out

1) From 17.3: A 1.00 V potential difference is maintained across a 10.0 Ω resistor for a period of 20.0 s. What total charge passes through the wire in this time interval?



- A) 4 C
- B) 1 C
- C) can’t determine
- D) 2 C

2) From 17.33: How many 100 W light bulbs can you use in a 120 V circuit without tripping a 15 A circuit breaker? (the potential difference across each bulb is 120 V.)

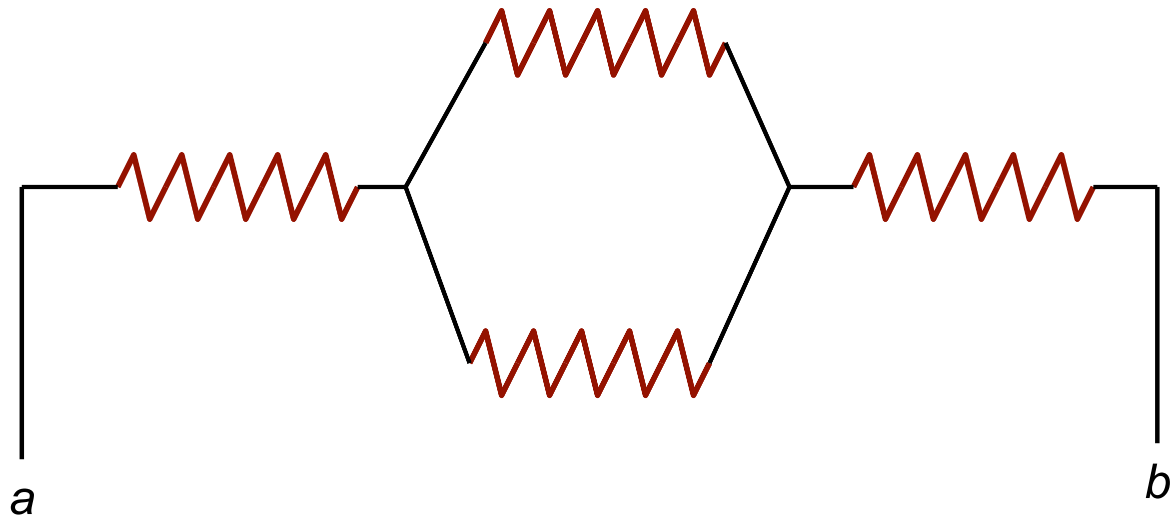


- A) 1 bulb
- B) 18 bulbs
- C) 9 bulbs
- C) 10 bulbs

3) From 17.9: If the current carried by a conductor is doubled what happens to the charge carrier density?

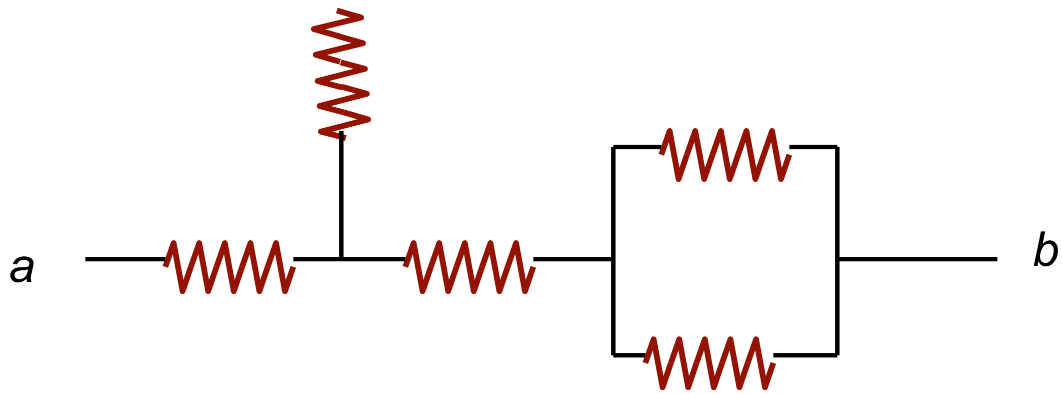
- A) doubled
- B) unchanged
- C) halved

4) From 18.5: Find the equivalent resistance between points *a* and *b* in the figure.



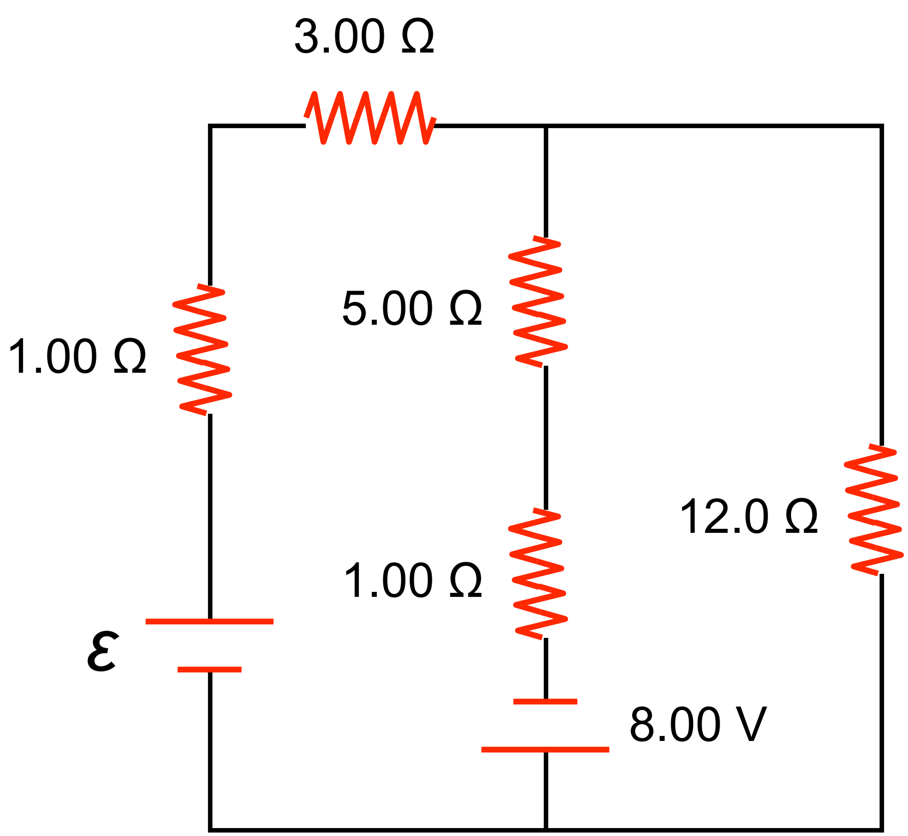
- A) 2.4 Ω
- B) 17.1 Ω
- C) 30 Ω
- D) 1.6 Ω

5) From 18.7: Find the equivalent resistance between points *a* and *b* in the figure. Each resistor has resistance R.



- A) 2.5 R
- B) 5 R
- C) 12.75 R
- D) R

6) From 18.21: What is the EMF of the battery in the following figure?



- A) 8.0 V
- B) 14.3 V
- C) 5.8 V
- D) 10.7 V