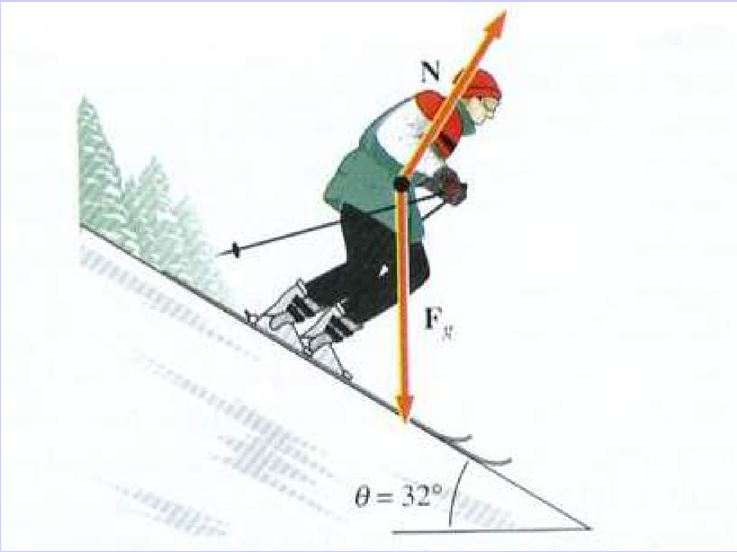
Today's Lecture

Lecture 10: Chapter 6,
Using Newton's Laws
Lots of Examples

First Example: Skier



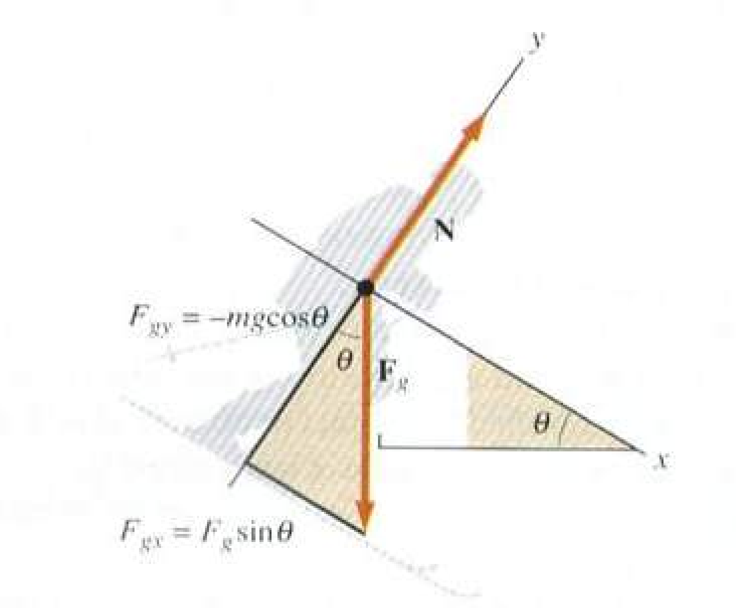
A skier with mass ***m = 65kg*** slides down A slope with an incline angle of **θ = 32º**. Find (a) skier's acceleration and (b) the magnitude of the force that the snow (slope) exerts on the skier.

We now have a choice of coordinate system. Do we choose the *x* axis to be horizontal or parallel to the slope?

Since the acceleration is parallel to the slope and there is no acceleration perpendicular to the slope, it is more convenient to choose our *x* axis to be **parallel to the slope** and subsequently the ***y*** **axis perpendicular to the slope**.

For this problem we choose to ignore any effects of friction and our Free-Body consists of only two forces, the force of gravity, $\boldsymbol{F_g}$, and the normal force, ***N***, of the snow on the skier.

First Example: Skier



A skier with mass ***m = 65kg*** slides down A slope with an incline angle of ***θ* = 32º**. Find (a) skier's acceleration and (b) the magnitude of the force that the snow (slope) exerts on the skier.

With this choice of coordinates the Free-Body diagram takes on the form shown in the figure.

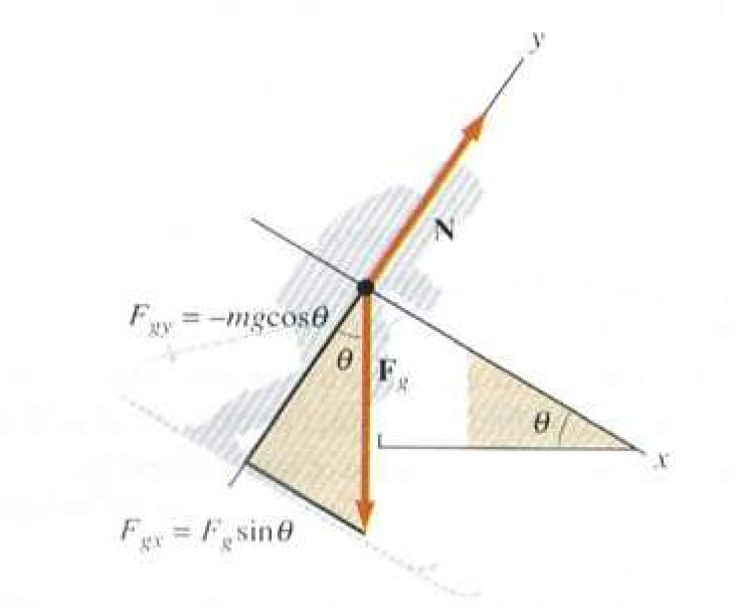
With the use of a little trigonometry the two EOM's (*x* and *y* directions) are

