Lecture 2

Particle collisions in plasma

Consider a completely ionized plasma with the ion charge Ze. Then in equilibrium plasma without external fields the main form of particle interactions is scattering in Coulomb fields. There are three types of elementary acts of scattering – scattering of electrons on electrons, electrons on ions and ions on ions.

Electron-ion collisions

In this case the scattering centers (ions) can be considered as motionless and probability of scattering over some angle is defined by Rutherford formula. An electron moving with some velocity v will scatter on many ions. Because of long range of Coulomb interaction, overwhelming majority of scattering acts take place at large distances, and lead to very small change of the trajectory direction (this is a feature of Rutherford scattering on point charges). The electric force acting on electron is a superposition of forces created by many ions and results in a very smooth change of its trajectory (Figure 1). This trajectory is strongly different from the test particle trajectory in non-ionized gas, where it consists of broken lines that connect the positions of collisions (Figure 2).

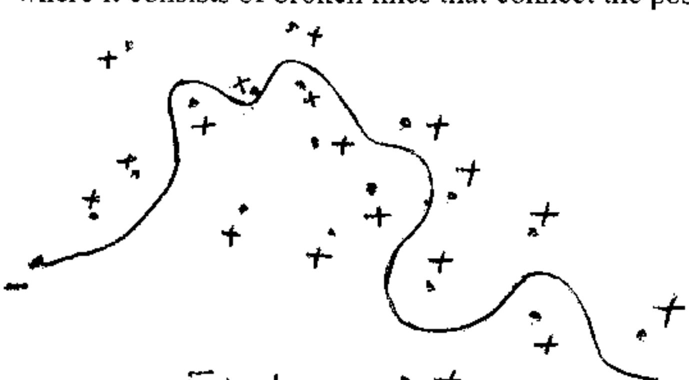


Fig 1

Fig. 2

Each scattering act due to Coulomb interaction of the test particle (electron) with scattering center leads to turn the particle trajectory on some angle θ, i.e. to decrease of the velocity component along initial direction of motion from v to $v\cos\theta$. It is natural to introduce the mean free path of the particle l_{mfp} as a distance at which the particle preserves the initial direction of its velocity. This definition is described by the equation

$$dv = -v\frac{dx}{l_{mfp}} \tag{1}$$

Here dv is an average change of the velocity component along initial direction of motion at the distance dx. Since $dx = vdt$, (1) can be written in the form of the equation with the effective friction force

$$m_e\frac{dv}{dt} = -m_e v\frac{v}{l_{mfp}} = -m_e v\nu \tag{2}$$

Here ν is so-called collision frequency. The friction force due to collisions is $F = -mv\nu$.

If the electron velocity vector turns at the angle θ due to one collisions, its component along initial direction decreases by $\Delta v = v(1-\cos\theta)$ (see Figure 3).

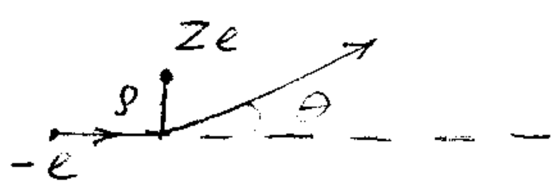


Fig.3

Since scatterings with small angles θ play the main role, we can write that $\Delta v \approx -v\theta^2/2$. The velocity change due to scattering on several (N) ions is equal

$$(\Delta v)_N = -\frac{v}{2}\sum_i \theta_i^2 \tag{3}$$

Where summation is carried out over all N scattering centers.

For small scattering angles the relation $\theta \approx v_\perp / v$ is correct, where $v_\perp$ can be found using the component of the equation of motion perpendicular to the initial direction of the electron velocity

$$m_e \dot{v}_\perp = Ze^2 \frac{\rho}{(\rho^2 + v^2 t^2)^{3/2}} \tag{4}$$

Here we assumed that the electron trajectory is almost straight line, ρ is an impact parameter.

