Physics 214 UCSD/225a UCSB

Lecture 5

- Symmetries & QCD
  - Finishing off Isospin et al.
  - SU(3)
- Draw heavily from H&M chapter 2 today.

Scattering

$$\frac{\sigma(pp \to \pi^+ d)}{\sigma(np \to \pi^0 d)} = \frac{\left|\left\langle \pi^+ d \middle| S \middle| pp \right\rangle\right|^2}{\left|\left\langle \pi^0 d \middle| S \middle| np \right\rangle\right|^2} \bullet PhSpR = \frac{\left|\left\langle 1 \| S \| 1 \right\rangle\right|^2}{\left|\left\langle 1 \| S \| 1 \right\rangle + \left\langle 1 \| S \| 0 \right\rangle\right|^2 \bullet \frac{1}{2}}$$

$$\left| pp \right\rangle = \left| \frac{1}{2}; + \frac{1}{2} \right\rangle \left| \frac{1}{2}; + \frac{1}{2} \right\rangle = \left| 1;1 \right\rangle_{NN}$$

$$\left| np \right\rangle = \left| \frac{1}{2}; - \frac{1}{2} \right\rangle \left| \frac{1}{2}; + \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} \left(\left| 1;0 \right\rangle_{NN} + \left| 0;0 \right\rangle_{NN} \right)$$

$$\left| \pi^+ d \right\rangle = \left| 1;1 \right\rangle_{\pi} \left| 0;0 \right\rangle_{d} = \left| 1;1 \right\rangle_{\pi d}$$

$$\left| \pi^0 d \right\rangle = \left| 1;0 \right\rangle_{\pi} \left| 0;0 \right\rangle_{d} = \left| 1;0 \right\rangle_{\pi d}$$

Isospin for anti-quarks

- Want to make anti-quark doublet with:
  - charge conserved -> anti-d must have T_3=+1/2 because Q = T_3 + B:
    - $Q_u = 1/2 + 1/3 = +2/3$ $\quad$ $Q_d = -1/2 + 1/3 = -1/3$
    - $Q_{anti\text{-}u} = -1/2 - 1/3 = -2/3$ $\quad$ $Q_{anti\text{-}d} = 1/2 - 1/3 = +1/3$
  - baryon number conserved
  - Same transformation properties as quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \xrightarrow{C} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

If you simply bar and flip then you get the wrong sign in front of the "sin" terms.

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = e^{\frac{1}{2} i\theta_y \sigma_y} \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} & \sin\frac{\theta_y}{2} \\ -\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} u + \sin\frac{\theta_y}{2} d \\ -\sin\frac{\theta_y}{2} u + \cos\frac{\theta_y}{2} d \end{pmatrix}$$

$$C\begin{pmatrix} u' \\ d' \end{pmatrix} = \begin{pmatrix} -\bar{d}' \\ \bar{u}' \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta_y}{2} \bar{u} - \cos\frac{\theta_y}{2} \bar{d} \\ \cos\frac{\theta_y}{2} \bar{u} + \sin\frac{\theta_y}{2} \bar{d} \end{pmatrix}$$

$$\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \rightarrow e^{\frac{1}{2} i\theta_y \sigma_y} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} & \sin\frac{\theta_y}{2} \\ -\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{pmatrix} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta_y}{2} \bar{u} - \cos\frac{\theta_y}{2} \bar{d} \\ \cos\frac{\theta_y}{2} \bar{u} + \sin\frac{\theta_y}{2} \bar{d} \end{pmatrix}$$

The point here is that you want to be able to derive the rotated doublet either via rotation to the quark doublet followed by Charge conjugation and flip, or by starting with the anti-q doublet and using the same rotation as for q doublet.

Quantum Numbers for Mesons

- J^{PC}

  J = total angular momentum = L+S

  P = parity

  C = charge conjugation

- Only neutral particles can be eigenstates of C, of course.

Generalized Pauli Principle

- The fermion-antifermion wave function must be odd under interchange of all coordinates (space, spin, charge).
  - Space interchange -> $(-1)^L$
  - Spin interchange -> $(-1)^{S+1}$
  - Charge interchange -> depends on eigenvalue of C
- Bottom line:

$(-1)^{L+S+1}\,C = -1 \quad \Rightarrow \quad C = (-1)^{L+S}\,;\ \ P = (-1)^{L+1}$

π^0 : $C = (-1)^{0+0} = 1$; $P = (-1)^{0+1} = -1$ => pseudoscalar meson
ρ^0 : $C = (-1)^{0+1} = -1$; $P = (-1)^{0+1} = -1$ => vector meson
b : $C = (-1)^{1+0} = -1$; $P = (-1)^{1+1} = +1$ => axial vector meson

Example: π Wave Function

$$\left|T = 1; T_3 = 1\right\rangle = -u\bar{d}$$

$$\left|S = 0\right\rangle = \uparrow\downarrow - \downarrow\uparrow$$ -> Antisymmetric under interchange

$$\left|color\right\rangle = R\bar{R} + B\bar{B} + G\bar{G}$$

$$\left|\pi^{+}\right\rangle = \sqrt{\frac{1}{6}} \sum_{a=R,G,B} \left|u_a \uparrow \bar{d}_{\bar{a}} \downarrow\right\rangle - \left|u_a \downarrow \bar{d}_{\bar{a}} \uparrow\right\rangle$$

Note: C=+1 for pi0 because pi0 -> 2 photons.
Photons are C=-1 because the are produced
by moving charges which have C=-1, of course.

Accordingly, pi0 -> 3photons is heavily suppressed.

SU(3)

- Start with general characteristics
  - Generators and fundamental representation
  - T,U,V spin; SU(2) embedded in SU(3)
  - Graphical way to construct multiplets
  - Applications:
    - Flavor SU(3)
    - Color SU(3)

SU(3) Generators

3^2-1=8 generators λ_i :

Interesting structure in that there are three spin-1/2 subspaces.

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix};\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix};\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix};\sqrt{\frac{1}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

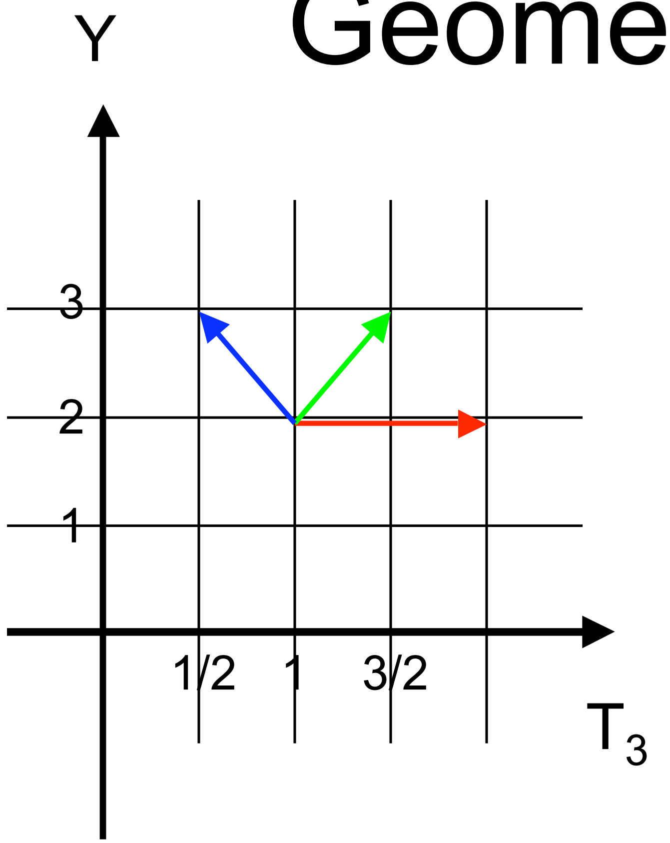
Rank = 2

$\Rightarrow$D(a,b) to classify multiplets.

