Today’s Lecture

Motion in More Than One-D
Circular Motion 2

Newton’s Laws of Motion

Circular Motion - Spin Up The CD

A 12.7cm diameter CD is spinning up with the rim undergoing a uniform tangential acceleration of $a_t = 91.4 cm/s^2$. Find the magnitude and direction of the acceleration for a point on the rim when $t = .341 \sec$.

To find the radial acceleration we need the tangential velocity.

$$v = a_t t = 91.4(.341) = 31.2 cm/s$$

This means that the radial acceleration is

$$a_r = \frac{v^2}{r} = \frac{(31.2)^2}{6.35} = 153.2 cm/s^2$$

Spin Up The CD

A 12.7cm diameter CD is spinning up with the rim undergoing a uniform tangential acceleration of $a_t = 91.4 cm/s^2$. Find the magnitude and direction of the acceleration for a point on the rim after $t = .341 \sec$.

The magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(153.2)^2 + (91.4)^2} = 178 cm/s^2$$

The direction is

$$\theta = \tan^{-1}\frac{153.2}{91.4} = 1.033 rad = 59.2^\circ$$

Measured from the tangent to the rim of the CD.

Spin Up The CD

Given a uniform tangential acceleration a_t derive the an expression for the time *t* when the acceleration points at 45° toward the direction of motion.

