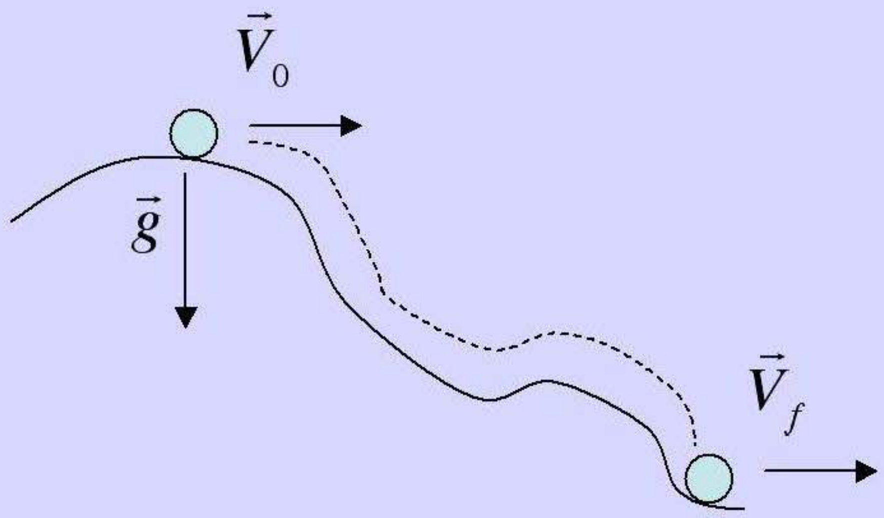
Today’s Lecture

Lecture 12: Chapter 7
Work, Energy, Power

Concepts of Work, Energy and Power are useful for Solving Complex Motion

Complex trajectories, such as a ball rolling down a bumpy hill, will have a complicated solution in the kinematic trajectory method.

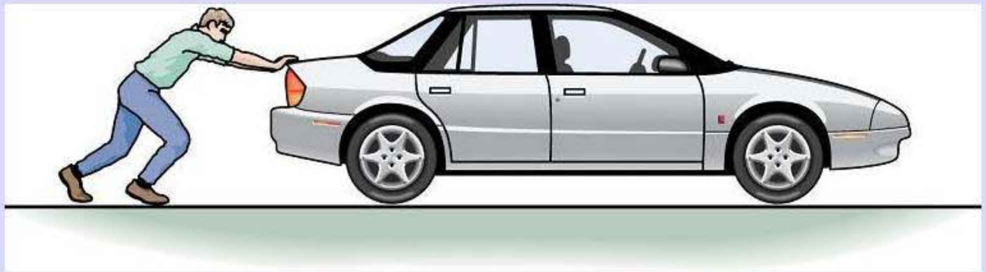


Using Work and Energy allows us to relate the final velocity to the initial velocity without needing to evaluate all of the kinematics in between.

What is Work?

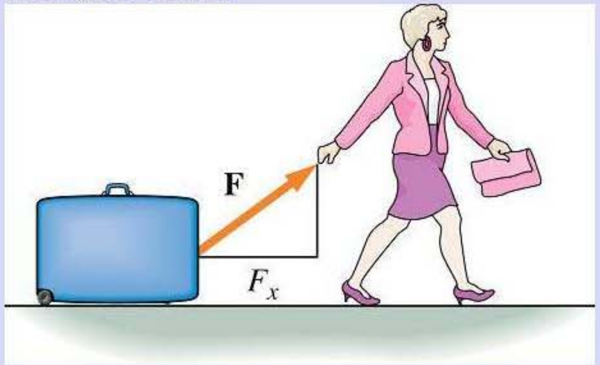
We in general define work as an amount of change in energy.

This can be energy imparted to an object to make it begin moving or change its movement.



The man does work on the car by pushing on it to make it roll (no friction). The energy is transferred to the motion of the car.

This can also be energy imparted to sustain a movement against a dissipative resistant force such as friction.



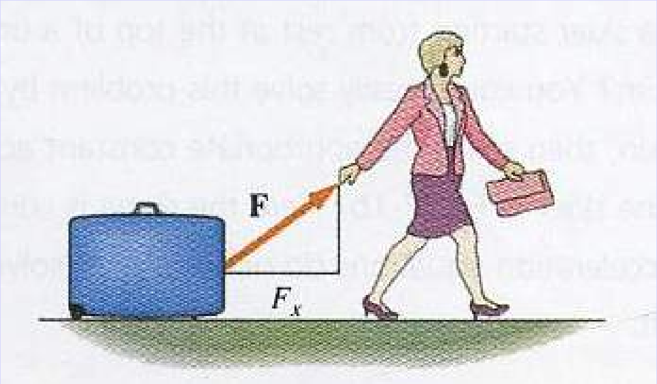
The woman does work on the suitcase by pulling on it to continue its roll. The energy is transferred into heat caused by friction.