Total change of $v_\perp$ in one collision is

$$\Delta v_\perp = \frac{2Ze^2}{m_e}\int_0^\infty \frac{\rho}{(\rho^2 + v^2 t^2)^{3/2}} dt \tag{5}$$

New variable $\tan y = \frac{v}{\rho} t$, $\qquad dt = \frac{\rho}{v}\frac{dy}{\cos^2 y}$

$$\Delta v_{\perp} = \frac{2Ze^2}{m_e \rho v} \int_0^{\pi/2} \cos y dy = \frac{2Ze^2}{m_e \rho v} \tag{6}$$

$$\theta \approx \frac{v_{\perp}}{v} = \frac{2Ze^2}{m_e \rho v^2} \tag{7}$$

At the distance dx, the electron meets many scattering centers with all possible impact parameters. We should carry out summation over all scattering centers to find total change of the velocity component along initial direction of motion dv at distance dx:

$$dv = -\frac{v}{2} \sum_i \theta_i^2 = -\frac{v}{2} dx n_0 \int \theta^2 dS_\rho \tag{8}$$

Here n_0 is the ion number density, dS_ρ is the area of the ring with radius ρ and width $d\rho$. All ions inside the ring have the same impact parameter (see Figure 4).

As a result, we obtain

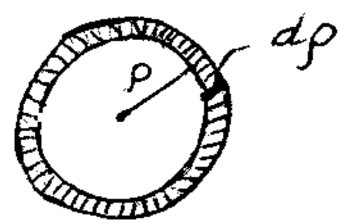


Fig. 4

$$dv = -\frac{v}{2} n_0 \frac{4Z^2 e^4}{m_e^2 v^4} dx 2\pi \int_{\rho_{\min}}^{\rho_{\max}} \frac{d\rho}{\rho} \tag{9}$$

Since integral over ρ diverges at both large and small ρ, it is necessary to use the cutting procedure. The values of $\rho_{\max}$ and $\rho_{\min}$ can be estimated from the following.

The ion electric field can be considered as Coulomb field only at distances less than Debye length λ_{De}. For larger distances the electric field decreases exponentially, so collisions with ions with the impact parameter larger than λ_{De} should be excluded from consideration and $\rho_{\max} = \lambda_{De}$.

As $\rho_{\min}$ we can choose the value of an impact parameter for which the scattering angle $\theta \approx 1$, so the approximation of small angles is broken. For such an impact parameter the electron kinetic energy is on the order of potential energy of Coulomb interaction

$$\frac{Ze^2}{\rho_{\min}} = \frac{m_e v^2}{2} \qquad \Rightarrow \qquad \rho_{\min} = \frac{2Ze^2}{m_e v^2} \tag{10}$$

Then

$$\int_{\rho_{\min}}^{\rho_{\max}} \frac{d\rho}{\rho} \approx \ln\left(\frac{\lambda_{De}}{Zb}\right) \approx \ln\left(\frac{2\pi}{Z} n_0 \lambda_{De}^3\right) = \ln\Lambda \tag{11}$$

($b = e^2/k_B T$ is classical distance of closest approach) is so called Coulomb logarithm. The quantity under logarithm is very large (in all cases from 10^5 to 10^{13}), and the value of the Coulomb logarithm is $\approx 10 \div 20$. Using the definition the mean free path (1) and relations (9), (11) one can obtain the expression for l_{mfp}:

$$l_{ei} = \frac{m_e^2 v^4}{4\pi Z^2 e^4} \frac{1}{n_0} \frac{1}{\ln\Lambda} \tag{12}$$

An average electron mean free path due to collisions with ions l_{ei} can be obtained by averaging (12) over the electron maxwellian distribution function

$$l_{ei} = 4.5 \times 10^5 \frac{T_e^{\circ 2}}{n_0} \frac{1}{Z^2} \frac{1}{\ln \Lambda} \tag{12'}$$

Here T_e° is the electron temperature measured in Kelvin degrees.

Other average characteristics of collision processes of electrons with ions:

a. Effective cross-section is determined by the relation

$$l_{ei} = \frac{1}{n_0 \sigma_{ei}}$$

b. The average time between two collisions $\tau_{ei} = l_{ei} / v_{Te}$, where v_{Te} is the electron thermal velocity.

c. The collision frequency $\nu_{ei} = 1/\tau_{ei}$. It can be calculated using the following formula

$$\nu_{ei} \approx 20 \frac{n_0 (cm^{-3})}{(T_e^\circ)^{3/2}} \tag{13}$$

Plasma parameter

Let us find the ratio of the plasma frequency to the electron-ion collision frequency

$$\frac{\omega_{pe}}{\nu_{ei}} = \frac{\omega_{pe} l_{ei}}{v_{Te}} \approx \frac{4\pi n_0}{Z^2 \ln \Lambda} \frac{v_{Te}^3}{\omega_{pe}^3} \approx \frac{4\pi}{Z^2 \ln \Lambda} N \tag{14}$$

Here the parameter

$$N = n_0 \lambda_{De}^3 \tag{15}$$

is approximately equal to the particle number in Debye shielding sphere. It is so called plasma parameter. If this parameter is large, then collisions don't affect electron plasma oscillations. In this case collective interactions of charge particles via fields created by these particles mainly determine dynamics of plasma. Collective behavior of charged particles is the most important feature of plasma.

