Lecture 3

3.1 Transport phenomena in plasma

After introducing the mean free path l_{mfp}, collision frequency ν etc. in plasma, we can construct a qualitative picture of transport processes in plasma. For example, motion of charged particles in plasma is of stochastic, diffusion type. We can introduce the diffusion coefficient as

$$D \approx v_T^2 \tau \qquad (1)$$

to describe diffusion of charged particles, e.g. the electron diffusion in weakly ionized plasma in which neutral particles play a role of scattering centers.

The most important feature of plasma is its ability to conduct current under influence of electric field. If spatial scales of plasma motion is much larger than the mean free path $L >> l_{mfp}$ and the characteristic time of motion is much larger than the collision time $T >> \tau$, collisions should play an important role.

We can consider the plasma ions as motionless and assume that current I is conducted by electrons. In the simplest case when $I = const$, an equilibrium should be established between the accelerating force (by electric field) and decelerating one (due to electron-ion collisions):

$$-e\vec{E} - m_e \nu_{ei} \vec{u} = 0 ===\Rightarrow \vec{u} = -\frac{e\vec{E}}{m_e \nu_{ei}} \qquad (2)$$

Then the current density is equal to

$$\vec{j} = -en_0\vec{u} = \frac{e^2 n_0}{m_e \nu_{ei}} \vec{E} \equiv \sigma\vec{E} \qquad (3)$$

The relationship (3) is the **Ohm's law** for plasma.

Here

$$\sigma = \frac{e^2 n_0}{m_e \nu_{ei}} \qquad \sigma \approx 10^7 (T_e^\circ)^{3/2} s^{-1} \tag{4}$$

is an electric conductivity of plasma.

The expression (4) is applicable for completely ionized single-charged-ion plasma ($Z = 1$, hydrogen plasma). For $T_e \approx 10^8$, $\sigma \approx 10^{19} s^{-1}$, is more than order of magnitude larger than σ_{copper} for the room temperature. The presence of ions with $Z > 1$ decreases strongly the plasma electric conductivity:

$$\sigma \approx 10^7 \frac{\sum \alpha_k Z_k}{\sum \alpha_k Z_k^2} (T_e^\circ)^{3/2} s^{-1} \tag{5}$$

Here $\alpha_1, \alpha_2, \ldots$ are relative concentrations of ions with charges $Z_1, Z_2, \ldots$.

In weakly ionized plasma, electrons collide more frequently with neutrals, and so

$$\sigma = \frac{e^2 n}{m_e \nu_{en}} \tag{6}$$

Since the electro-neutral collision frequency is proportional to the density of neutrals n_0, the plasma conductivity is proportional to the ratio n/n_0 and depends strongly on the ionization degree of plasma.

The Ohm's law is applicable if equilibrium is established in plasma between electric forces and friction forces acting on electrons. However, should such equilibrium be establish in any conditions?

Let us consider a completely ionized plasma. We know that the decelerating force acting on an electron in completely ionized plasma is smaller for faster electrons. Let us consider the behavior of an electron that belongs to the tail of Maxwellian distribution with energy $w_e >> k_B T_e$ in electric field. The velocity increment of electron u between two collisions with ions is equal:

$$u \approx \frac{eE}{m_e}\tau_{ei} \tag{7}$$

If the electron velocity v is sufficiently large, the velocity increment u can be of the order of or even larger than v. In this situation the simplified model, in which electron acquires relatively small velocity on mean free path and loses it during the collision, doesn't work.

In real plasma, acceleration and deceleration take place at the same time. Electric field seeks to straighten the electron trajectory while Coulomb scattering on ions bend it. If the velocity increment is not compensated by scattering, then the equilibrium of forces is not established and electron should become continuously accelerated. Its energy increases and the friction force decreases, hence electron will continue to accelerate. These electrons are so-called run-away electrons. Electrons will become run-away electrons, if on the mean free path they acquire velocity u larger than their initial velocity

$$eE > m_e \nu_{ei} v \quad \Rightarrow \quad \frac{eE}{m_e}\tau_{ei} > v \tag{8}$$