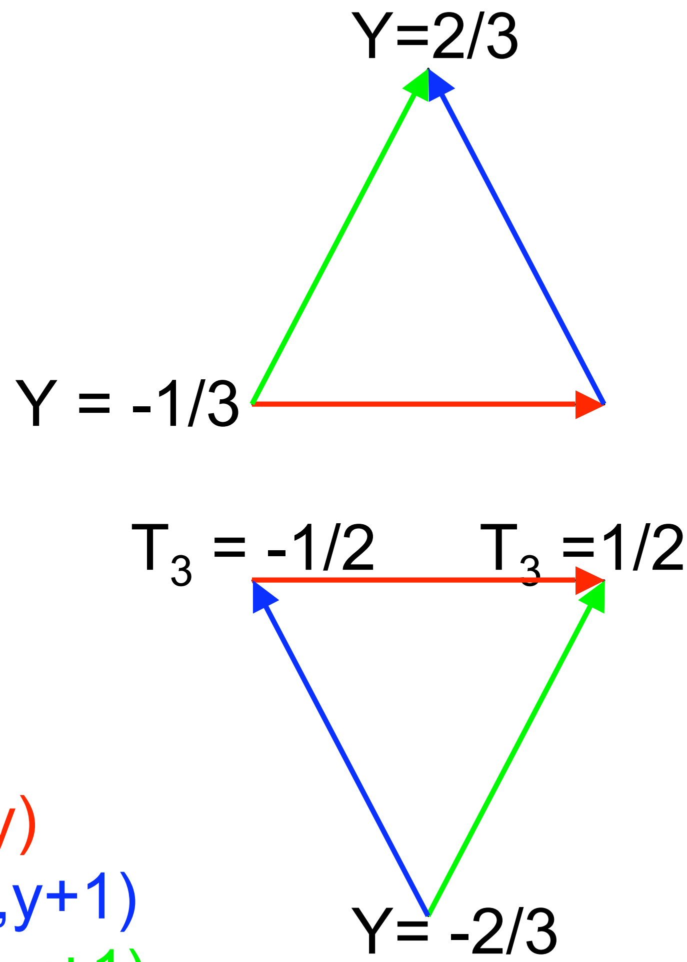
$\Rightarrow T_3$ and Y Q# within multiplet.

$$T_3 = \lambda_3 /2$$
$$Y = \lambda_8 /\sqrt{3}$$

Geometric Construction



Fundamental Representations:



D(0,1)=$[\underline{3}]$

D(1,0)=[3]

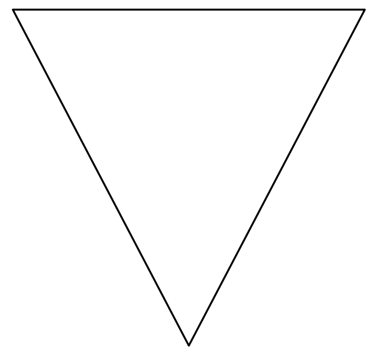
Lines of increasing:

Isospin: T_+ (m,y) = (m+1,y)
U-spin: U_+ (m,y) = (m-1/2,y+1)
V-spin: V_+ (m,y) = (m+1/2,y+1)

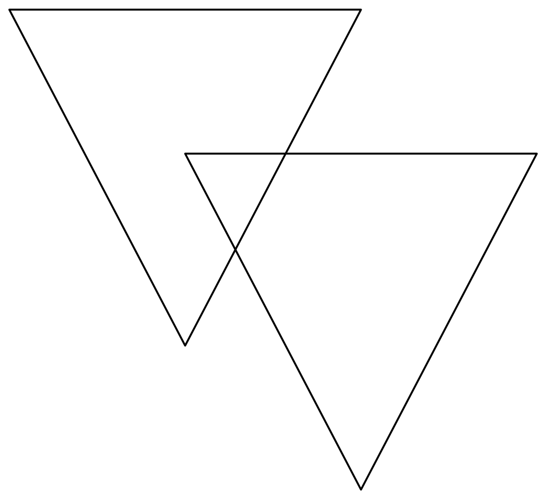
Three additional rules:

- The outer most ring can only occupy one state at each point.
- Going inwards, you get multiple occupation per site.
  - Going one ring in, add one more state per site on that ring.
- If you get more points than would fit on that ring at that site, then collect left-overs, because they will form a separate multiplet.