What is Work?

For an object moving in one dimension, the work, ***W***, done on the object by a **constant** force is:

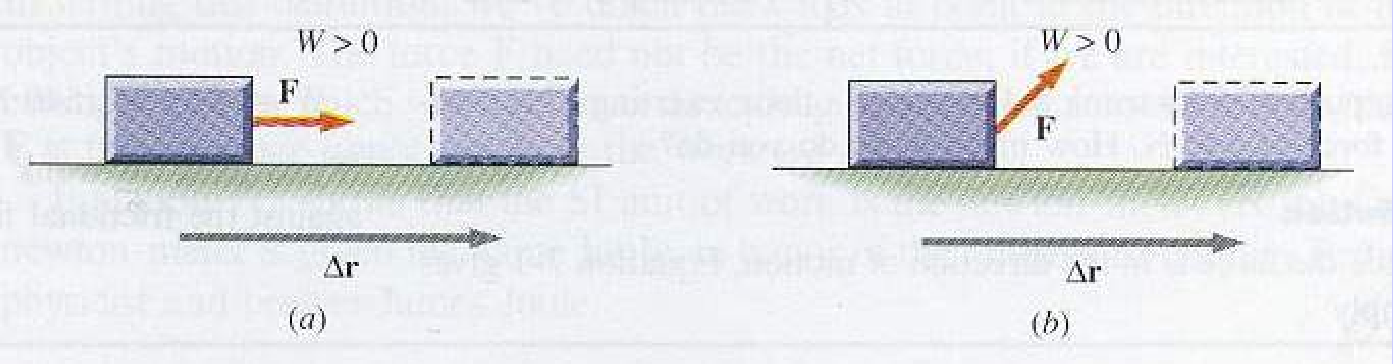
$$W = F_x \Delta x,$$

Where $\boldsymbol{F_x}$ is the component of the force in the direction of the object's motion and $\boldsymbol{\Delta x}$ is the object's displacement.



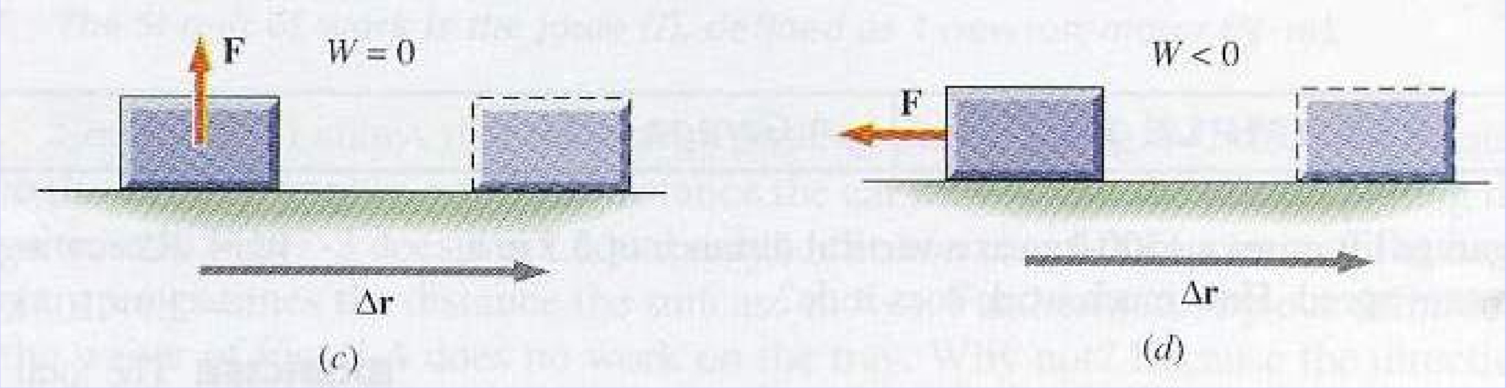
Since the woman is moving horizontally only the *x* component of the force, $\boldsymbol{F_x}$, contributes to the work.

Work Can Be Positive



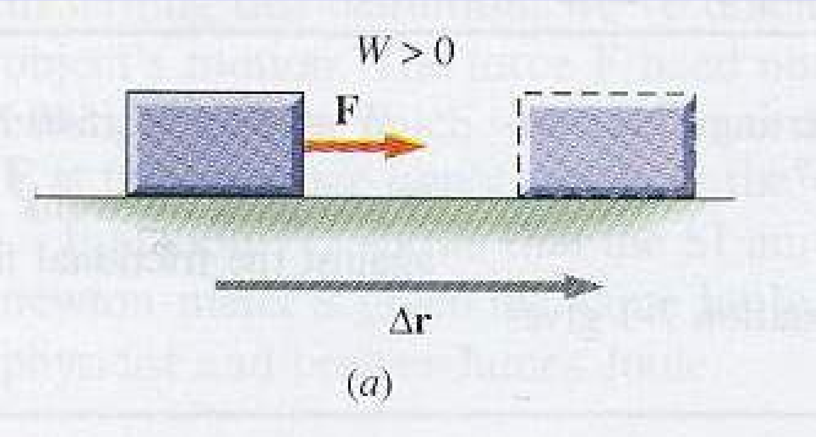
When there is a component of a force acting in the same direction as the motion of an object the work is positive.