Since $\tau_{ei} \sim \frac{v^3}{n}$, continuous acceleration takes place when the quantity

$$\frac{E w_e}{n} > C$$

C is its critical value. For hydrogen plasma this condition is:

$$\frac{E(V/cm)\,w(eV)}{n(cm^{-3})} > 3\times 10^{-12} \tag{9}$$

In laboratory experiments with plasma the relationship (9) is fulfilled for electrons with energy much larger than $k_B T_e$, which are a small part of total electrons. Since the main input in the current is due to thermal electrons, Ohm's law is applicable to such plasmas. For larger values of E/n, condition (9) will be fulfilled even for thermal electrons and in this case condition (9) can be written in the form

$$E > E_{Dreicer}, \tag{10}$$

where $$E_{Dreicer} = \frac{m_e \nu_{ei}(T_e)}{e} v_{Te} \tag{10'}$$

Condition (10) has obvious physical meaning. For $E > E_{Dreicer}$, electron acquires the energy larger than its thermal energy during the time between two collisions. As a result, the electron-ion collision frequency decreases ($\nu_{ei} \sim 1/v^3$) and the friction force decreases as well. This force is not capable to hinder the electric force eE and all electrons turn into run-away electrons.

More profound analysis of behavior of accelerated electrons shows that such electron flow can excite waves in plasma giving them part of electron energy. As a result, new mechanism of electron deceleration appears after electrons acquire some energy of directed motion. So-called anomalous resistivity will be created due to this mechanism of electron deceleration.

3.2 Plasma in a high frequency field

Peculiar features of plasma can be seen if we consider its behavior in a high frequency field. Let us study the plasma motions in a high frequency electric field of a plane electromagnetic wave $E_x = E_0 \exp(-i\omega t + ikz)$.

If $\omega/\nu_{ei} >> 1$, we can neglect the friction force. We assume that ions are motionless.

For electrons:

$$m_e \ddot{x} = -eE_0 e^{-i\omega t + ikz} \tag{11}$$

where x -axis is in direction of electric field.

Integrating (11) we have

$$u_e \equiv \dot{x} = -i\frac{eE_0}{m_e\omega}\exp(-i\omega t + ikz) \qquad (12)$$

The current density

$$j = -en_0u_e = i\frac{n_0e^2}{m_e\omega}E \qquad (13)$$

Let us compare (13) with formula (3)

$$j = \frac{n_0e^2}{m_e\nu_{ei}}E$$

that was obtained for collisional plasma in constant electric field. In contrast to the tatter case, the electric field in the former case is shifted over phase at -90° in comparison with current. This means that plasma possesses a self-inductance due to the electron inertia.

By integrating (12) over time we obtain the electron displacement

$$x = \frac{e}{m_e\omega^2}E_0e^{-i\omega t+ikz} \qquad (14)$$

that is shifted over phase at 180° relative to the acting force ($F = -eE$).

With such a shift the polarization of the medium directed against the field and permittivity $\varepsilon < 1$

$$\varepsilon = 1 + \frac{4\pi P}{E} \tag{15}$$

$$P = -n_0 e x \qquad \Rightarrow \qquad \varepsilon = 1 - \frac{\omega_{pe}^2}{\omega^2} \tag{16}$$

Refraction index for an electromagnetic wave:

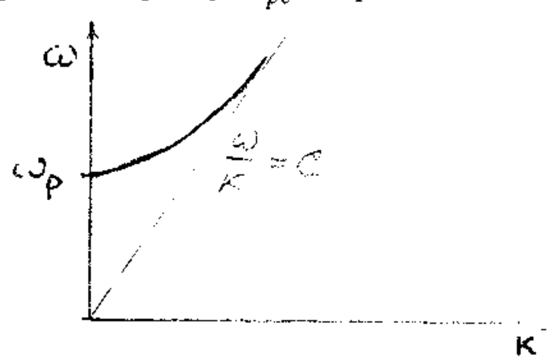
$$N = \sqrt{\varepsilon} = \left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)^{1/2} \tag{17}$$

where $N = kc/\omega$

Equation (17) that can be written in the form

$$\omega^2 = \omega_{pe}^2 + k^2 c^2 \tag{17'}$$

is a dispersion relation for a high frequency electromagnetic wave in plasma. It follows from it that electromagnetic wave cannot propagate in plasma if its frequency is less than electron plasma frequency ω_{pe}. Dispersion curve is shown in Figure 1.