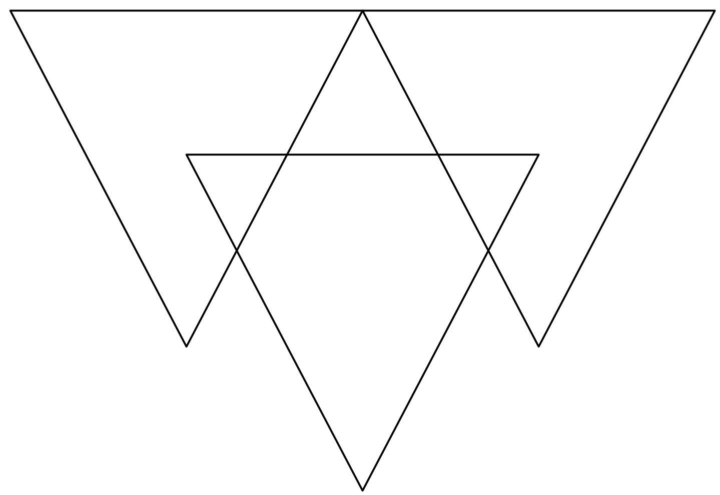
Constructing 3 x 3



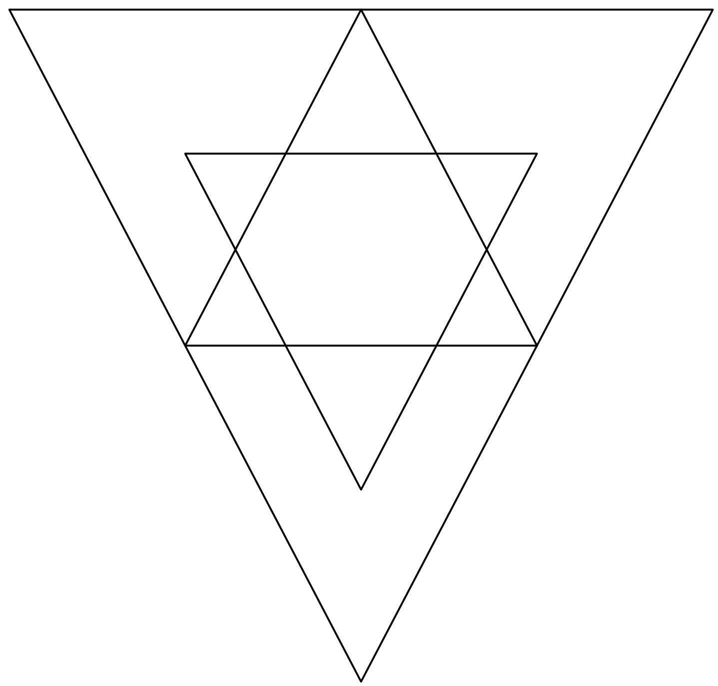
Constructing 3 x 3



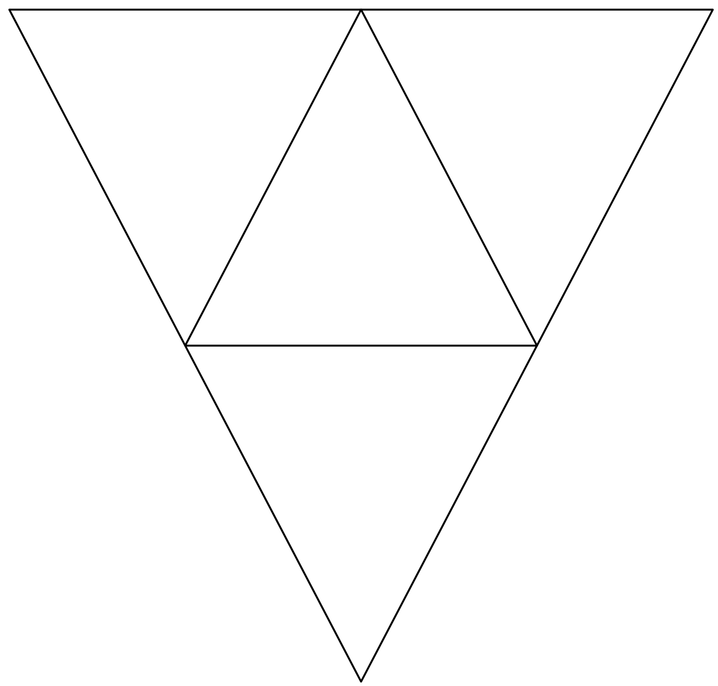
Constructing 3 x 3



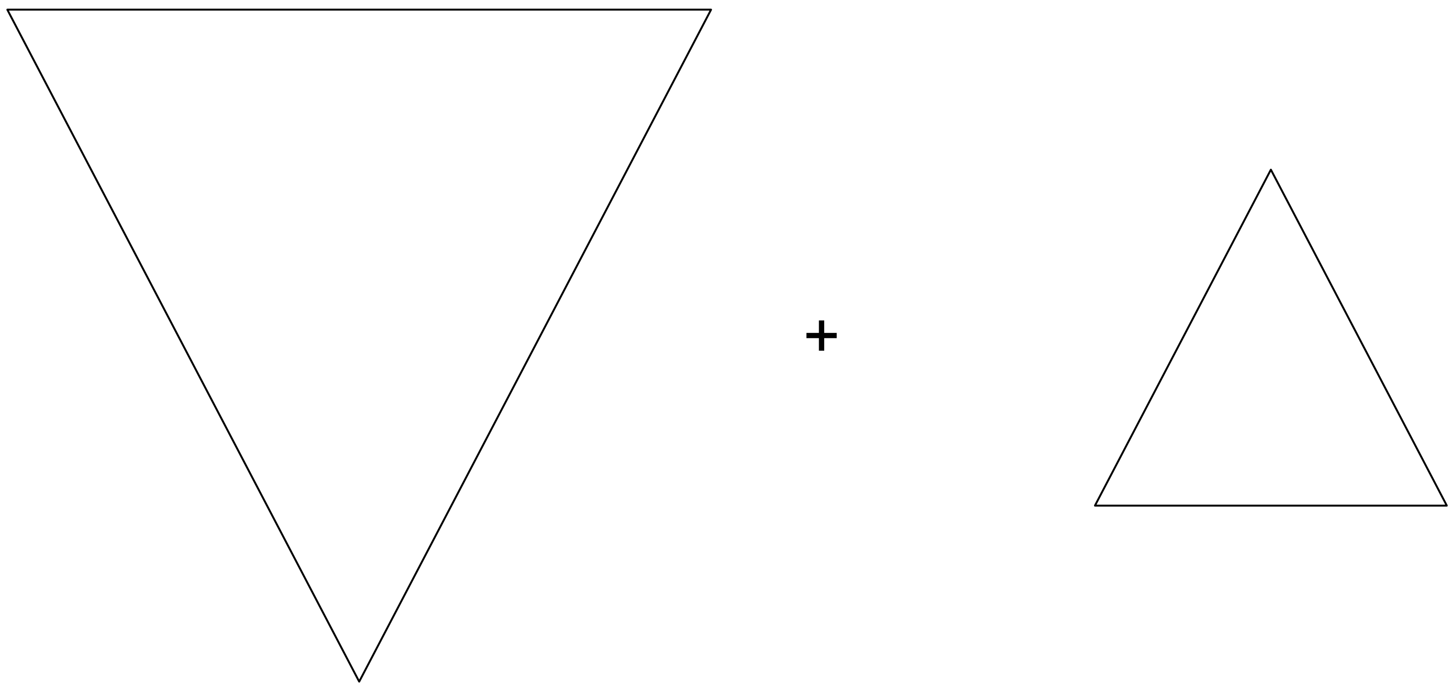
Constructing 3 x 3



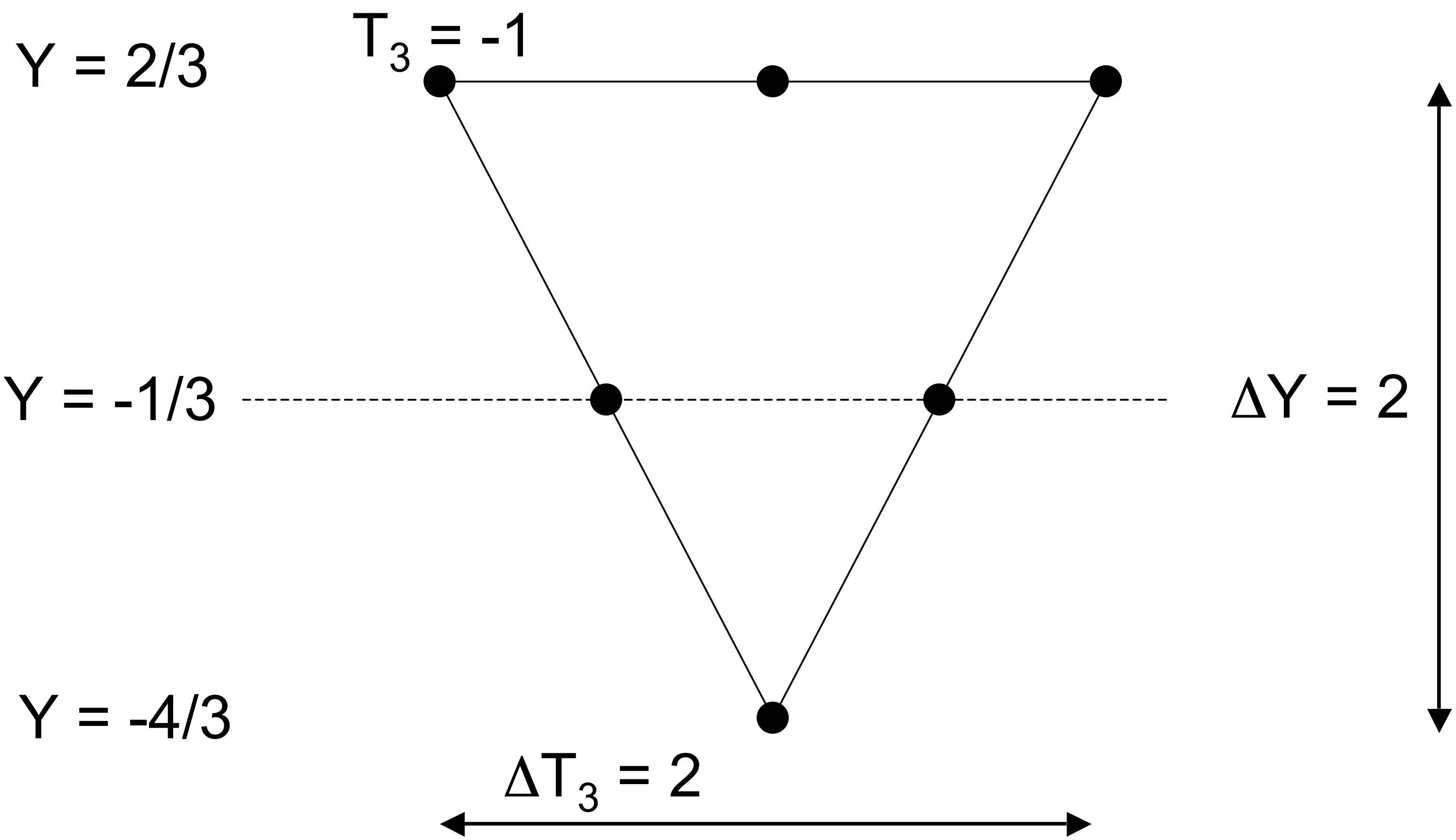
Constructing 3 x 3



$3 \times 3 = 6 + \underline{3}$



This is a sextuplet, or in group notation
D(2,0) = [6]

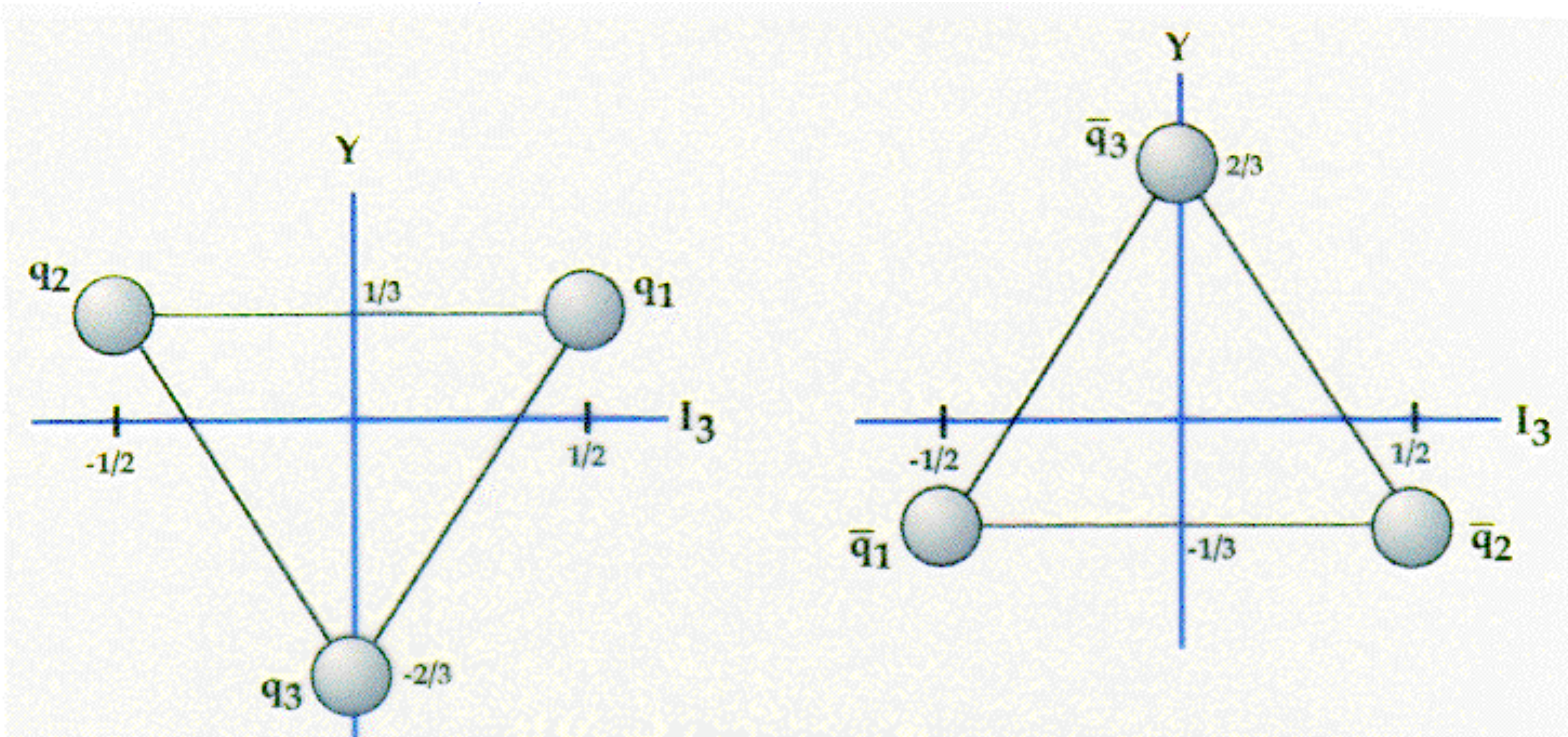


All multiplets of SU(3) can be constructed in this fashion.