Work Can Be Zero or Negative



A force acting 90º to the direction of motion does no work. A force that opposes the motion (friction) does negative work.

Moving a Mass in One Dimension



When we push an object against friction, a force is required to maintain or increase its velocity. The work done is

$$W = F_x \Delta x$$

The work has been converted into heat energy via friction and any change in energy. For example the work against friction to move a block of mass ***m=2kg*** a distance **2** meters with a coefficient of kinetic friction $\boldsymbol{\mu_k = .2}$ is:

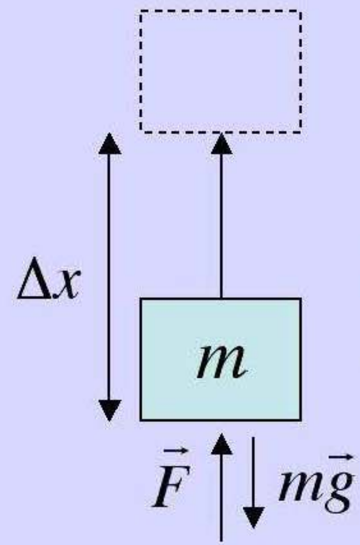
$$F_x = \mu_k N = \mu_k mg = .2(2)(9.8) = 3.92N$$
$$W = F_x \Delta x = (3.92N)(2m) = 7.84Nm$$

The SI unit of work is the Joule where **1J = 1 N-m**.

Example: Raising an Object

When an object is raised at constant speed, the applied force is exactly countering against the gravity.

The work in this case is therefore the magnitude of the applied force to hold the object against gravity times the distance it moved.

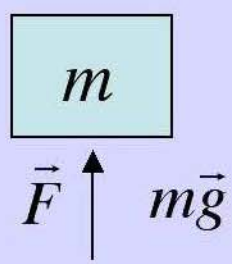


$$W = F\Delta x$$

$$W = mg\Delta x$$

THAT'S EASY!

Note: Work to hold an object still agianst gravity is zero (Δx=0)



Work is The Scalar Product Between Force and Displacement

Generalizing the result from the previous example

$$W = \vec{F} \cdot \Delta\vec{r} = \mathbf{F}\boldsymbol{\Delta}\mathbf{r}\cos\theta, \text{ where } \vec{\mathbf{F}} \text{ is a constant vector}$$

We discussed scalar products at the beginning of the course. In "ij" notation:

$$W = \vec{F} \cdot \Delta\vec{r} = (F_x \Delta x) + (F_y \Delta y) + (F_z \Delta z)$$ **Scalars!**

Work does not have a direction, but it can be **negative or positive**. This relates to whether work was done by or to an object respectively.

Note: the work done in one direction can be negative while the total is positive. It is (typically) the total that matters.

Example: Pulling a Glider

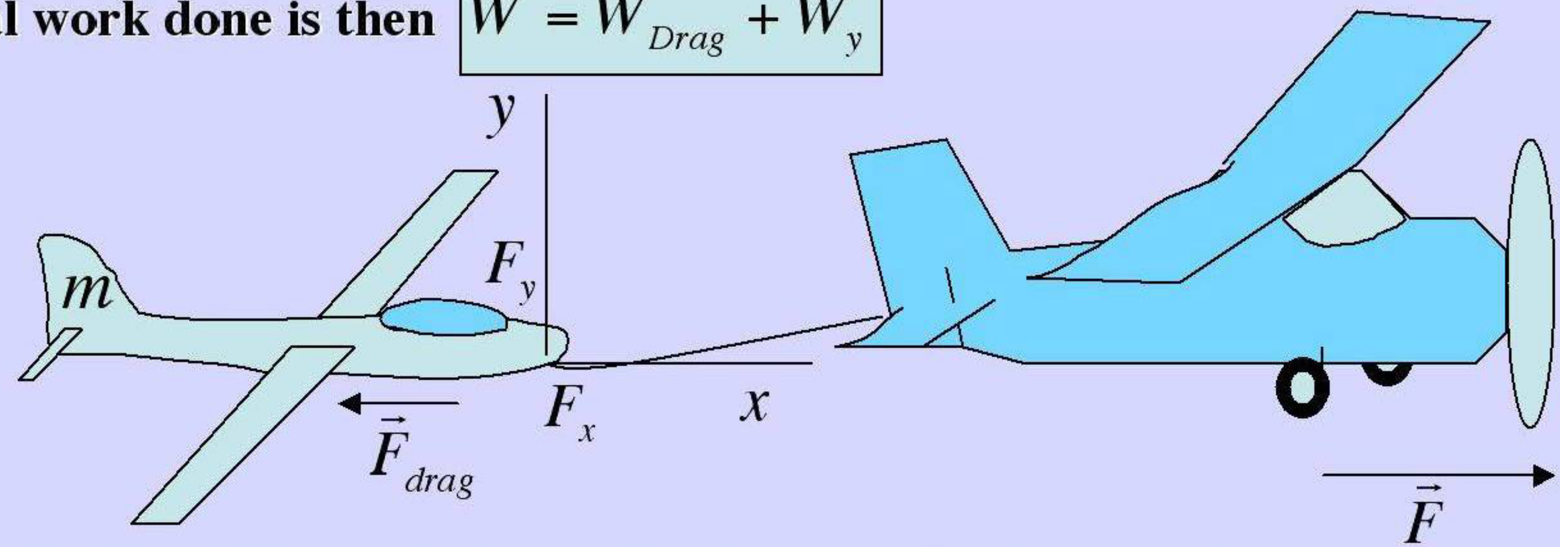
From our work with vector mechanics we know that the x and y directions are separable in force. But, Work is a scalar, and therefore the parts from different directions are simply summed to get the total. ie