$$F_g = \sin\theta = mg\sin\theta = ma_x$$

and

$$N - F_g\cos\theta = N - mg\cos\theta = ma_y$$

First Example: Skier



A skier with mass *m = 65kg* slides down A slope with an incline angle of $\boldsymbol{\theta} = \mathbf{32^o}$. Find (a) skier's acceleration and (b) the magnitude of the force that the snow (slope) exerts on the skier.

Recognizing that there is no acceleration in the *y* directon, $\boldsymbol{a_y = 0}$, the two EOM's,

$$F_g = \sin\theta = mg\sin\theta = ma_x$$

and

$$N - F_g\cos\theta = N - mg\cos\theta = 0$$

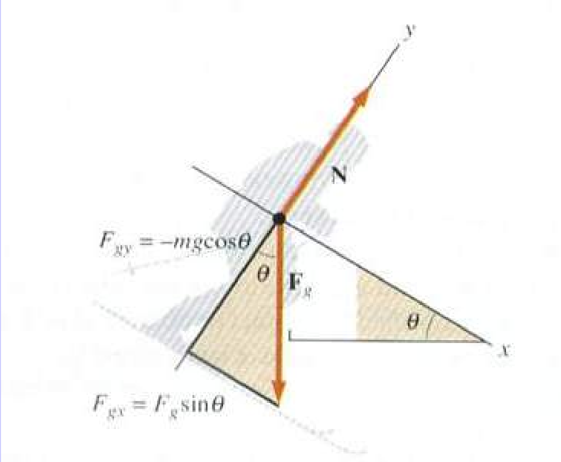
yield the solutions:

(a) $a_x = g\sin\theta = 9.8\sin 32^\circ = 5.19 m/s^2$

(b) $N = mg\cos\theta = 65(9.8)\cos 32^\circ = 540N$

Like a falling body, the acceleration down the hill is independent of mass!

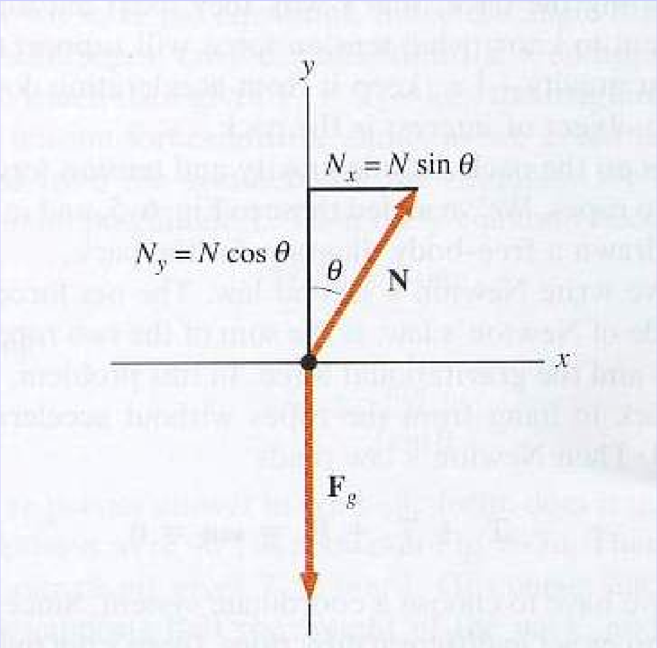
First Example: Skier



It is worthwhile to reemphasize that there is only one vector equation here, namely:

$$\vec{N} + \vec{F}_g = m\vec{a}$$

We could have solved this problem with a coordinate system in which the *x* axis is horizontal and the *y* axis vertical.



