Physics 215A – Problem Set #1
due Wednesday Oct. 10, 2007

Read Srednicki 1,2

1. Consider a unitary linear operator U and an antiunitary antilinear operator U_A. Verify the multiplication table:

$$\begin{array}{ccc} & U & U_A \\ U & U & U_A \\ U_A & U_A & U \end{array}$$

i.e. the product of two unitary operators is a unitary operator, the product of an antiunitary and unitary operator is an antiunitary operator, etc.

2. Demonstrate that

$$\left(\Lambda^{-1}\right)^{\mu}{}_{\nu} = \Lambda_{\nu}{}^{\mu}$$

for a boost by $v\hat{z}$:

$$\Lambda^{\mu}{}_{\nu} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma v & 0 & 0 & \gamma \end{pmatrix}$$

and for a rotation about the $\hat{z}$ axis by angle θ:

$$\Lambda^{\mu}{}_{\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

An arbitrary Lorentz transformation Λ can be written as the product of an acceleration (boost) A and a rotation R,

$$\Lambda = AR \qquad .$$

Show that an arbitrary Lorentz transformation satisfies

$$\left(\Lambda^{-1}\right)^{\mu}{}_{\nu} = \Lambda_{\nu}{}^{\mu},$$

provided that A and R do separately.

3. The algebra of the generators $J^{\mu\nu}$ of the Lorentz group is

$$\left[J^{\mu\nu}, J^{\rho\sigma}\right] = i\left(\eta^{\nu\rho} J^{\mu\sigma} - \eta^{\mu\rho} J^{\nu\sigma} - \eta^{\nu\sigma} J^{\mu\rho} + \eta^{\mu\sigma} J^{\nu\rho}\right).$$

Show that the generators of rotations J^i and boosts K^i, defined by

$$J^i = \frac{1}{2}\epsilon^{ijk} J^{jk},$$

$$K^i = J^{0i},$$

satisfy the commutation relations

$$\left[J^i, J^j\right] = i\epsilon^{ijk} J^k,$$

$$\left[J^i, K^j\right] = i\epsilon^{ijk} K^k,$$

$$\left[K^i, K^j\right] = -i\epsilon^{ijk} J^k.$$

Note: Srednicki uses the notation $M^{\mu\nu} \equiv J^{\mu\nu}$ for the Lorentz generators.