$$W = \vec{F} \cdot \Delta\vec{r}$$

The work to raise the glider to an altitude (at a constant F_y) is $W_y = F_y \Delta y$

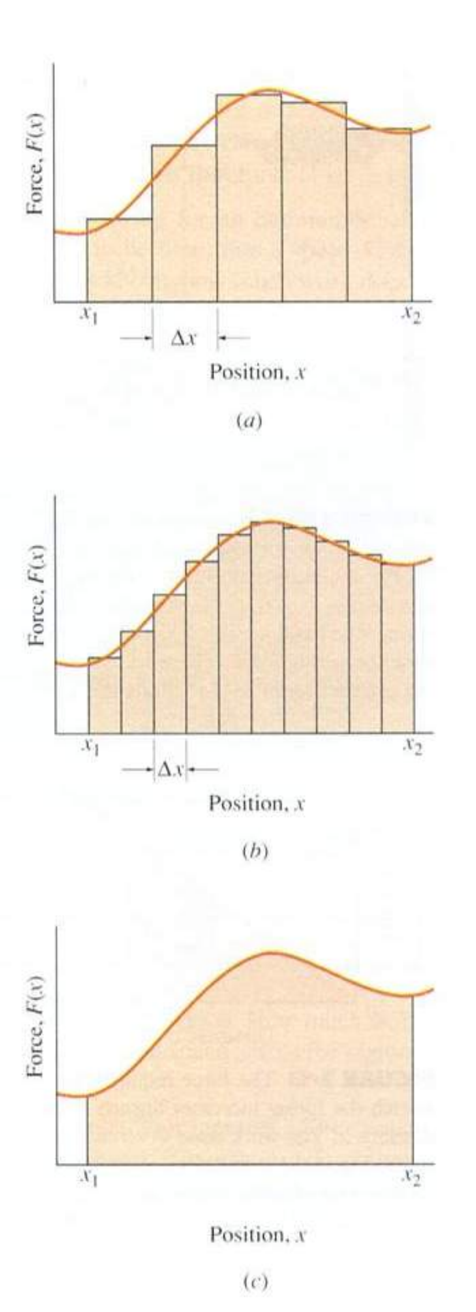
The work done against drag during this time is $W_{Drag} = F_{drag} \Delta r$

The total work done is then $W = W_{Drag} + W_y$



Work to hold a glider against gravity at a a constant altitude is zero, work to pull against the drag force is still nonzero.

Work for a Spatially Varying Force



With a varying force, consider summing the work done over small displacements as shown in (a). The work done for each displacement $\boldsymbol{\Delta x}$ is $\boldsymbol{\Delta W_i}$. To find the total work we sum $\boldsymbol{\Delta W_i}$.