Significance to Physics
Flavor SU(3)

- Identify:
    - T = isospin
    - Y = B + S = baryon number + strangeness

    => Charge = Q = T_3 + Y/2
- Identify 3 with quarks u,d,s and $\underline{3}$ with antiquarks $\underline{u}$, $\underline{d}$, $\underline{s}$
- Not all SU(3) multiplets are physically meaningful !!!
    – A physical state needs to simultaneously satisfy SU(3) flavor and SU(3) color, and has to have the appropriate overall symmetry under interchange.
    – The two symmetries operate on completely separate hilbert spaces. The fact that both are SU(3) is an accident of nature.



SU(3)-Tripletts der Quarks und Antiquarks

$d = q_2 = (-1/2,\ 1/3)$ $Q = -1/2 + 1/6 = -1/3$

$u = q_1 = (+1/2,\ 1/3)$ $Q = +1/2 + 1/6 = +2/3$

$s = q_3 = (0,\ -2/3)$ $Q = 0 - 1/3 = -1/3$

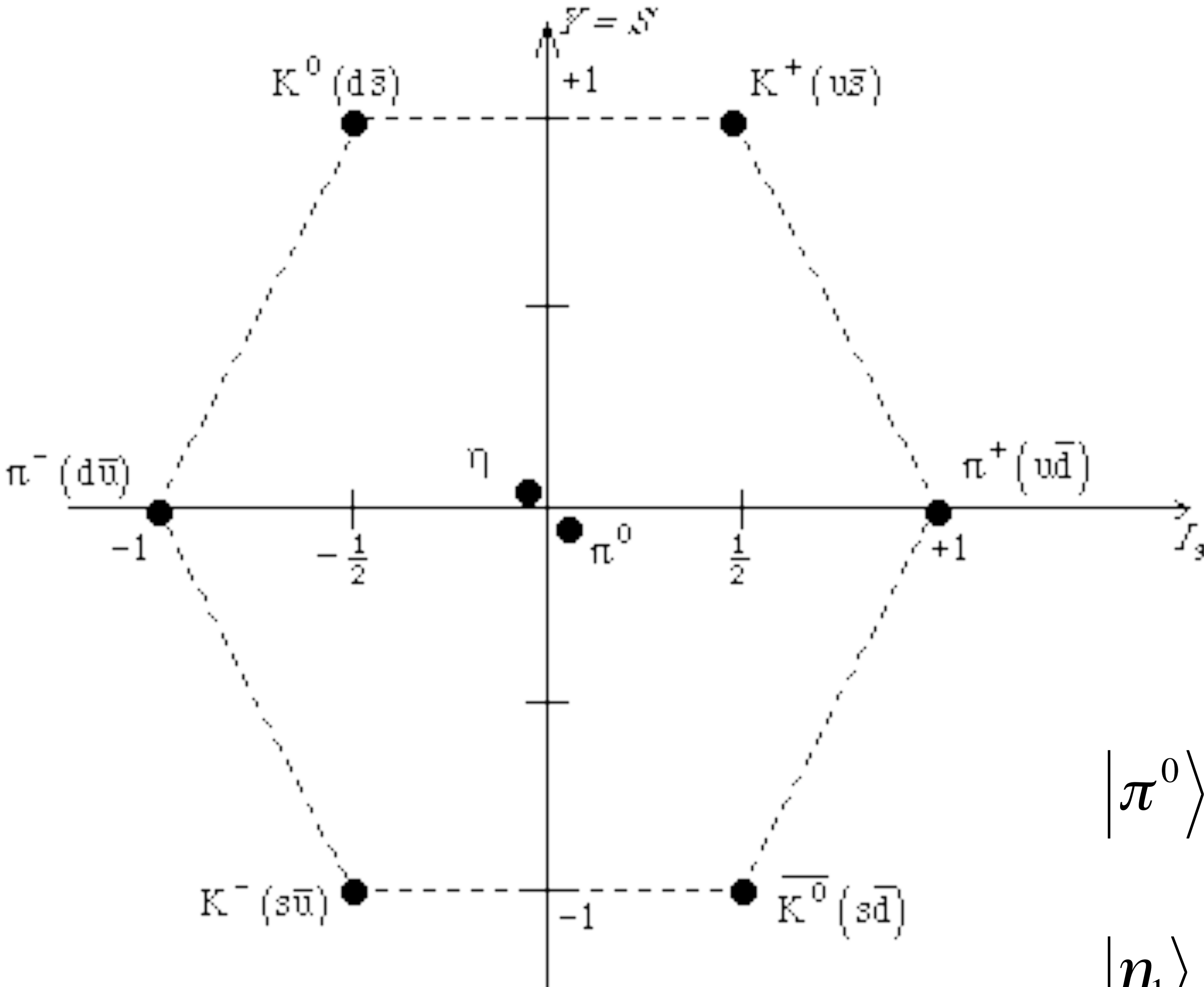
Examples:

- Mesons: $3 \times \underline{3} = 8 + 1$
  - Works for ground state as well as excited states.
- Baryons: $3 \times 3 \times 3 = (6 + \underline{3}) \times 3$

$$= 10 + 8 + (3 \times \underline{3})$$

$$= 10 + 8_S + 8_A + 1$$

- Note: The $\underline{3}$ in $(6 + \underline{3})$ is different from the quark triplet. It is the antisymmetric di-quark triplet.

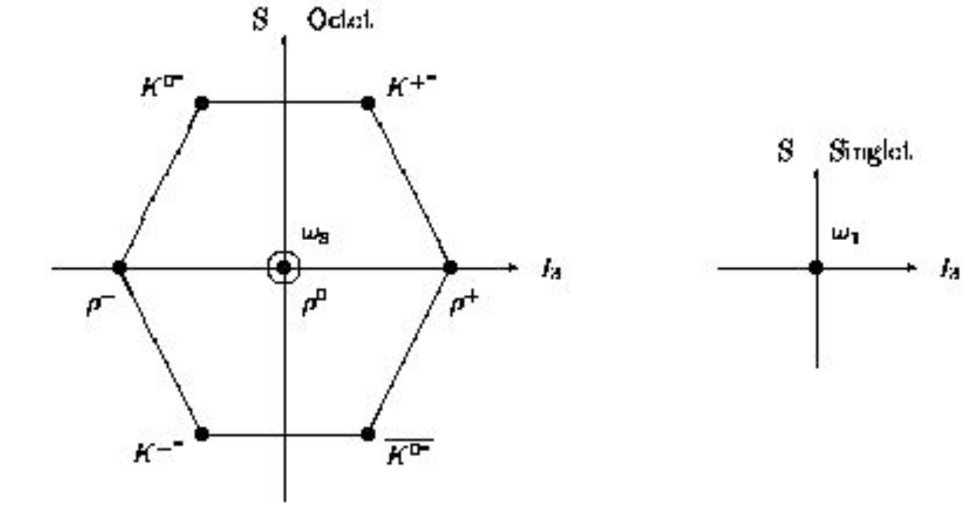


$$\left|\pi^0\right\rangle = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$

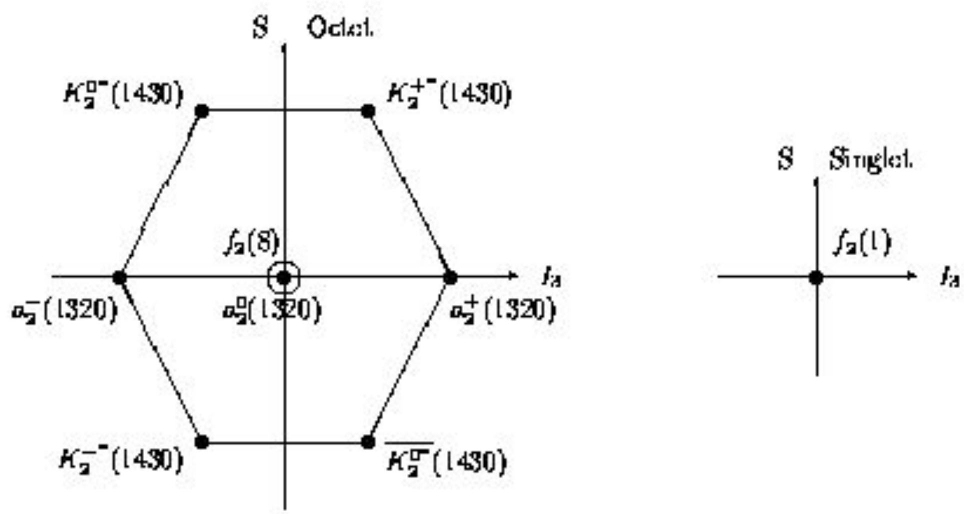
$$\left|\eta_1\right\rangle = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}}$$

$$\left|\eta_8\right\rangle = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}$$