Sometimes the plasma parameters is defined as

$$g = \frac{1}{N} = \frac{1}{n_0 \lambda_{De}^3} \tag{15'}$$

The parameter g should be small for collective interaction of plasma particles to control the plasma behavior.

Large value of ratio (14) doesn't mean that collisions are not important in plasma dynamics. They are important for low frequency oscillations of plasma, heating of plasma and its diffusion. For example in tokamak plasma ($n_0 \approx 10^{13}, T \approx 10^7, \nu_{ei} \approx 10^4$) there are several thousands of collision during confinement time (~1 sec).

Classification of different plasmas

Parameters N and g characterize also the ideality of plasma as an ionized gas. Compare the kinetic thermal energy with an average energy of electrostatic interaction. The average distance to the neigbouring particle $r \approx (1/n_0)^{1/3}$, therefore the interaction energy is approximately equal to $e^2 n_0^{1/3}$. Ratio of this energy to the thermal energy

$$\frac{e^2 n_0^{1/3}}{k_B T} = \frac{1}{4\pi}\frac{4\pi e^2 n_0}{T}\frac{1}{n_0^{2/3}} = \frac{1}{4\pi}\frac{1}{N^{2/3}} \tag{16}$$

It is much less than unity if N is sufficiently large. So plasma with $N >> 1$ ($g << 1$) can be considered as ideal gas where potential energy of electrostatic interaction is much smaller than kinetic energy of thermal motion. Such types of ideal plasmas will be subject of our study.

If N<<1, plasma is non-ideal ionized gas with strong correlation between particles created by collisions. Such a plasma is similar to liquid that has a very complicated or even unknown equation of state.

We can find the boundary between ideal and non-ideal plasma by putting

$$n_0 \frac{v_{Te}^3}{\omega_{pe}^3} = 1$$

Using approximate relations $v_{Te} \approx 3\times 10^5 (T^\circ)^{1/2}$ and $\omega_{pe} \approx 5.6\times 10^4 (n(1/cm^3))^{1/2}$, we obtain that the boundary is determined by the relation

$$T^\circ \approx \frac{1}{1.5}(n(cm^{-3}))^{1/3}$$

With further increase of plasma density, plasma is going to metallic state.

Quantum effects in plasma

1. de Broglie wave length can be comparable with the mean distance between particles

$$\lambda \approx \frac{\hbar}{m_e v_{Te}} \approx n_0^{-1/3}$$

For smaller temperatures or larger densities electron gas in plasma starts to be degenerate. Electrons are described by Fermi-Dirac distribution instead of Maxwellian distribution. The boundary between classical and degenerate plasmas:

$$T^\circ \approx 1.5 \times 10^{-10} n_0^{2/3} (cm^{-3})$$

2. The energy quantum of plasma oscillations is compared with thermal (Fermi) energy

$$\hbar\omega_{pe} \approx k_B T \qquad \Rightarrow \qquad \frac{\hbar}{m_e v_{Te}} \approx \lambda_{De}$$

In this case de Broglie wave length is comparable with Debye length. The relationship between the plasma temperature and density can be written in the following form