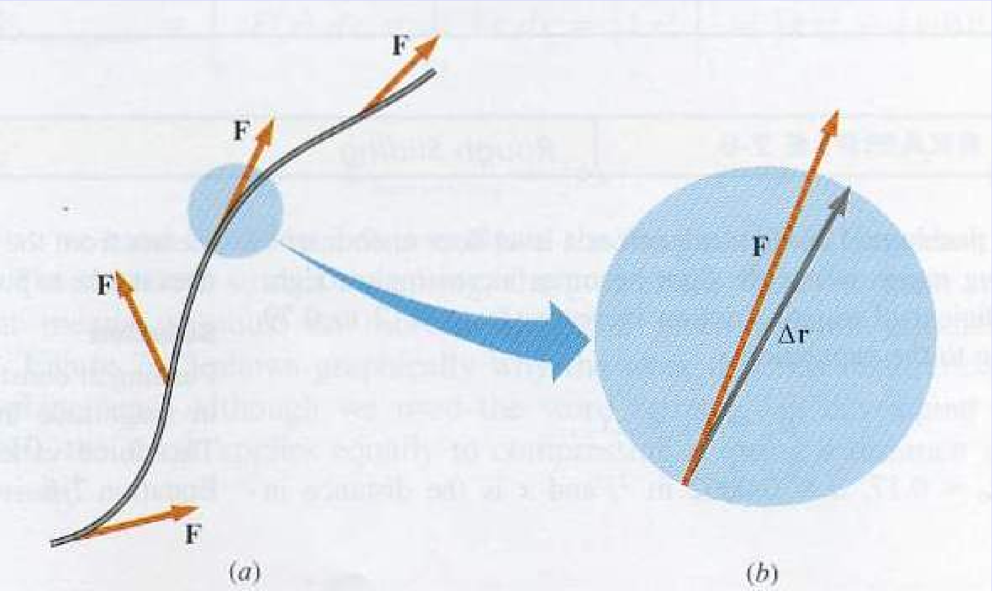
$$W = \sum_{i=1}^{N} \Delta W_i = \sum_{i=1}^{N} F(x_i)\Delta x$$

This is an approximation. Taking the limit as $\boldsymbol{\Delta x}$ approaches zero we obtain:

$$W = \lim_{\Delta x \to 0} \sum_{i=1}^{N \to \infty} F(x_i)\Delta x = \int_{x_1}^{x_2} F(x)dx$$

This is the integral form of the work done in one-dimension by a varying force.

Work for a Spatially Varying Force in 3D



What about a force that varies in direction? Or what if the path is curved? For this more general case

$$\Delta W_i = \vec{F} \cdot \Delta\vec{r}_i$$

Again we sum $\boldsymbol{\Delta W_i}$ and in the limit of small $\boldsymbol{\Delta r_i}$:

$$W = \lim_{\Delta r_i \to 0} \sum_{i=1}^{N\to\infty} \vec{F}(\vec{r}_i) \cdot \Delta\vec{r}_i = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r}$$

This is the integral form of the work done by a varying force in three dimensions. This integral is taken over a specific path. This type of integral is called a **line integral** and in general is path dependent!

Example – Line Integral

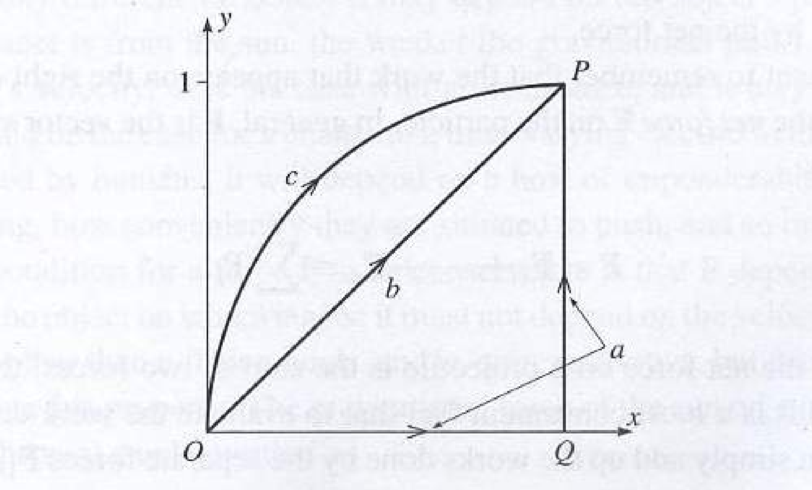


Figure 4.2 Three different paths, a, b, and c, from the origin to the point $P = (1, 1)$.

Consider the line integral for the work along path ***a*** and path ***b*** from the origin to the point **(1,1)** for the force:

$$\vec{F} = y\hat{i} + 2x\hat{j}$$

Along path ***a*** the work is:

$$W_a = \int_{(a)} \vec{F} \cdot d\vec{r} = \int_0^1 F_x(x,0)dx + \int_0^1 F_y(1,y)dy$$

$$W_a = 0 + 2 = 2$$

Along path ***b*** the work is:

$$W_b = \int_{(b)} \vec{F} \cdot d\vec{r} = \int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r} = \int_{(0,0)}^{(1,1)} (F_x(x,x)dx + F_y(y,y)dy)$$

$$W_b = \int_0^1 xdx + 2\int_0^1 ydy = \frac{3}{2}$$

For this particular force the work done is **Path Dependent**! We shall see that there are forces for which the work done is **Path Independent**.