The vector mesons $J^{PC} = 1^{--}$



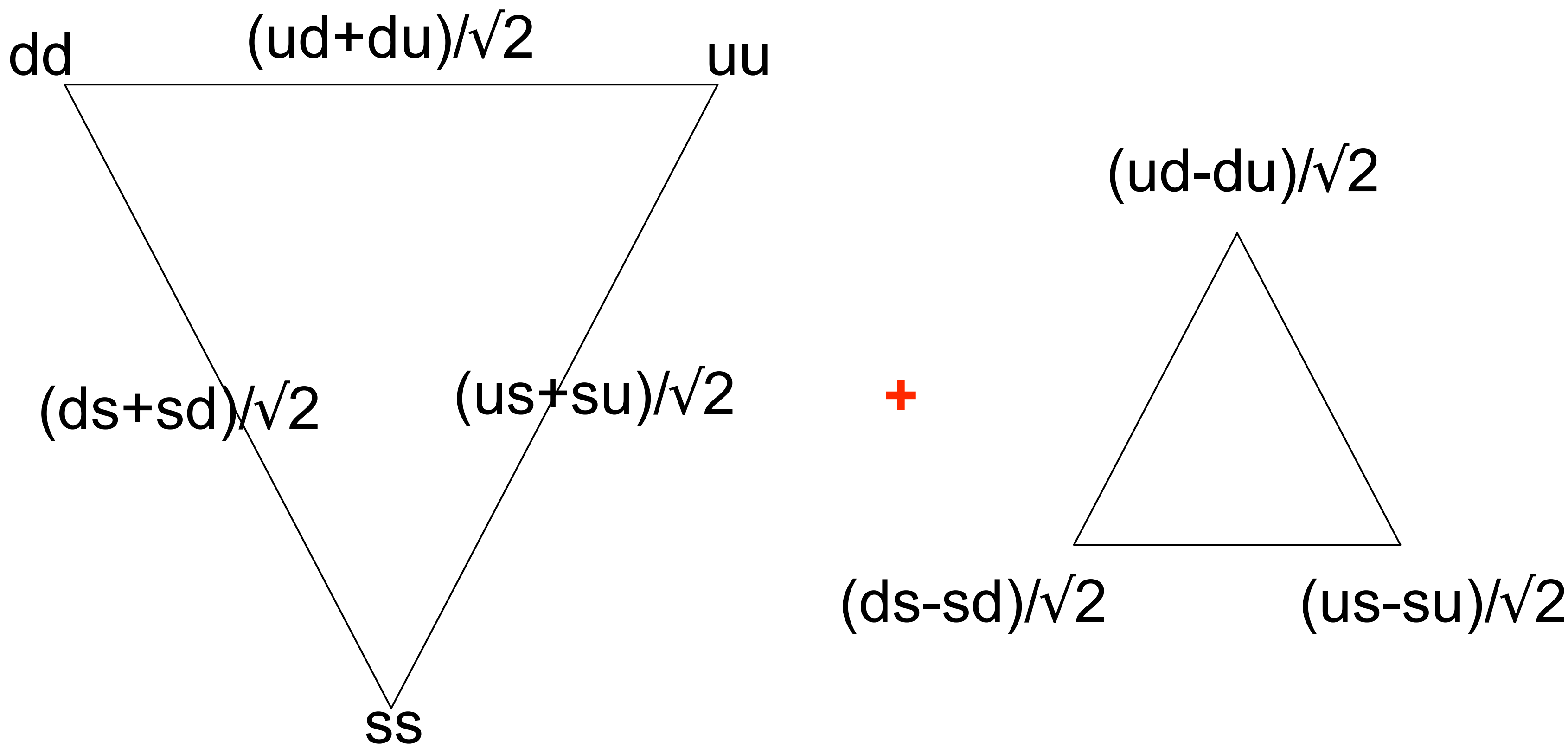
The tensor mesons $J^{PC} = 2^{++}$



Aside:

- SU(3) flavor is strongly dynamically broken in nature.
  - Masses within a multiplet depend on s-quark content.
  - Physical states with T=0 mix across singlet and octet. For vector mesons the physical states are indeed flavor orthogonal rather than flavor symmetric.
- Flavor SU(3) most important to order particles into multiplets, and to show that color must exist, and have (at least some) SU(3) properties.

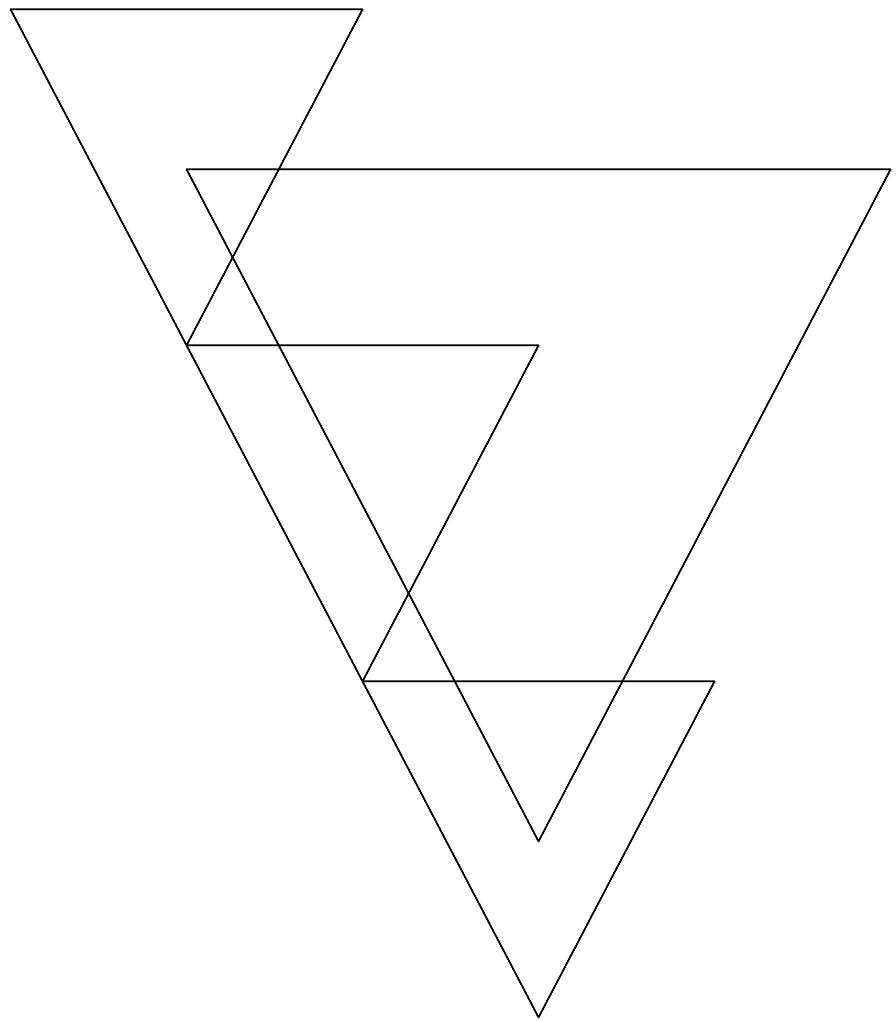
$3 \times 3 = 6 + \underline{3}$ for di-quarks