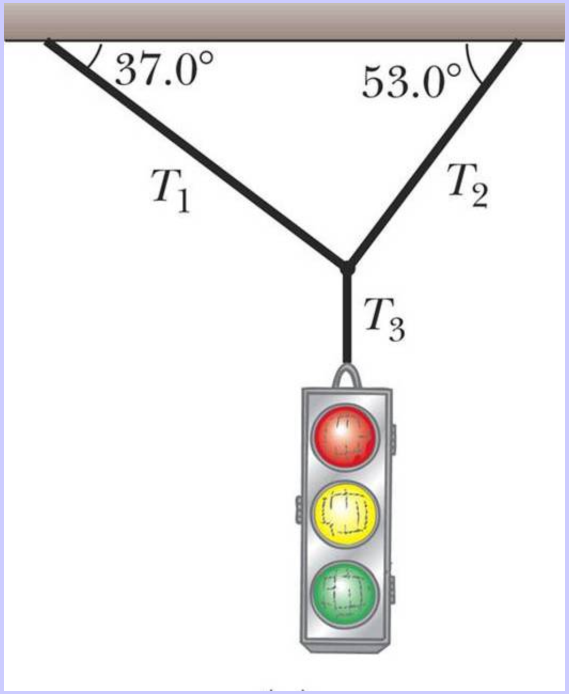
Now the gravitational force simplifies but both The normal force and acceleration have components in both directions:

$$N_x = N\sin\theta = ma_x = ma\cos\theta$$

$$N_y - mg = ma_y = -ma\sin\theta$$

Our original choice of coordinates clearly simplified the problem, $\boldsymbol{N_x}$ and $\boldsymbol{a_y}$ were zero!

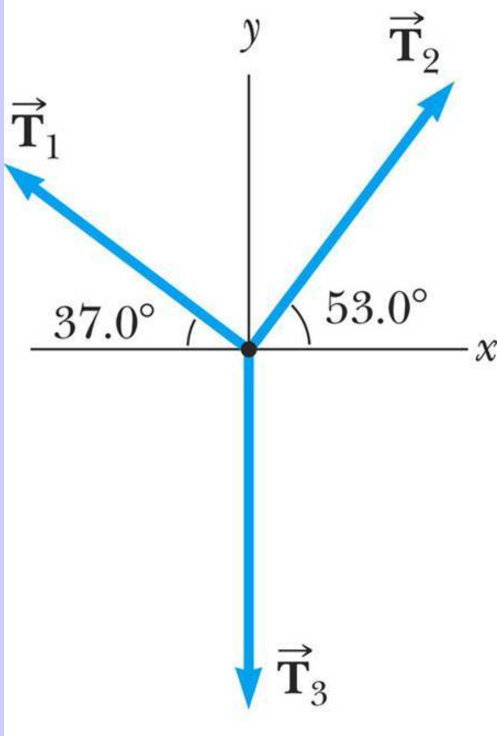
Second Example: Equilibrium in 2-D



For this case there is no acceleration and Newton's second law for the forces at the knot reads:

$$\vec{T}_1 + \vec{T}_2 + \vec{T}_3 = m\vec{a} = 0$$

Free-body diagram at the knot:



$$T_{2x} - T_{1x} = T_2 \cos 53^\circ - T_1 \cos 37^\circ = 0$$

$$T_{2y} + T_{1y} - T_3 = T_2 \sin 53^\circ + T_1 \sin 37^\circ - T_3 = 0$$

There are two equations and three unknown tensions. Did we miss something?

Of course, the free body diagram of the stoplight itself.