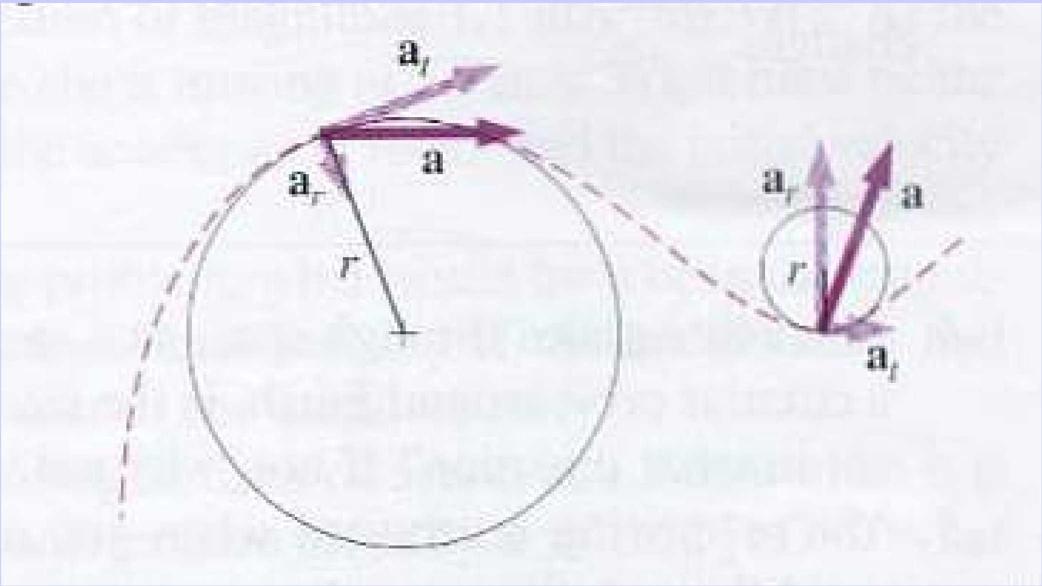
This condition requires that

$$a_t = a_r \rightarrow a_t = \frac{v^2}{r} = \frac{a_t^2 t^2}{r}$$

Solving this algebraic equation for *t*

$$t = \sqrt{r/a_t}$$

Nonuniform Circular Motion

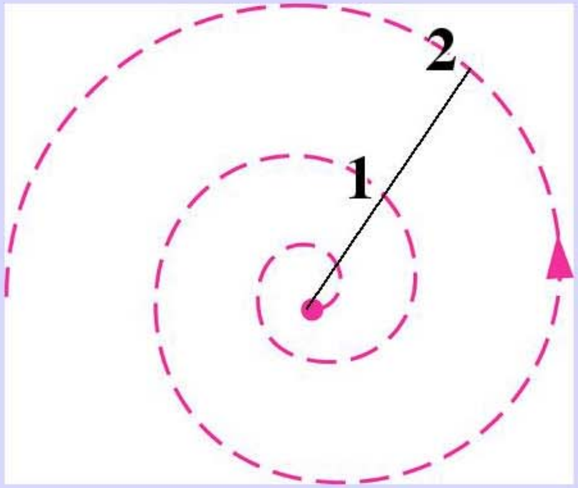


Each point along the path can be characterized by a radius of curvature, *r*. An object with speed *v* has a radial acceleration of v^2/r and a tangential acceleration of magnitude dv/dt . In general both *v* and *r* change as the object moves.

$$\vec{a} = a_t\hat{\phi} - a_r\hat{r} \quad \text{with} \quad a_t = \frac{dv}{dt} \quad \text{and} \quad a_r = \frac{v^2}{r} = \omega^2 r$$

Example: Nonuniform Circular Motion

Question 21: An object moves outward along a spiral path shown in Fig. 4-26. Must its speed increase, decrease, or remain the same if the magnitude of its radial acceleration is to remain constant?



First, pick one radial line, and examine two intersections of that radius. At point 1, the radial acceleration is

$$a = \frac{v_1^2}{r_1}$$

At point 2, it should be the same

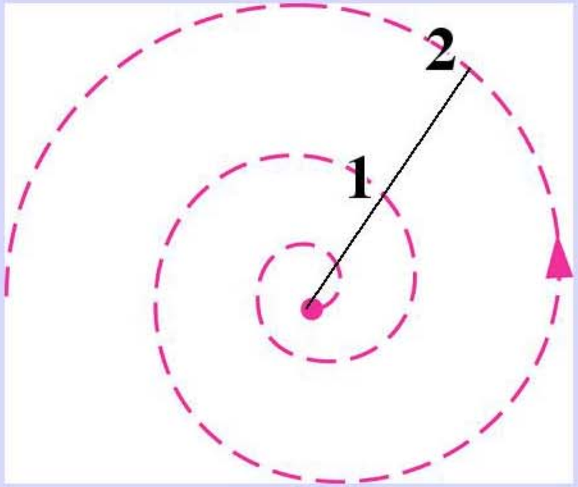
$$a = \frac{v_2^2}{r_2} = \frac{v_1^2}{r_1}$$

Therefore $\frac{v_2^2}{v_1^2} = \frac{r_2}{r_1}$ **Since** $r_2 > r_1$ **we know** $v_2^2 > v_1^2$

Thus, it must increase $\boxed{v_2 > v_1}$

Example: Nonuniform Circular Motion

Question 21 extension: **What if the tangential velocity remains constant**, **does the radial acceleration increase or decrease**?



At point 1: $a_1 = \dfrac{v_1^2}{r_1}$

At point 2: $a_2 = \dfrac{v_2^2}{r_2}$ **But in this example** $v_2 = v_1 = v$

So, $v^2 = a_1 r_1 = a_2 r_2$

Therefore $\dfrac{a_1}{a_2} = \dfrac{r_2}{r_1}$ **Since** $r_2 > r_1$ **we know** $\boxed{a_1 > a_2}$

It decreases.

Kinematics vs. Dynamics

Kinematics are a description of motion

- What we have studied so far
- Displacement, Velocity, Acceleration

Dynamics are a description of the cause of change in motion

- Require New Concepts: **Force** and **Mass**
- Newton's Three Laws of Motion
  - Obtained from first principles
  - Fundamental
  - Based on experimental observations of motion
  - Not a derivation

Mass and the Momentum Vector

Up to now we have only discussed the velocity with which an object moves given an acceleration. The mass of an object was not considered. Two objects with the same velocity but different mass will have different **dynamics**.

What is Mass?

A truck going 20m/h is very different than a baseball going 20m/h.

Mass is a measure of the amount of matter contained in a body. It determines the response of a body to a given force. In SI units mass is measured in kg.

What is Momentum?

The momentum is the mass times the velocity. It is the fundamental quantity describing motion.