Work-Kinetic Energy Theorem

The kinetic energy for a single particle of mass ***m*** traveling at speed ***v*** is defined as:

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \vec{v} \bullet \vec{v}$$

The time derivative of this expression is easily evaluated as

$$\frac{dK}{dt} = m\vec{v} \bullet \frac{d\vec{v}}{dt} = m\frac{d\vec{v}}{dt} \bullet \vec{v} = \vec{F}_{net} \bullet \frac{d\vec{r}}{dt}$$

Now we can multiply this expression by *dt* to find $dK = \vec{F}_{net} \bullet d\vec{r}$
Integrating this expression along the path of the particle we find

$$\Delta K = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{net} \bullet d\vec{r} = W$$

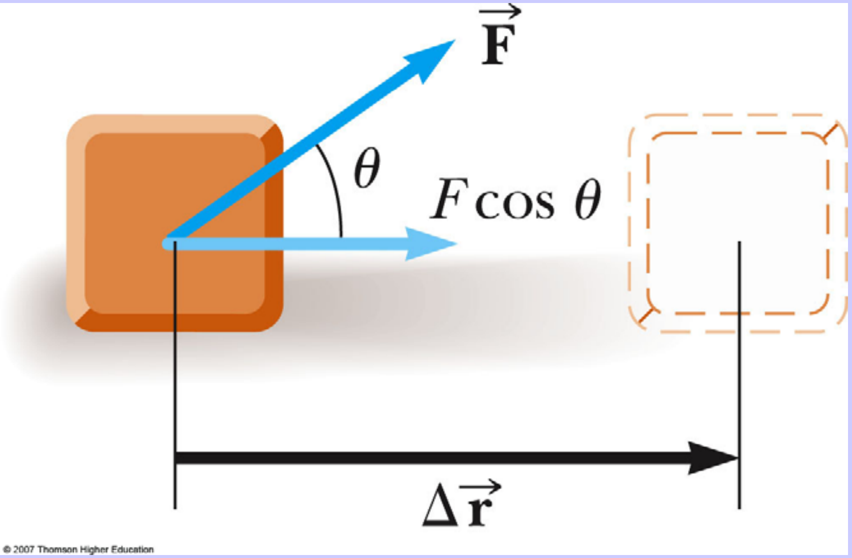
This is a statement of the **Work-KE theorem**,
"The change in a particle's kinetic energy between two points is equal to the work done by the net force along the path between the two points."

Kinetic Energy and Work

From the Work-Kinetic Energy theorem:

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{net} \cdot d\vec{r} = W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

It is only the component of the net force that is parallel to the displacement (or vice versa) that contributes to the work done and consequently the change in kinetic energy.



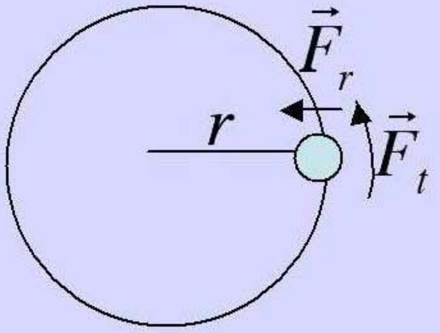
In this figure it is only the horizontal component of the force that contributes to the work.

When pulling a suitcase on rollers why doesn't the suitcase speed up?

Frictional forces do work as well only they oppose the displacement and are < 0.

In Uniform Circular Motion No Work is Being Done.

When a mass is in uniform circular motion. The centripetal force is always perpendicular to the direction of motion. Thus:



$$\vec{F}_r \cdot d\vec{l} = 0$$

and no work is done by $\vec{F}_r$.

Any tangential acceleration will relate to a **tangential force** $\vec{F}_t$ which **does do work**.

$$W = \int \vec{F}_t \cdot d\vec{l} \neq 0$$

Example: friction

Example: Kinetic Energy and Work

It is time to pass. In order to pass a slower car a ***1400kg*** car accelerates from ***70*** to ***95 km/h***. (a) How much work was done on the car? (b) If the car then brakes to a stop, how much work is done on the car?

From the work-kinetic energy theorem we know

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{net} \bullet d\vec{r} = W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

(a) The change in kinetic energy after the acceleration period is

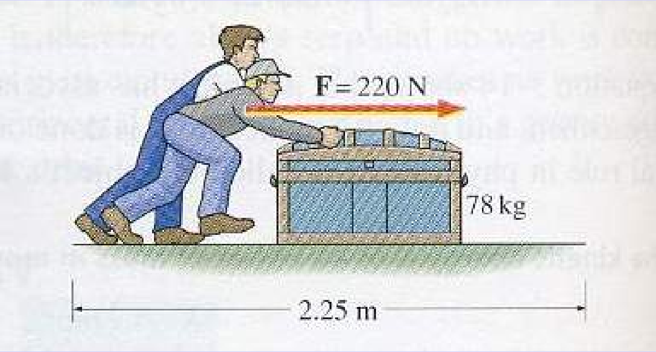
$$\Delta K = \frac{1}{2}1400\left[\left(\frac{95 \times 10^3}{3600}\right)^2 - \left(\frac{70 \times 10^3}{3600}\right)^2\right] = 223kW$$

(b) The change in kinetic energy after the de-acceleration period is

$$\Delta K = \frac{1}{2}1400\left[0 - \left(\frac{70 \times 10^3}{3600}\right)^2\right] = -487kW$$

Do these signs make sense? What provides the force to do the work?