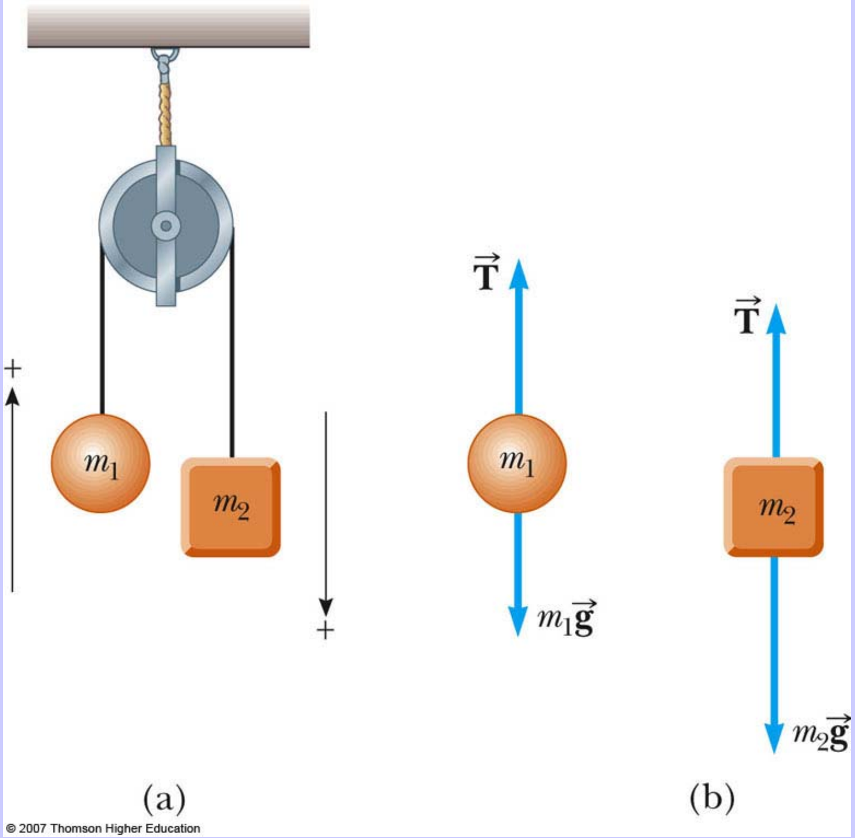
$$\boldsymbol{T_3 = mg}$$

Third Example: Atwood's Machine

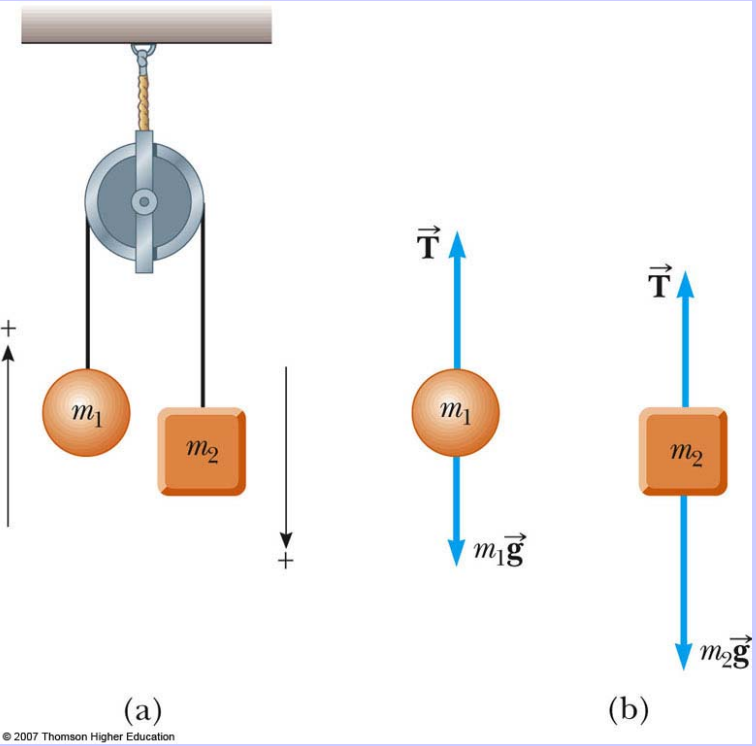
Forces acting on the objects:

- Tension (same for both objects, one string)
- Gravitational force

Each object has the same acceleration since they are connected



Third Example: Atwood's Machine



Applying Newton's 2nd for each object (often called the equations of motion or EOM) yields (note signs in each equation):

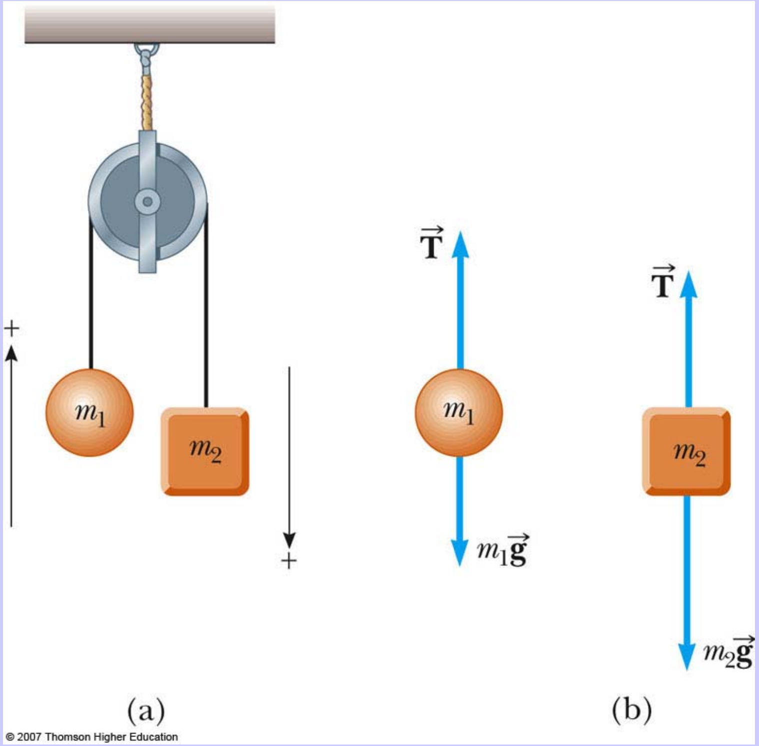
$$m_2 g - T = m_2 a \quad \text{and} \quad T - m_1 g = m_1 a$$

To solve this system of equations we start by summing them to eliminate ***T***,

$$(m_2 - m_1)g = (m_2 + m_1)a \rightarrow a = \frac{m_2 - m_1}{m_2 + m_1} g$$

Does this result make physical sense?