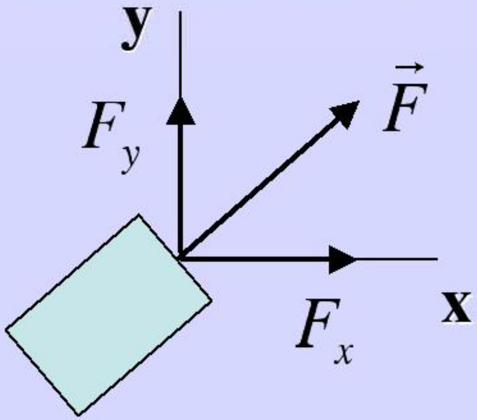
$$\vec{p} = m\vec{v}$$

The Momentum and Force Vectors

Force is the "push" or "pull" on some body, causing a change in the momentum.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$$

Force and Momentum are vectors. They have magnitude and direction, and can be expressed as components along coordinate axes.



In SI units (m, s, etc.) the units of force are called Newtons.

1N=1kg m/s²

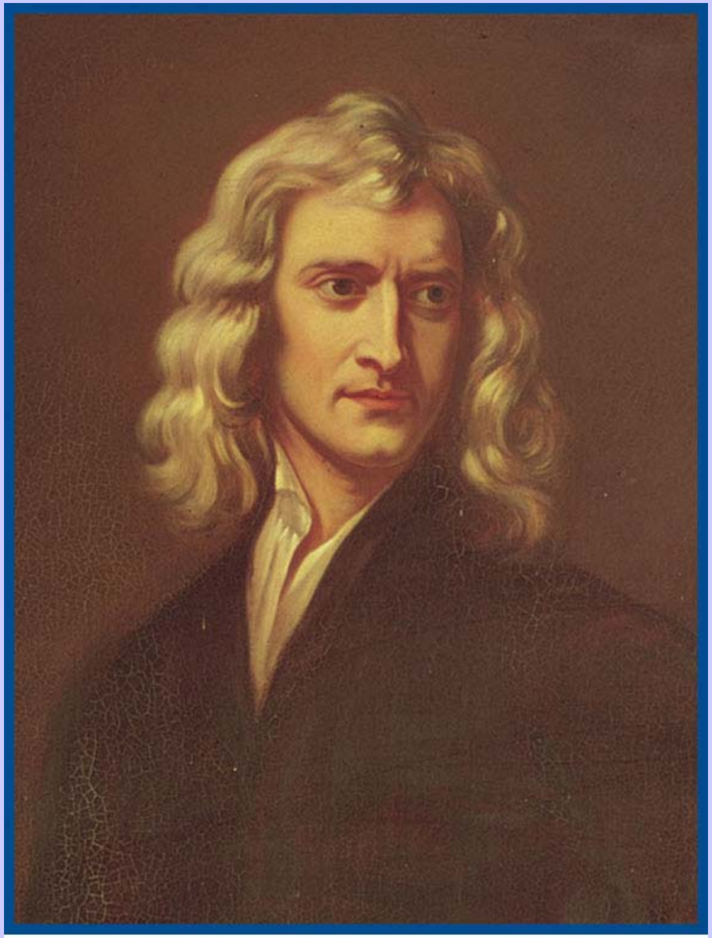
The Net Force is the sum of all force vectors acting on a body

$$\vec{F}_{net} = \sum \vec{F}_i$$

Newton's Laws of Motion

Sir Isaac Newton

- 1642 – 1727
- Formulated basic laws of mechanics
- Discovered Law of Universal Gravitation
- Invented form of calculus
- Many observations dealing with light and optics



Newton's Laws of Motion

1. **The Law of Inertia:** The velocity of a body will not change unless a net force acts on that body.