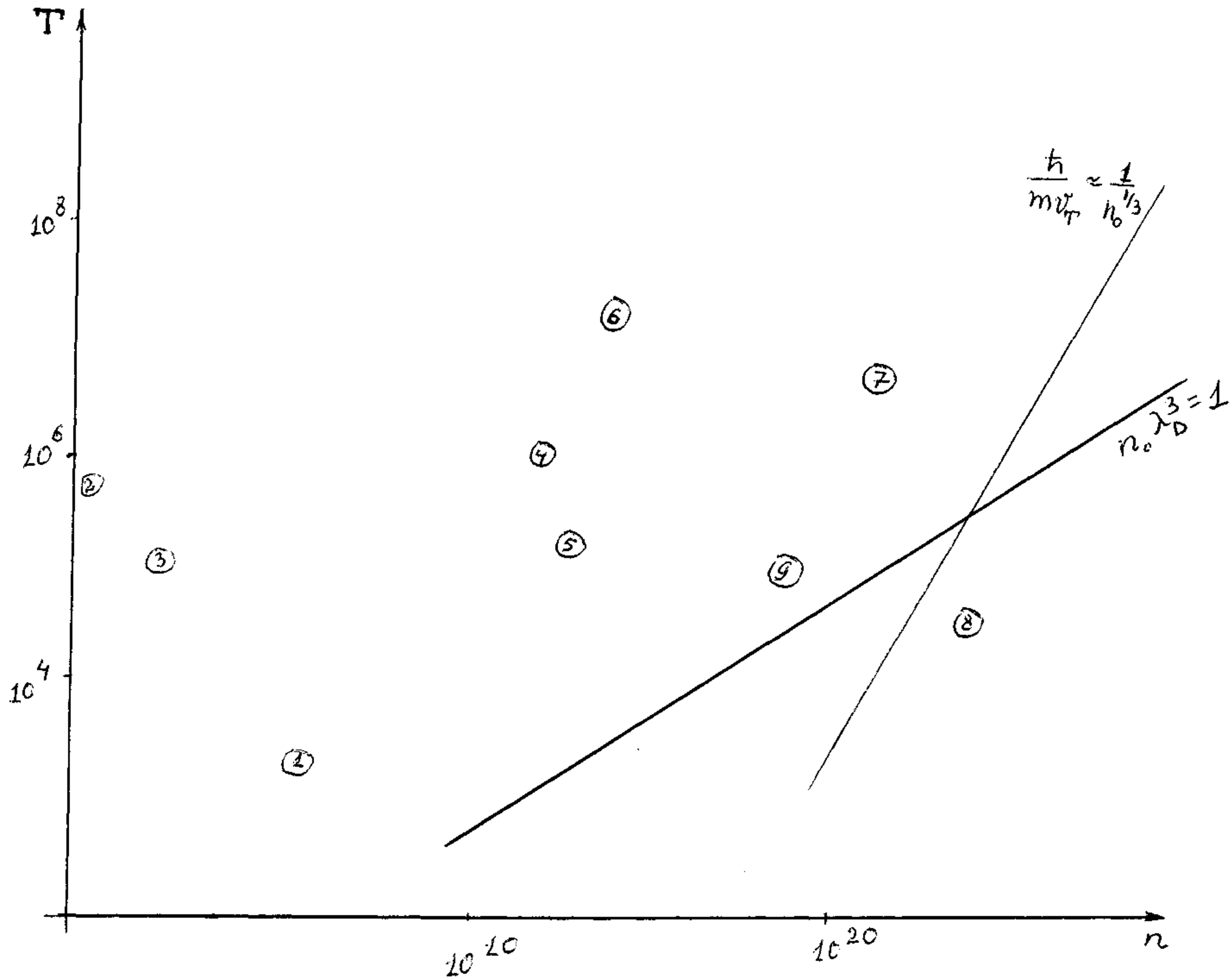
$$T^\circ \approx 2.7 \times 10^{-6} \sqrt{n_0 (cm^{-3})}$$

According to Pauli's principle, two electrons with the same spins cannot be located at the same point in space. Therefore potential energy of electrostatic repulsion as well as a returning force in plasma oscillations somewhat decrease.

As a result the dispersion relation for plasma waves is slightly different

$$\omega^2 = \omega_{pe}^2 + \frac{3}{5}(1 - a r_s) k^2 v_F^2$$

Here v_F is the Fermi velocity, a is a numeric coefficient (~0.06), r_s is dimensional parameter that characterizes the ratio of the interaction energy to Fermi energy.



T
10^8
10^6
10^4
10^{10}
10^{20}
n
$\frac{\hbar}{m v_T} \approx \frac{1}{n_0^{1/3}}$
$n_0 \lambda_D^3 = 1$
1
2
3
4
5
6
7
8
9

Space plasmas

1 – ionosphere

2 - hot interstellar media

3 - solar wind plasma

4 – solar corona

"Earth" plasmas

5 - gas discharge, typical plasma of laboratory experiment

6 – tokamak plasma

7 – inertial fusion plasma

8 – plasma in metals (gas of free electrons)

9 – plasma in semiconductors

Particle scattering by waves

The similar scattering of the test electron takes place in electric fields of space charge oscillations excited in plasma by some source. As before we can introduce the mean free path using the same formula (1) where dv is determined with the same relation

$$dv = v\cos\theta \approx -\frac{v}{2}\theta^2, \qquad \theta^2 \approx \frac{v_\perp^2}{v^2}$$