Third Example: Atwood's Machine



Now that we have found the acceleration
The tension can be found from Newton's 2nd (or the EOM) for either mass.

$$T = m_2(g - a) \quad \text{or} \quad T = m_1(g + a)$$

Do these results make physical sense?

Inserting our result for the acceleration we find that the tension is given by

$$T = 2\frac{m_1 m_2}{m_2 + m_1} g$$

Again, do these results make physical sense?

Fourth Example: Incline With Friction

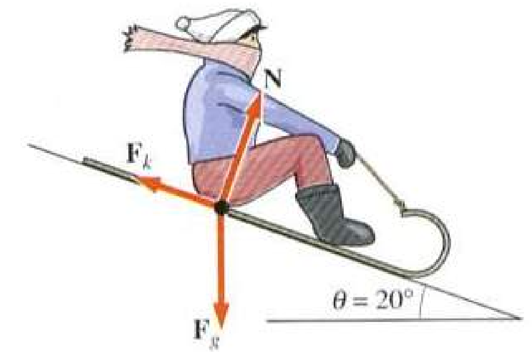
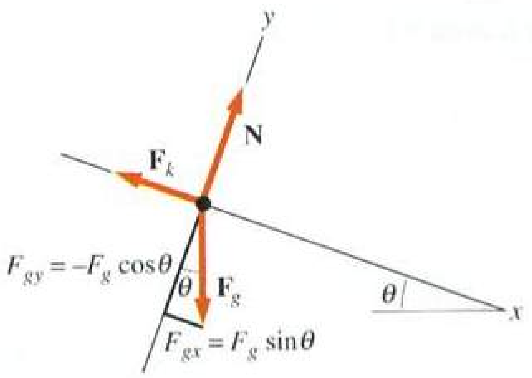


FIGURE 6-33 Forces on the child and sled.



A child slides down a **20º** slope with a coefficient of kinetic friction $\boldsymbol{\mu_k}$ **= .085**.
(a) What is the child's acceleration?

From the Free-Body diagram, the vector equation for the sled is:

$$\vec{F}_g + \vec{F}_k + \vec{N} = m\vec{a}$$

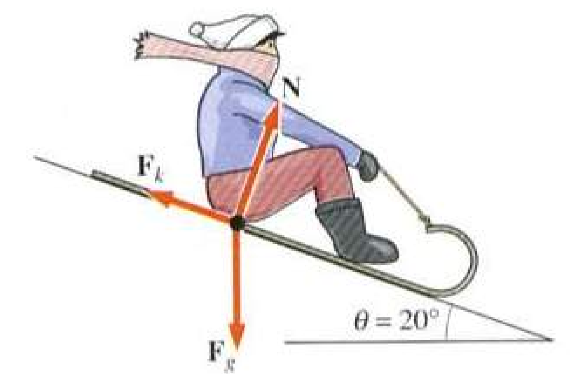
From the Free-Body diagram we see that the component equations are:

$$F_g \sin\theta - F_k = mg\sin\theta - \mu_k N = ma$$
$$N - mg\cos\theta = 0$$

Substituting for the normal force in the *x* component equation followed by dividing by ***m*** yields:

$$a = g(\sin\theta - \mu_k \cos\theta) = 9.8(\sin 20^\circ - .085\cos 20^\circ) = 2.57 m/s^2$$

Fourth Example: Incline With Friction



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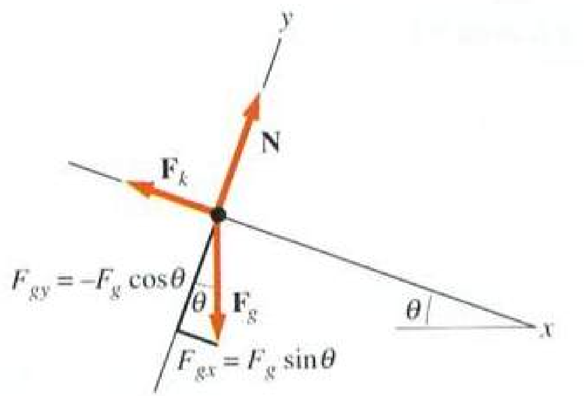


A child slides down a **20º** slope with a coefficient of kinetic friction $\boldsymbol{\mu_k}$ **= .085**.
(b) At what angle will the child velocity remain constant?