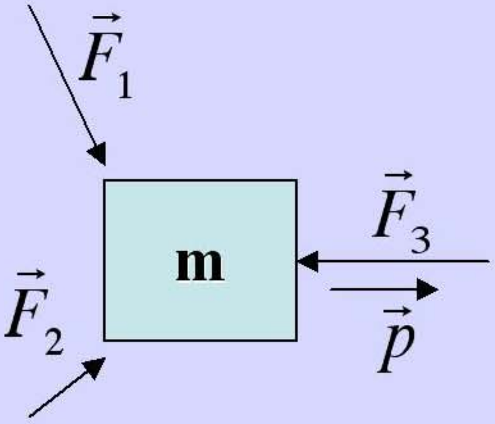
2. **The Law of Force, Mass and Acceleration:** The force on a body is equal to its change in momentum with time. For constant mass, this is the mass times the acceleration.

3. **Law of Action and Reaction:** For each force acting on body B from body A, there is an equal and opposite force acting on body A from body B.

Newton's First Law of Motion

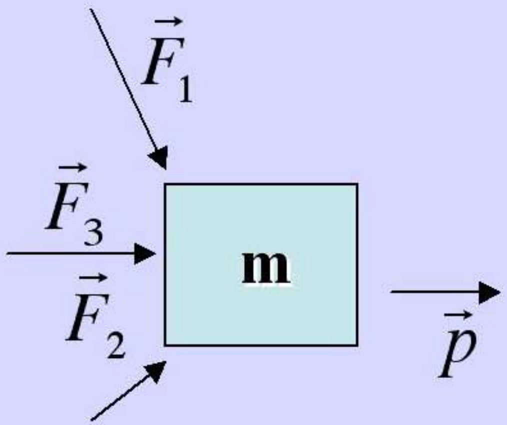
The Law of Inertia: The velocity of a body will not change unless a net force acts on that body.

$$\vec{F}_{net} = \sum \vec{F}_i = 0 \Rightarrow \vec{p} = const.$$



If $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

Then $\vec{p}$ won't change.



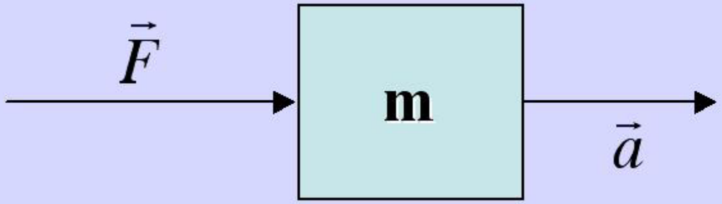
If $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_{net}$

Then $\dfrac{d\vec{p}}{dt} = \vec{F}_{net}$

Newton's Second Law of Motion

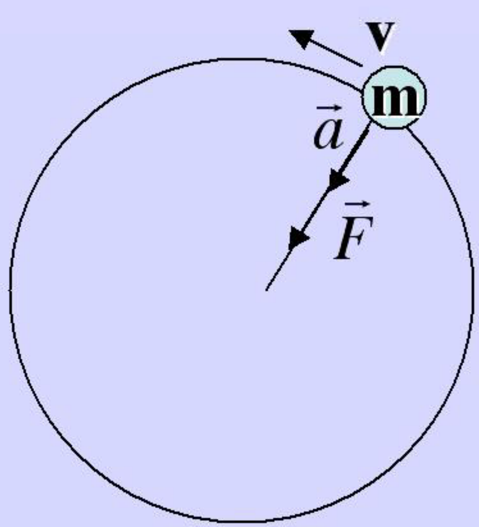
The Law of Force, Mass and Acceleration: The force on a body is equal to its change in momentum with time. For constant mass, this is the mass times the acceleration.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$$ or $$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$$ For m constant



$$\vec{F} = m\vec{a}$$

For a body in linear motion this accelerates the body.



In uniform circular motion there is a radial force inward.