Example: Kinetic Energy and Work
Rough Sliding



Movers are pushing a ***78kg*** trunk across ***2.25m*** of rough floor with a coefficient of kinetic friction of $\boldsymbol{\mu_k}$ **= *.295***. If they push with a force of ***220 N*** what is the speed of the trunk at the end of the rough stretch if the initial speed was ***.71m/s***?

For this example we know the net force

$$F_{net} = F - \mu_k mg = 220 - .295(78)9.8 = -5.50N$$

The work done is

$$W = \int_1^2 \vec{F}_{net} \cdot d\vec{r} = F_{net}\Delta x = (-5.50)(2.25) = -12.37J$$

The change in kinetic energy is equal to the work. Solving for v_2 we find

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \rightarrow v_2 = \sqrt{v_1^2 + 2W/m} = \sqrt{.71^2 - 2(12.37)/78} = .43m/s$$

Could we have found this velocity another way? Kinematics anyone?

Example: Kinetic Energy and Work

Rough Sliding Again

Workers push a ***180kg*** trunk slowly across a level floor. For a ***10m*** section the coefficient of kinetic friction increases from ***0.17*** to ***0.79*** via the relation:

$$\mu_k = 0.17 + .0062x^2$$

Since there is no change in the kinetic energy of the trunk the work done by the workers is equal and opposite to the friction. The work they do is

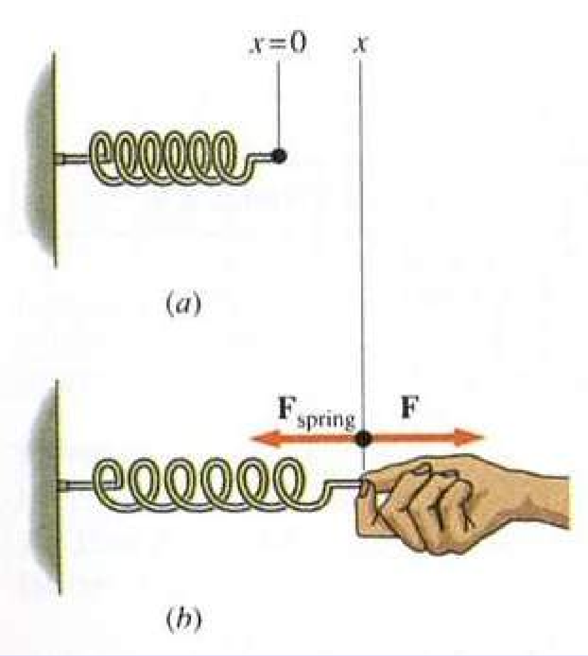
$$W = \int_0^{10} \mu_k mg\,dx = \int_0^{10} (0.17 + .0062x^2)180 \times 9.8\,dx$$

$$W = (1.7 + 6.2/3)1764 = 6.64kJ$$

In the absence of friction the increase in velocity of the trunk would be

$$v = \sqrt{2W/m} = \sqrt{2(6.64 \times 10^3)/180} = 8.6m/s$$

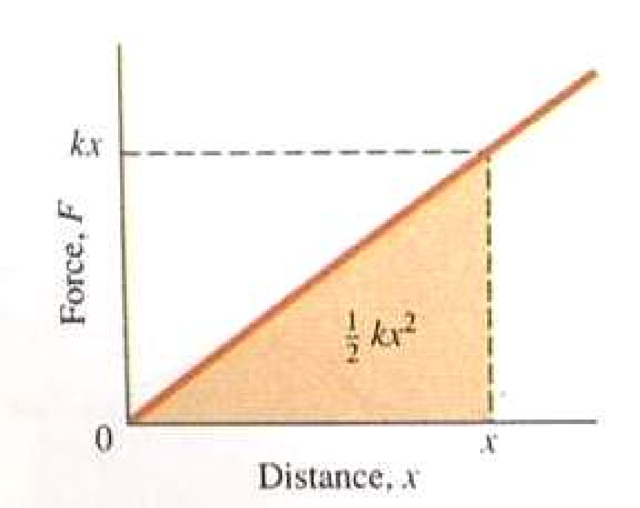
Example: Work to Stretch a Spring



From Hook's Law a spring exerts a force proportional to its displacement from equilibrium:

$$F = -kx$$

This is the force by the spring on the hand stretching it. From Newton's 3rd, the force exerted by the hand is ***kx***. The work done by the hand is the integral:



$$W = \int_0^x kx'\,dx' = \frac{1}{2}kx^2$$

What would the work be if the hand compressed the spring?