From the results for a general angle we saw

$$a = g(\sin\theta - \mu_k \cos\theta)$$

The acceleration vanishes when

$$\sin\theta = \mu_k \cos\theta \rightarrow \mu_k = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

What happens when the angle of the incline, $\boldsymbol{\theta}$, approaches $\boldsymbol{\pi/2}$?

What happens when the angle of the incline, $\boldsymbol{\theta}$, is less than $\mathbf{tan^{-1}(\boldsymbol{\mu_k})}$?

Fifth Example: Problem 16

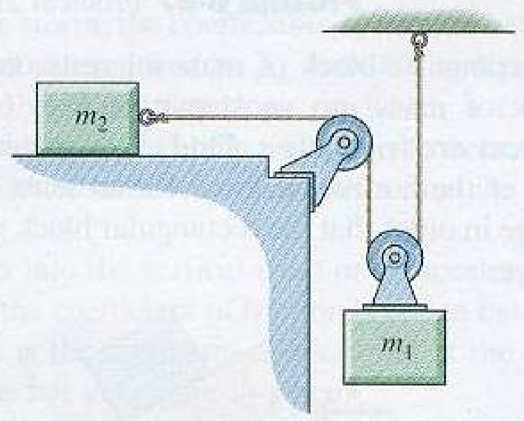


FIGURE 6-66 Problem 16.

Neglecting friction and assuming that the cord is inextensible, find the expressions for the accelerations of the blocks.

The vector EOM's for both masses are

$$\vec{N} + \vec{T} = \vec{F}_{g2} = m_2 \vec{a}_2 \text{ and } 2\vec{T} + \vec{F}_{g1} = m_1 \vec{a}_1$$

Before we write down the component equations it is useful to consider the constraints. First for $\boldsymbol{m_2}$ only horizontal motion is possible and for $\boldsymbol{m_1}$ only vertical motion is possible. Next, since the length of the cord is unchanged any change in the horizontal position of $\boldsymbol{m_2}$ results in ½ of that change in the vertical position of $\boldsymbol{m_1}$. This means that $\boldsymbol{a_1 = a_2/2}$. Now we can write the Component equations as:

$$T = m_2 a \text{ and } m_1 g - 2T = m_1 a/2$$

Fifth Example: Problem 16

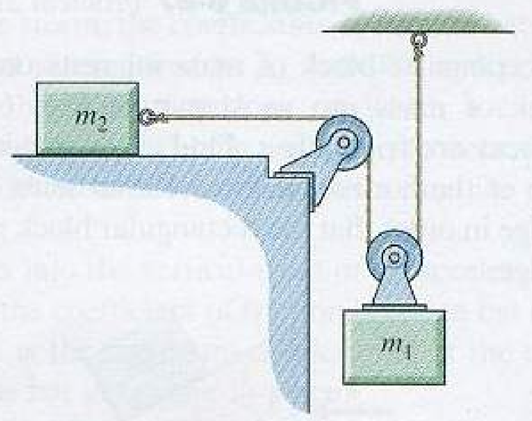


FIGURE 6-66 Problem 16.

Neglecting friction and assuming that the cord is inextensible, find the expressions for the accelerations of the blocks.

The component EOM's for both masses are

$$T = m_2 a \quad \text{and} \quad m_1 g - 2T = m_1 a/2$$

Substituting for ***2T*** in the equation for $\boldsymbol{m_1}$ yields:

$$m_1 g - 2m_2 a = m_1 a/2 \rightarrow \frac{1}{2}(m_1 + 4m_2)a = m_1 g$$

Solving for the acceleration of $\boldsymbol{m_2}$ and $\boldsymbol{m_1}$ we find

$$a = \frac{2m_1 g}{m_1 + 4m_2} \quad \text{and} \quad a/2 = \frac{m_1 g}{m_1 + 4m_2}$$

Sixth Example: Problem 22

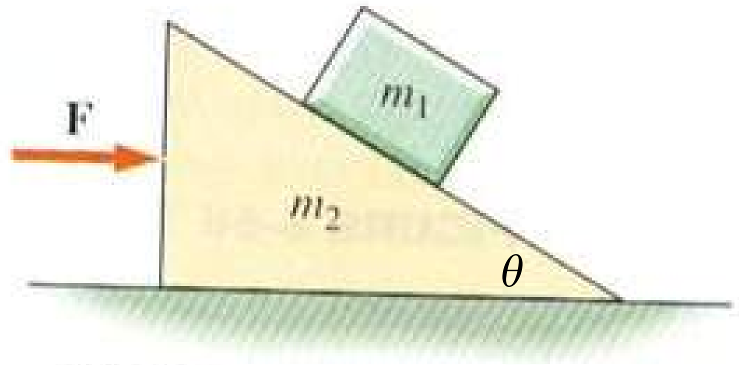


FIGURE 6-68 Problem 22.