More About Newton's Second Law

- $\sum \vec{F}$ is the net force
  - This is the vector sum of all the forces acting on the object
- Newton's Second Law can be expressed in terms of components:
  - $\Sigma F_x = m\, a_x$
  - $\Sigma F_y = m\, a_y$
  - $\Sigma F_z = m\, a_z$

Newton's Second Law of Motion

Food for thought

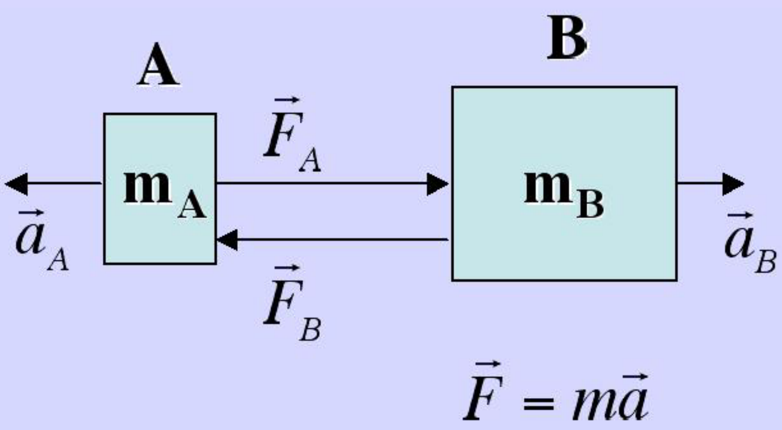
We have well defined definitions for mass and acceleration but how exactly do we define a force? We are very conscious of forces that we exert ourselves, but we somehow need to define what we mean by a force.

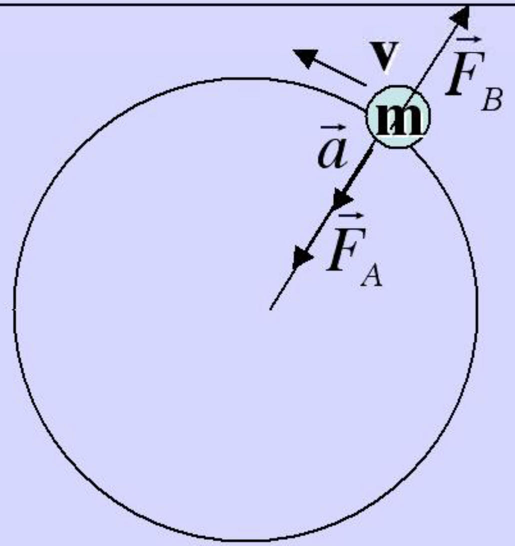
The route preferred by most philosophers of science is to use Newton's second law, $\vec{F} = m\vec{a},$ as the definition of a force. The unit we will adopt is the newton (N), which is the magnitude of a single force that accelerates a standard kilogram mass with an acceleration of $1\ m/s^2$.

Newton's Third Law of Motion

Law of Action and Reaction: For each force acting on body B from body A, there is an equal and opposite force acting on body A from body B.

$$\vec{F}_A = -\vec{F}_B$$





Both bodies move, the lighter mass accelerates faster.

Uniform circular motion F_B is outward on the string.

A Guide to Problem Solving in NLM

Not always the case, but a good guide for typical cases.

1. Always identify all the forces acting on each body first.

2. Add all forces as vectors acting on each body.

3. Equate forces to ma for each body, typically determining a.

4. Use the kinematics you have learned in chapters 1-4 to solve motion.

5. Calculate any final quantities, such as momentum.

Example - Newton's Second Law of Motion

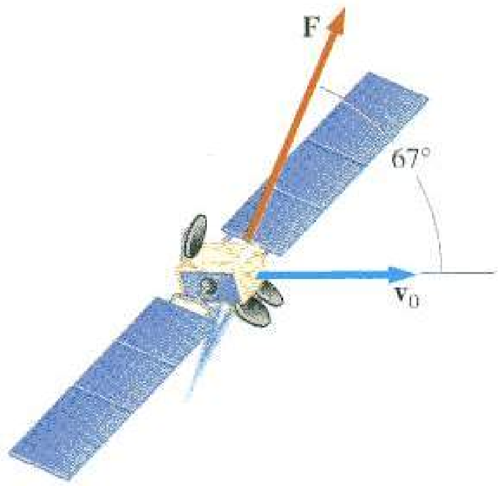


FIGURE 5-13 A rocket thrust.

A 940kg spacecraft is moving uniformly at 4.8 km/s when it fires a rocket that exerts a 4.5×10^{3} N force at 67^{o} to the initial direction for 120s.

(a) How far does the craft move during the firing?