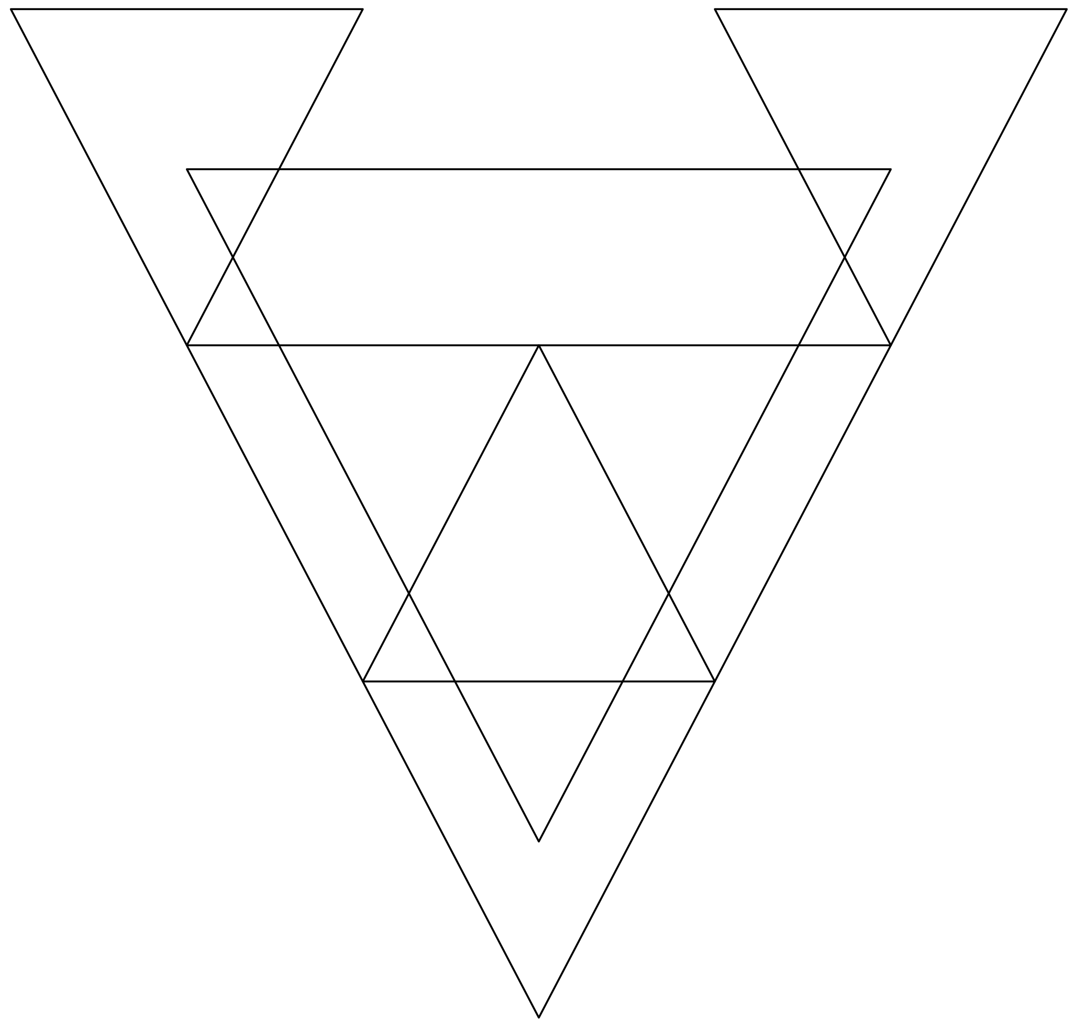
Symmetric Sextet

Antisymmetric triplet

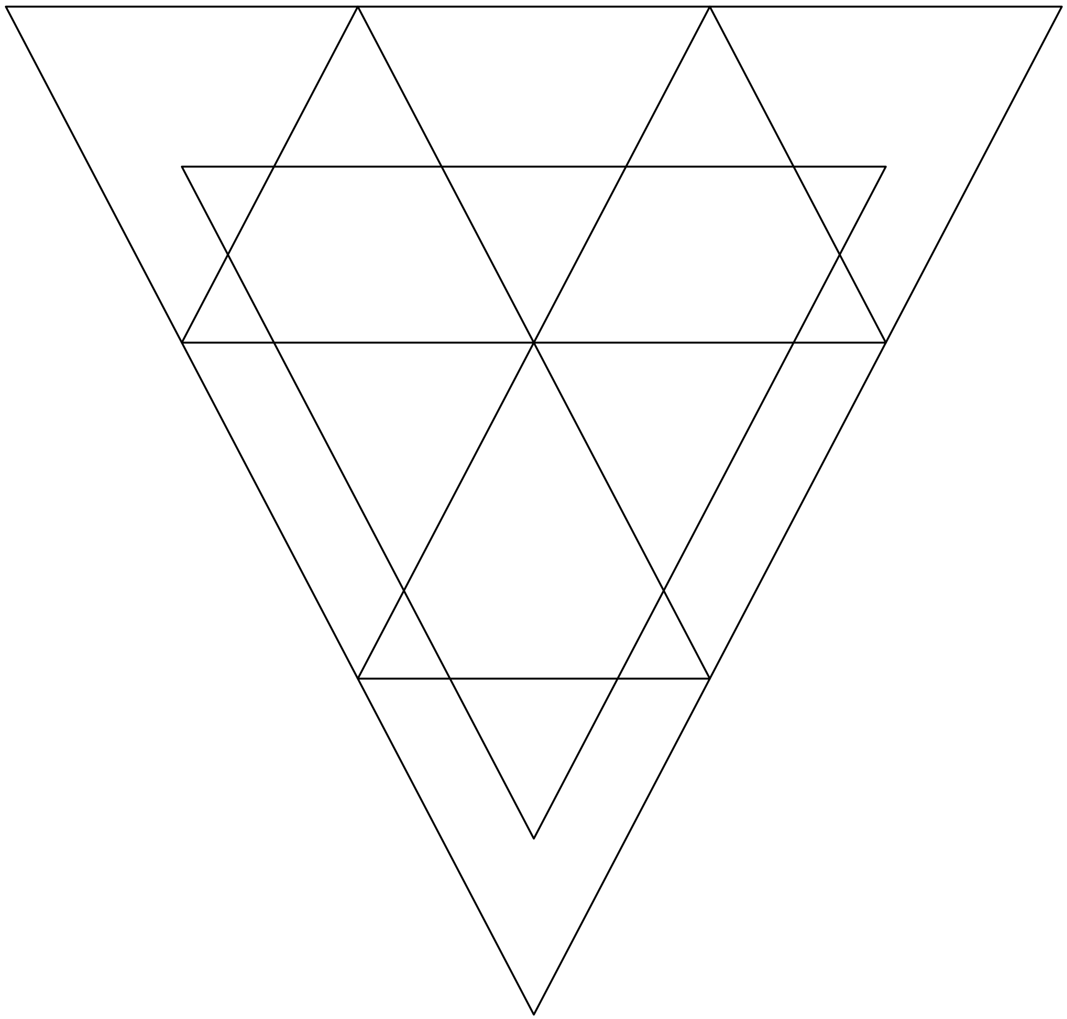
Symmetric di-quarks to triquarks



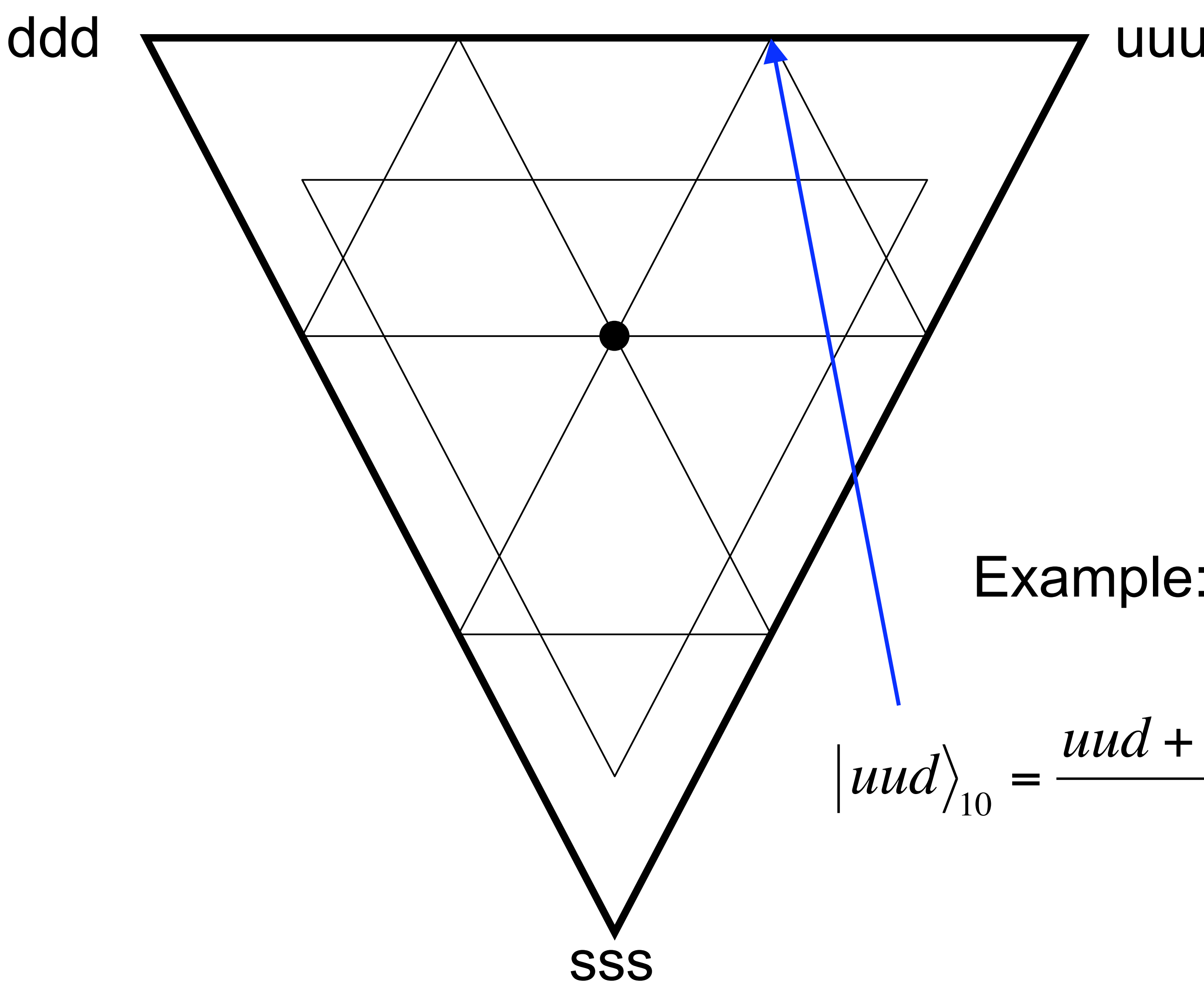
Symmetric di-quarks to triquarks



Symmetric di-quarks to triquarks