Find the expression for the force *F* that must be applied to the wedge so that block will not slide along the frictionless wedge.

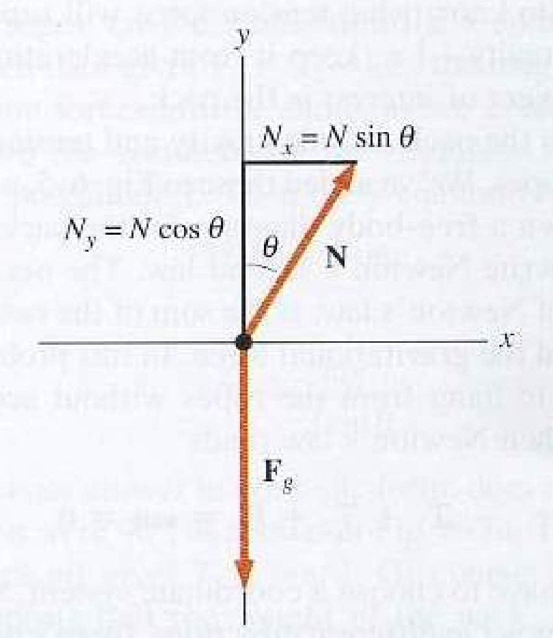
For this problem we need to consider our choice of coordinate system. If the block is not going slide along the wedge then both its horizontal and vertical accelerations must match that of the wedge. This means that the coordinate system should match that of the wedge.

In the coordinates of the wedge the free body diagram for the block is given in the figure. The two component EOM for the block:

$$N\sin\theta = m_1 a, \quad \text{and} \quad N\cos\theta - m_1 g = 0$$

The *x* component EOM for the wedge

$$F - N\sin\theta = m_2 a$$



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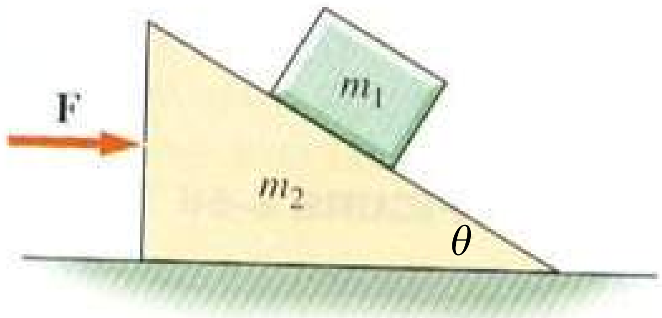


FIGURE 6-68 Problem 22.

Find the expression for the force ***F*** that must be applied to the wedge so that block will not slide along the frictionless wedge.

The three equations are:

$$\begin{aligned} N\sin\theta &= m_1 a \\ N\cos\theta &= m_1 g \\ F - N\sin\theta &= m_2 a \end{aligned}$$

To solve for the acceleration, we divide the first equation by the second and find ***a*** **= gtan(*θ*).** From the third equation we know that

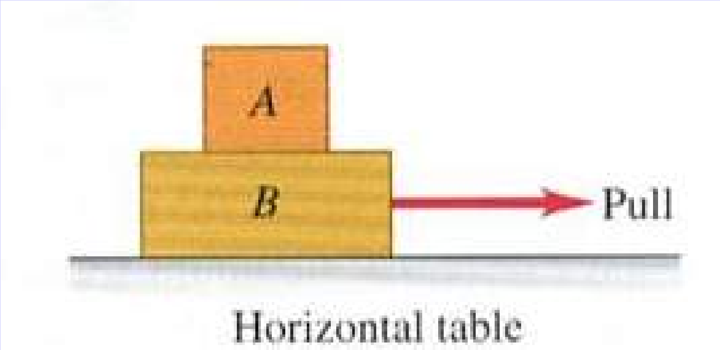
$$F = (m_1 + m_2)a$$

Hence the required force is

$$F = (m_1 + m_2)g\tan\theta$$

Does these results make physical sense?

Seventh Example: Block on Block



Assume a frictionless surface between block B and the table, and a coefficient of static friction μ_s between block A and block B. What is the maximum force that you can pull on block B before block A slides off ?