But now scattering of the test electron is conducted by electric fields, oscillating with electron plasma frequency.

$$\frac{dv_\perp}{dt} = \frac{e}{m_e}E_\perp \quad => \quad v_\perp = \frac{eE_\perp}{m_e\omega_{pe}}$$

$$\left\langle\theta^2\right\rangle = \frac{e^2\left\langle E_\perp^2\right\rangle}{m_e^2\omega_{pe}^2 v^2} = \frac{2}{3}\frac{e^2\left\langle E^2\right\rangle}{m_e^2\omega_{pe}^2 v^2} \tag{21}$$

Brackets mean averaging over fast oscillations. We assumed here isotropy of plasma oscillations. It is easy to find that $\left\langle E^2\right\rangle = E^2/2$.

As a result we obtain for dv the following relation:

$$dv = -\frac{v}{2}\left\langle\theta^2\right\rangle = -\frac{v}{6}\frac{e^2E^2}{m_e^2\omega_{pe}^2 v^2} = -v\frac{dx}{l_{mfp}} \tag{22}$$

The minimal distance at which such interaction will take place is on the order of several Debye lengths, i.e. of the order of the shortest wavelength of plasma waves $dx = v/\omega_{pe}$.

Then

$$l_{mfp} = 6\frac{m_e^2 \omega_{pe} v^3}{e^2 E^2} \tag{23}$$

the stronger oscillations are the faster is the test particle scattering.

Scattering by thermal noise

In a state of thermal equilibrium there are always fluctuations of space charge in plasma that lead to existence of plasma oscillations at very low level, so called thermal noise. Electric field of oscillations is determined by equation

$$V\frac{E_{noise}^2}{8\pi} = \frac{k_B T}{2} n_p \tag{24}$$

Here V is plasma volume, $k_B T/2$ is the energy of every quantum of plasma oscillations (plasmon). We used here hypothesis about energy equipartition – in the thermal equilibrium, the energy of every degree of freedom (particle or wave) is $k_B T/2$.

In formula (19) n_p is a number of plasma waves

$$n_p = \frac{(4\pi/3)k_{\max}^3}{(\Delta k)^3} \tag{25}$$

The numerator is the total volume of oscillations in k-space, where $k_{\max}$ is maximal wave number. As we discussed before, the phase velocity of electron plasma oscillations must be larger than electron thermal velocity to avoid strong Landau damping on thermal particles

$$\frac{\omega}{k} > v_{Te} \quad \rightarrow \quad k < \frac{1}{\lambda_{De}} \quad \rightarrow \quad k_{\max} \approx \frac{1}{3\lambda_{De}}$$

The denominator in formula (25) is the smallest volume in k-space for plasmon with $\Delta k = 2\pi/a$, a is the linear dimension of plasma.

$$n_p \approx \frac{1}{50\pi^2}\frac{V}{\lambda_{De}^3} \tag{25'}$$

Using (24) and (25') we obtain the energy density of thermal noise

$$\frac{E_{noise}^2}{4\pi} \approx \frac{1}{50\pi^2}\frac{k_B T}{\lambda_{De}^3} \tag{26}$$

that is much less than thermal energy density of plasma

$$\frac{E_{noise}^2}{8\pi n_0 k_B T} \approx \frac{1}{n_0\lambda_{De}^3} = g << 1 \quad (27$$

Substituting E_{noise}^2 in equation (23) for the electron mean free path, we obtain for scattering of the thermal electron ($v \approx v_{Te}$)

$$l_{mfp} \approx 60\pi \frac{m_e^2 \omega_{pe} v_{Te}^2}{e^2}\frac{\lambda_{De}^3}{k_B T} \approx 60\pi \frac{m_e^2 v_{Te}^4}{4\pi n_0 e^4} = 60 l_{ei} \ln\frac{\lambda_D}{\rho_{\min}} >> l_{ei} \tag{28}$$

Thus, electron scattering on thermal fluctuations is weaker than Coulomb collisions.

If there is some energy source for the excitation of plasma waves (e.g. electron beam) then energy of plasma waves will exceed that of thermal noise by many order of magnitude. Then scattering of particles by waves will become much stronger than Coulomb collisions. This gives an explanation of so-called Langmuir paradox – fast scattering of electron beam in plasma by self-excited plasma waves.