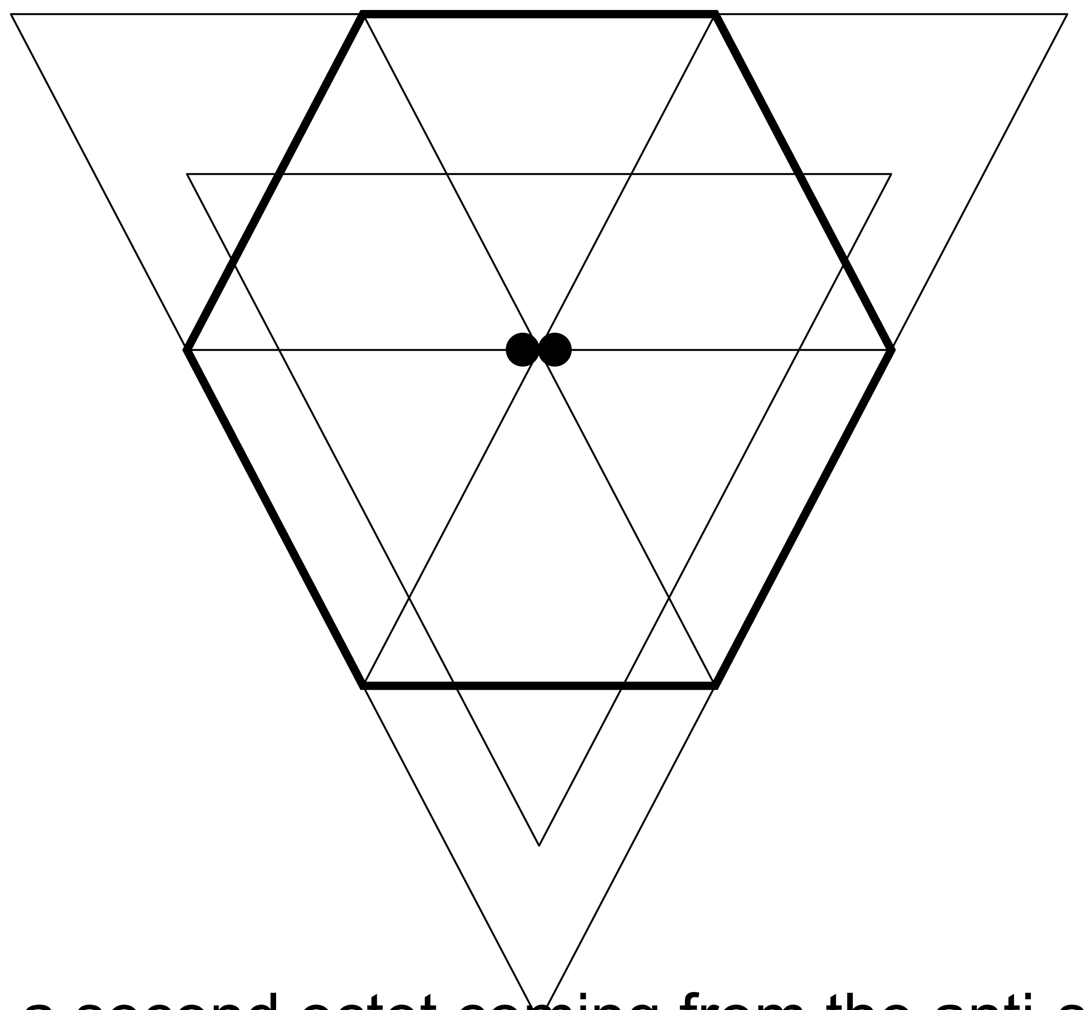
Symmetric 10-plet



Example:

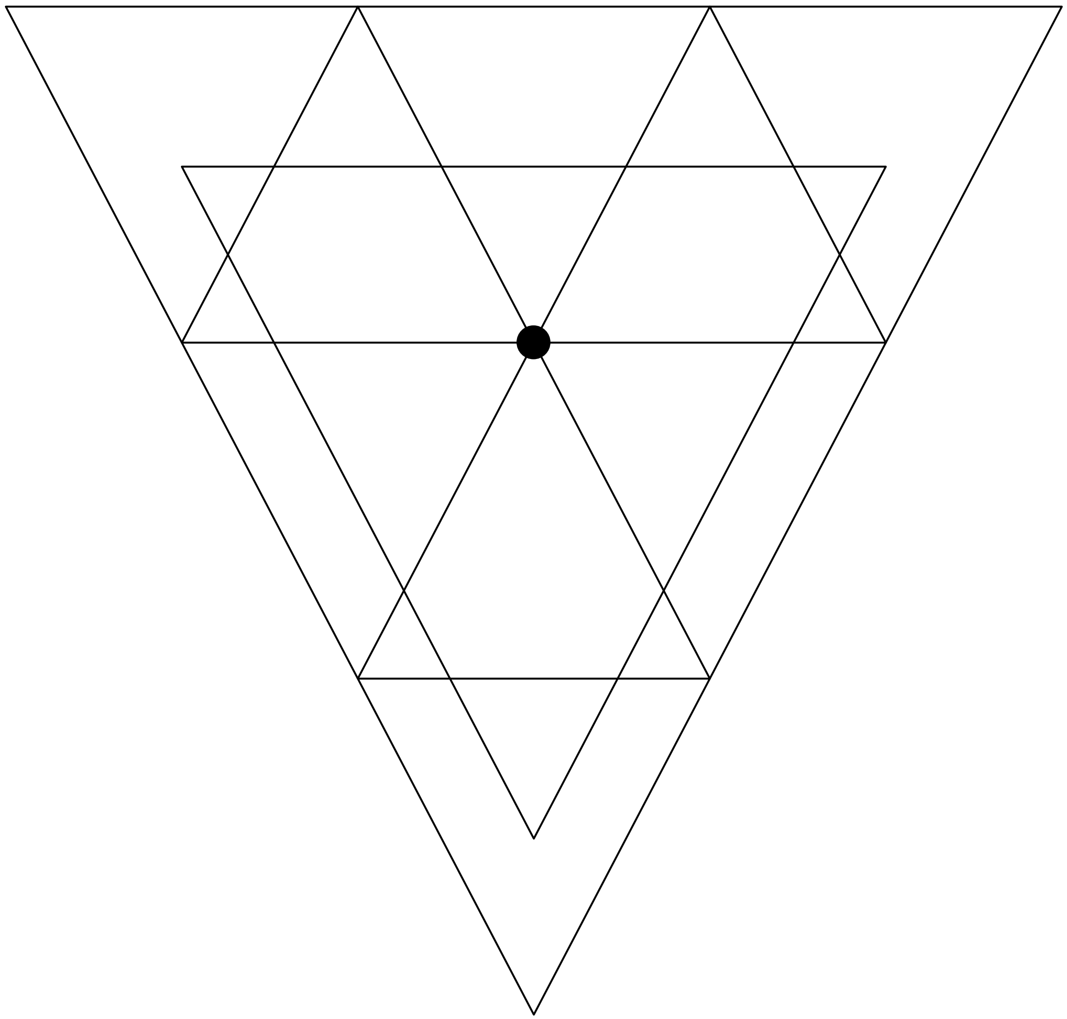
$$\left|uud\right\rangle_{10} = \frac{uud + udu + duu}{\sqrt{3}}$$

Octet with symmetric di-quarks



There’s a second octet coming from the anti-symmetric Di-quark triplet x quark triplet.