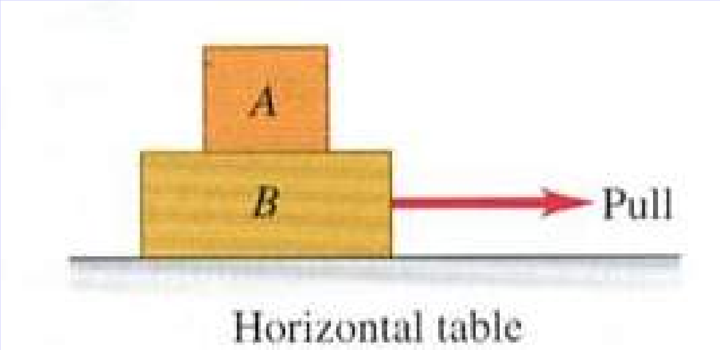
Again we need to consider the constraints to this problem before we consider a Free-Body diagram and the resulting EOM's. The maximum frictional force exerted on block A is $\mu_s N$ and it is this force that allows block A to be accelerated along with block B. Newton's third law tells us that this frictional force retards the acceleration of block B. Hence our vector EOM's are:

$$\vec{F} - \vec{F}_f = m_B \vec{a} \quad \text{and} \quad \vec{F}_f = m_A \vec{a}$$

When the frictional force is maximum the component form of these EOM's are:

$$F - \mu_s N = F - \mu_s m_A g = m_B a \quad \text{and} \quad \mu_s N = \mu_s m_A g = m_A a$$

Seventh Example: Block on Block



Assume a frictionless surface between block B and the table, and a coefficient of static friction μ_s between block A and block B. What is the maximum force that you can pull on block B before block A slides off ?

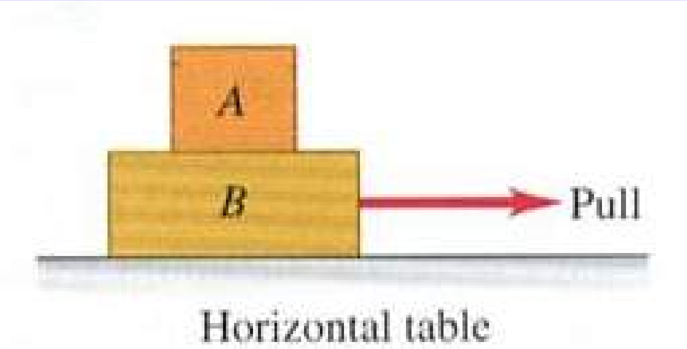
$$F - \mu_s N = F - \mu_s m_A g = m_B a \quad \text{and} \quad \mu_s N = \mu_s m_A g = m_A a$$

First we note that solving for F yields $F = (m_A + m_B)a$ which is exactly what we would expect. Since the acceleration is given by $a = \mu_s g$, the maximum force is

$$F = (m_A + m_B)\mu_s g$$

Something interesting (??) happens when the force pulling block B increases beyond this limit. At that point the frictional force is reduced to $\mu_k N$ and block A will slide backward at an accelerating rate relative to block B. Also the acceleration of block B will increase.

Seventh Example: Block on Block



When the force pulling block B increases beyond this limit, the frictional force is reduced to $\mu_{\mathbf{k}} N$. Block A then slides backward at an accelerating rate relative to block B. Also the acceleration of block B will increase. (a) Find the new acceleration of block B and compare it to the acceleration at the moment block A begins to slide. (b) How long will it take for block A to slide back a distance ***l*** after it begins to slide?

(a) The EOM for block B at threshold is:

$$F - \mu_k m_A g = m_B a_B \rightarrow (m_A + m_B)\mu_s g - \mu_k m_A g = m_B a_B$$

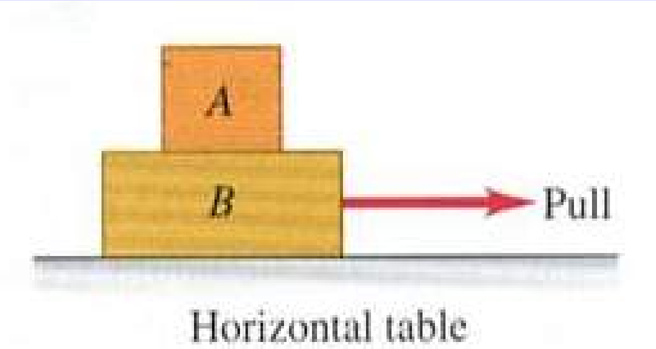
Solving for the acceleration of block B we find and we see that its acceleration has increased!

$$a_B = \mu_s g + \frac{m_A}{m_B}(\mu_s - \mu_k)g > \mu_s g$$

(b) The EOM for block A at threshold is:

$$\mu_k m_A g = m_A a_A \rightarrow a_A = \mu_k g$$

Seventh Example: Block on Block



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$$a_B = \mu_s g + \frac{m_A}{m_B}(\mu_s - \mu_k)g > \mu_s g$$

$$\mu_k m_A g = m_A a_A \rightarrow a_A = \mu_k g$$

The relative acceleration between the blocks is:

$$a = a_B - a_A = \mu_s g + \frac{m_A}{m_B}(\mu_s - \mu_k)g - \mu_k g$$
$$a = (\mu_s - \mu_k)\frac{m_A + m_B}{m_B}g$$

To find the time, it is now a simple matter of substituting this acceleration into:

$$t = \sqrt{2l/a}$$