First we determine the acceleration and then use the kinematic equations. The acceleration vector is:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{F\cos\theta}{m}\hat{i} + \frac{F\sin\theta}{m}\hat{j}$$

The components of the acceleration are:

$$a_x = \frac{4.5 \times 10^3}{940}\cos 67^\circ = 1.87 m/s^2 \quad \text{and} \quad a_y = \frac{4.5 \times 10^3}{940}\sin 67^\circ = 4.41 m/s^2$$

Example - Newton's Second Law of Motion

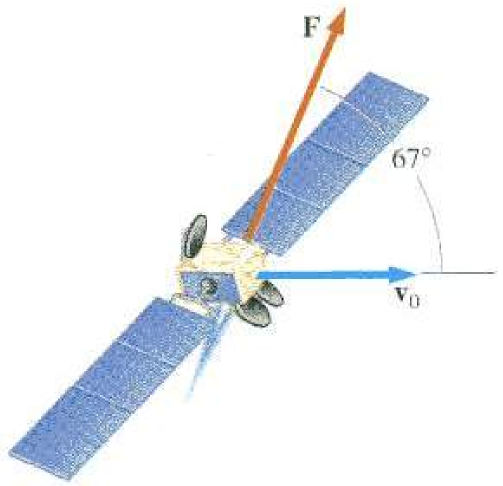


FIGURE 5-13 A rocket thrust.

A 940kg spacecraft is moving uniformly at 4.8 km/s when it fires a rocket that exerts a $4.5\text{x}10^3$ N force at 67° to the initial direction for 120s.

(a) How far does the craft move during the firing?

First we determine the acceleration and then use the kinematic equations. The displacements are

$$\Delta x = v_0 t + \frac{1}{2}a_x t^2 = 4.8\times 10^3(120) + \frac{1}{2}\times 1.87\times (120)^2 = 589.5km$$

$$\Delta y = 0 + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}\times 4.41\times (120)^2 = 31.75km$$

The net distance the craft moves while the rockets fire is

$$r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(589.5)^2 + (31.75)^2} = 590km$$

Example - Newton's Second Law of Motion

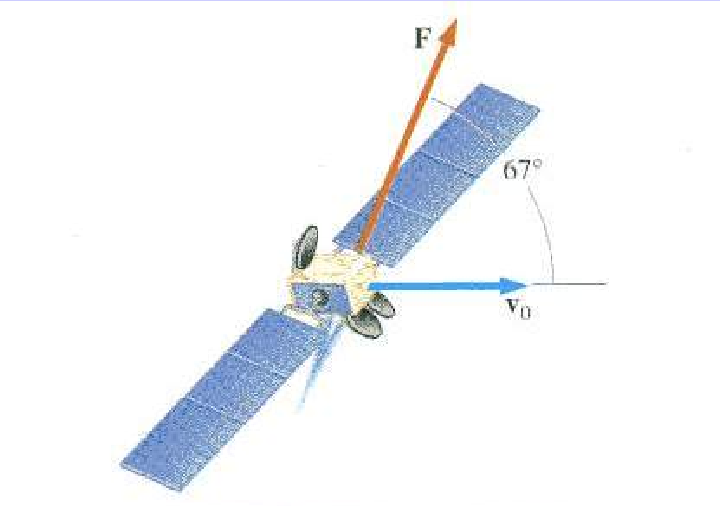


FIGURE 5-13 A rocket thrust.

A 940kg spacecraft is moving uniformly at 4.8 km/s when it fires a rocket that exerts a 4.5x10**³** N force at 67**°** to the initial direction for 120s.

(b) What is the final velocity of the spacecraft?

First we determine the acceleration and then use the kinematic equations. The velocity components are

$$v_x = v_0 + a_x t = 4.8 \times 10^3 + 1.87 \times 120 = 5.024 km/s$$
$$v_y = 0 + a_y t = 0 + 4.41 \times 120 = .529 km/s$$

The final speed and direction of the craft is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(5.024)^2 + (.529)^2} = 5.05 m/s \text{ with } \theta = \tan^{-1}\frac{.529}{5.024} = 6.01^\circ$$