Singlet



Total wave function symmetry must be anti-symmetric with interchange of any two fermions

- (baryon) = (flavor) (spin) (space) (color)
- (color) = antisymmetric singlet of SU(3)

  ⇒(flavor) (spin) (space) = symmetric
- Spin:
  - $2 \times 2 \times 2 = (3_s + 1_A) \times 2 = 4_s + 2_{s12} + 2_{A12}$
- Ground state => L=0, symmetric
- Need to combine symmetric states from spin and flavor. I.e. not all combos valid.

Symmetric Spin-Flavor combos

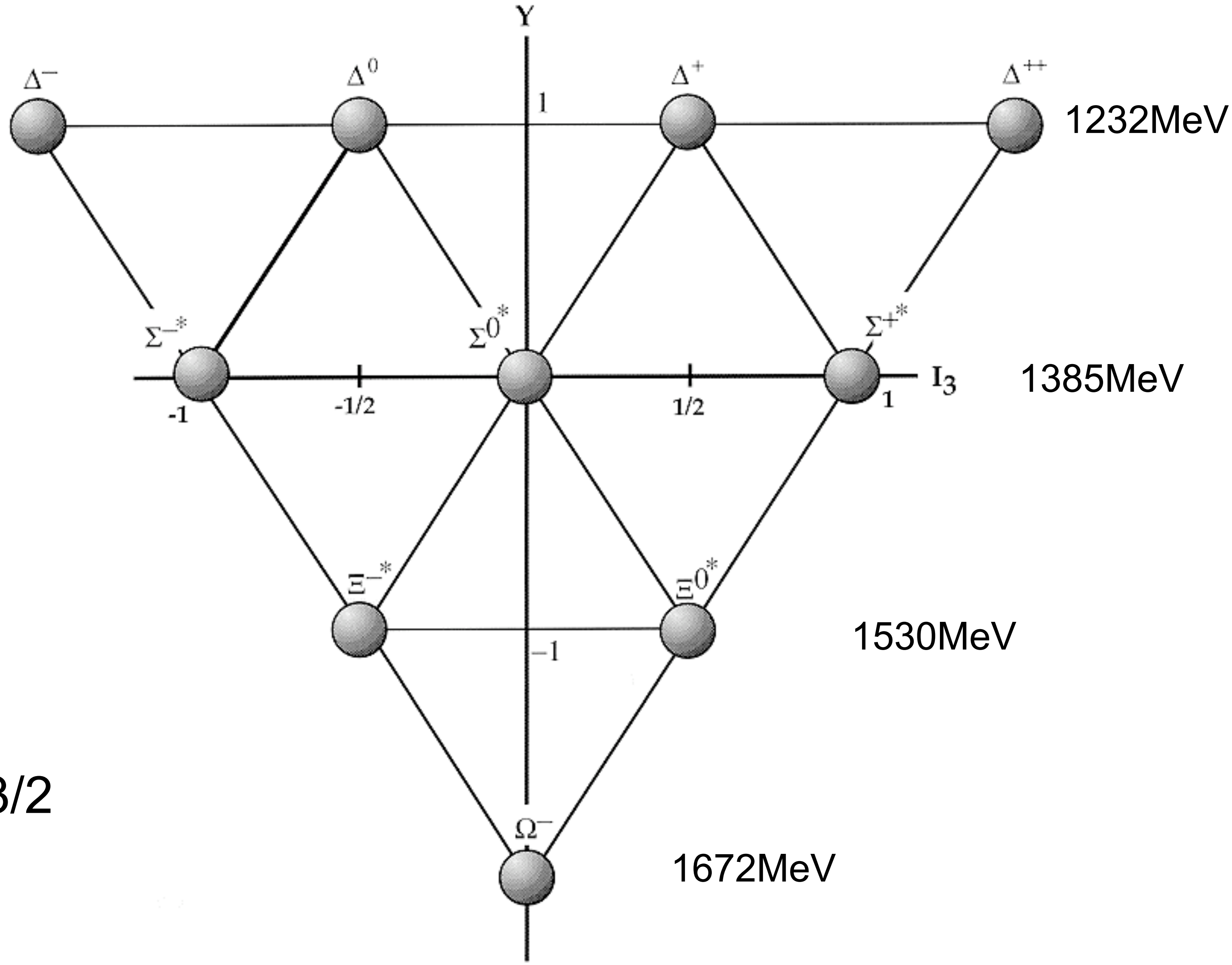
- 10 = symmetric
- 8 = S12
- 8 = A12
- 1 = symmetric

- 4 = symmetric
- 2 = S12
- 2 = A12

Possible options are thus:
Spin 3/2 decouplet
Spin 1/2 octet fully symmetric 1<->2
Spin 1/2 octet fully antisymmetric 1<->
Spin 1/2 spin 1<-> symmetric singlet

Color is necessary

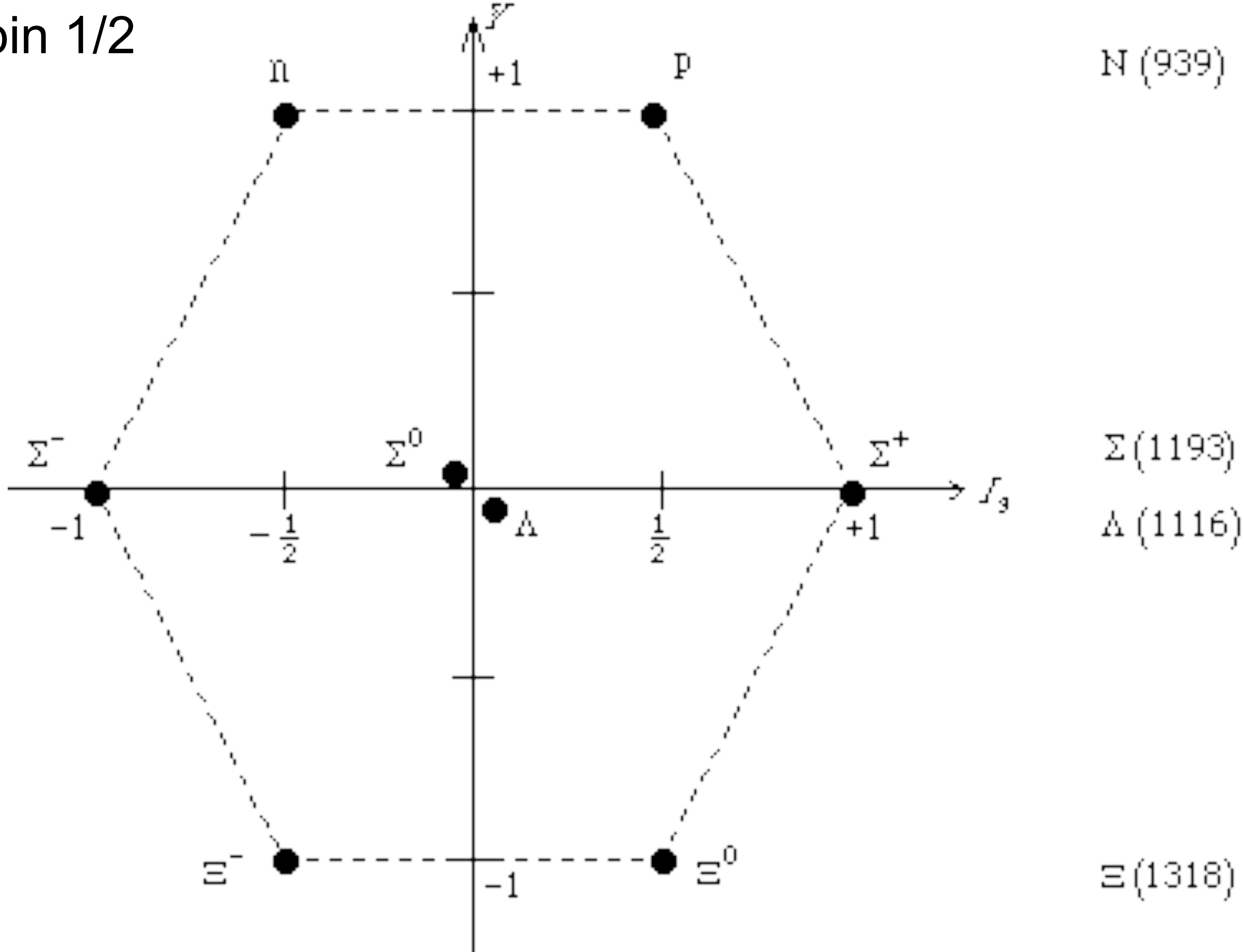
- If color did not exist then the Decouplet would have to be combined with the anti-symmetric spin 1/2.
- This would predict the uuu, ddd, sss baryons to have spin 1/2 instead of spin 3/2.



Spin 3/2

SU(3)- Dekuplett von Baryonen

Spin 1/2



Singlet: Spin 1/2 = /\(1405)