Example: Kinetic Energy and Springs

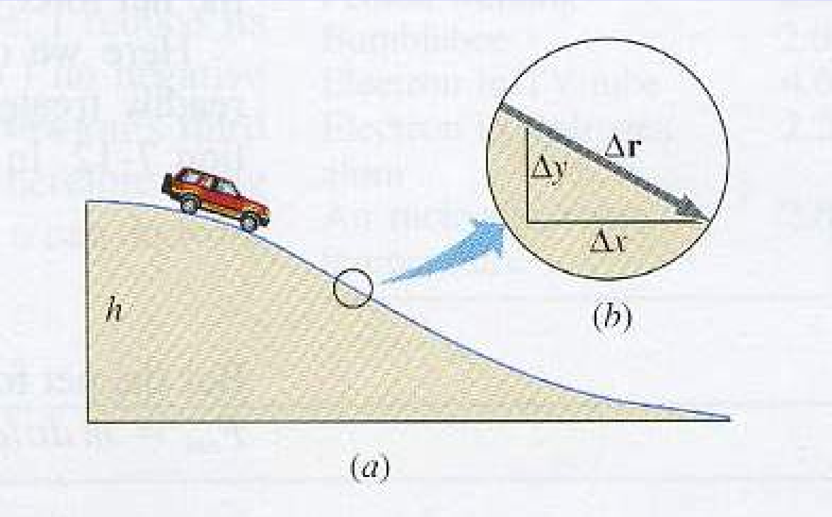
A spring with spring constant ***k*** is compressed a distance ***A*** and while being attached to an object of mass ***m***. The spring is then released. What is the speed of the object when the spring returns to its original equilibrium position?

The work done by the spring on the object is $\boldsymbol{W = \frac{1}{2} k A^2}$. From the Work-Kinetic Energy Theorem:

$$W = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{k}{m}}\,A$$

The details of using the force of the spring to find the acceleration and then using kinematics to find the velocity are not required. The Work-Kinetic Energy Theorem solves the problem with minimal effort.

Example: Work and the Gravitational Force



How much work does the force of gravity do on a car as it drives from the top of the hill, (***y*=*h***) to the bottom (***y*=*0***)?

The force of gravity is $\vec{F}_g = -mg\hat{j}$.

The path integral for the work done by gravity,

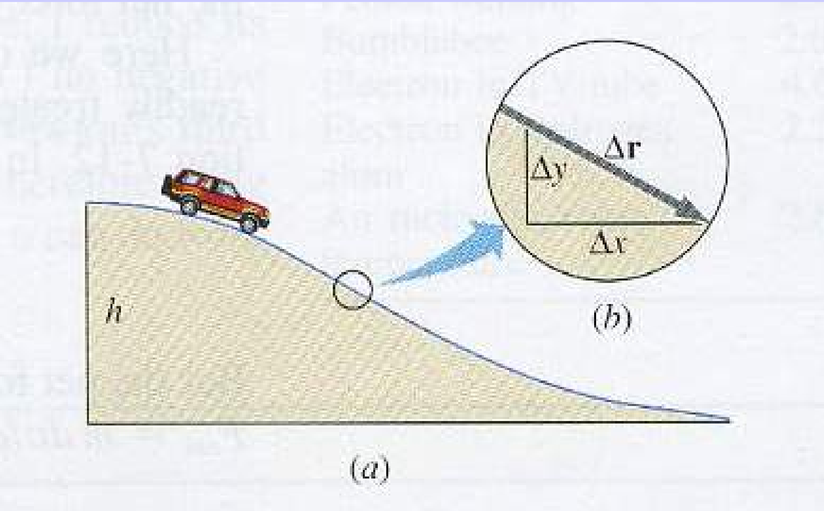
$$W = \int_1^2 \vec{F}_g \cdot d\vec{r} = -\int_1^2 mg\hat{j} \cdot d\vec{r} = -\int_h^0 mgdy$$

$$W = -mg(0-h) = mgh$$

Note that the details of the path didn't matter for this problem, only the change in height, ***h***, was relevant.

Does the sign make sense for this result?

Example: Work and the Gravitational Force



Assuming that the car started from rest, how fast is the car traveling when it reaches the bottom of the hill (ignoring friction)?

The work done by gravity as the rolls down the hill was found to be:

$$W = mgh$$

From the work-energy theorem: $$W = mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh}$$

Again the work-energy theorem solves this problem with minimal effort!