There is no Landau damping of electromagnetic wave in plasma since its phase velocity

$$\frac{\omega}{k} = \frac{c}{(1 - \omega_{pe}^2/\omega^2)^{1/2}}$$

is larger than speed of light, therefore there is no resonant particles for such a wave.

Mechanism of t damping of electromagnetic waves in plasma is electron deceleration due to collisions. By adding the friction force in r.h.s. of (14), we obtain

$$m_e\ddot{x} = -eE_0 e^{-i\omega t+ikz} - m_e\dot{x}\nu_{ei} \tag{18}$$

Assuming $x = A\exp(-i\omega t + ikz)$, we get

$$x = \frac{e}{m_e\omega(\omega + i\nu_{ei})}E_0 e^{-i\omega t+ikz} \tag{19}$$

Using (15) and (16), we obtain

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu_{ei})} \tag{20}$$

In ideal plasma $\omega >> \nu_{ei}$. Hence, (20) can be written in the form

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega^2} + i\frac{\omega_{pe}^2\nu_{ei}}{\omega^3} \tag{21}$$

Until now we considered the electron motion in the wave electric field in linear approximation. In this approach, electrons oscillate in direction of the electric field with velocity (12) and displacement (14). In the next order over the wave amplitude, the averaged effect of oscillations in the field of the wave with non-homogeneous amplitude leads to expulsion of electrons out of the regions occupied by electromagnetic field. The physical meaning of this expulsion can be explained in the following way.
Electromagnetic field leads to electron oscillations with velocity $u(t)$ and create additional high-frequency pressure

$$P_{h.f.} \sim nm_e u^2$$

As a result of this, electrons move to the field minimum.

Consider a standing electromagnetic wave. The wave electric and magnetic fields are in plane perpendicular to the direction of non-homogeneity (z -axis):

$$E_x = \frac{1}{2} E_x(z) e^{-i\omega t} + c.c.$$

$$B_y = \frac{1}{2} B_y(z) e^{-i\omega t} + c.c. \qquad (22)$$

From $\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \frac{dE_x}{dz} = i\frac{\omega}{c} B_y$ (23)

Electrons participate in two types of motion (i) fast oscillations along $\vec{E}$ and (ii) slow motion along non-homogeneity:

$$\vec{r} = \frac{1}{2}\vec{e}_x x e^{-i\omega t} + c.c. + \vec{e}_z z(t) \qquad (24)$$

Equation o motion along z-axis:

$$m_e \ddot{z} = \left\langle -\frac{e}{c} u_x B_y \right\rangle = -\frac{1}{4}\frac{e}{c}\left\langle (u e^{-i\omega t} + c.c.)(B_y e^{-i\omega t} + c.c.) \right\rangle \qquad (25)$$

We took into account that

$$u = \frac{1}{2} u(z) \exp(-i\omega t) + c.c.$$

where $u(z)$ is defined by (12).

Averaging in the r.h. side of (25) is over fast oscillations

$$\langle f(t) \rangle = \frac{1}{T}\int_0^T f(t) dt, \qquad T = \frac{2\pi}{\omega}$$

By averaging the r.h. side of (25), we obtain

$$m_e \ddot{z} = -\frac{e^2}{4m_e\omega^2}\left(E_x \frac{\partial E_x^*}{\partial z} + c.c.\right) = -\frac{e^2}{4m_e\omega^2}\frac{\partial |E_x|^2}{\partial z} \tag{26}$$

This equation means that in the field of an electromagnetic wave with non-homogeneous amplitude, a force of high-frequency pressure acts on electrons. This force is antiparallel to the electric field gradient.

$$P_{h.f.} = \frac{e^2}{4m_e\omega^2}|E_x|^2 \tag{27}$$

$$F_{h.f.} = -\frac{\partial P_{h.f.}}{\partial z} \tag{28}$$