Forces and Motion

Newton’s Laws
Review
and
Examples

Lecture 7

Kinematics vs. Dynamics

Kinematics are a description of motion

- What we have studied so far
- Displacement, Velocity, Acceleration

Dynamics are a description of the cause of change in motion

- Require New Concepts: **Force** and **Mass**
- Newton's Three Laws of Motion
  - Obtained from first principles
  - Fundamental
  - Based on experimental observations of motion
  - Not a derivation

Mass and the Momentum Vector

Up to now we have only discussed the velocity with which an object moves given an acceleration. The mass of an object was not considered. Two objects with the same velocity but different mass will have different dynamics.

What is Mass?

A truck going 20m/h is very different than a baseball going 20m/h.

Mass is a measure of the amount of matter contained in a body. It determines the response of a body to a given force. In SI units mass is measured in kg.

What is Momentum?

The momentum is the mass times the velocity. It is the fundamental quantity describing motion.

$$\vec{p} = m\vec{v}$$

What is a Force?

Force is the "push" or "pull" on some body, causing a **change in the momentum.**

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\vec{a} \quad \text{definition of } \vec{F}$$

All of the forces we experience arise from a combination of a small set of fundamental forces between atomic particles in the universe; electromagnetic, nuclear, gravitational etc.

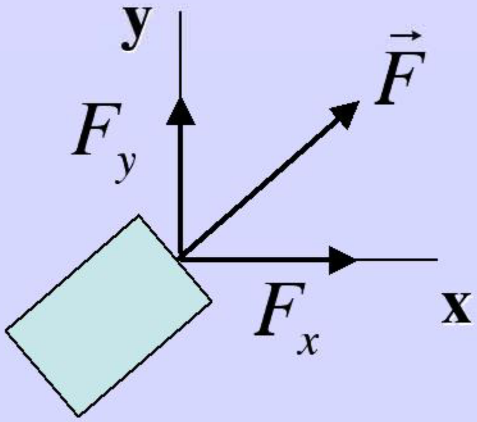
We will deal only with:

Gravity
Friction
Collision
Elasticity

The Momentum and Force Vectors

Force and Momentum are vectors. They have magnitude and direction, and can be expressed as components along coordinate axes.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$$



In SI units (m, s, etc.) the units of force are called Newtons.

1N=1kg m/s^2

The Net Force is the sum of all force vectors acting on a body

$$\vec{F}_{net} = \sum \vec{F}_i$$

Newton's Laws of Motion

1. **The Law of Inertia:** The velocity of a body will not change unless a net force acts on that body.

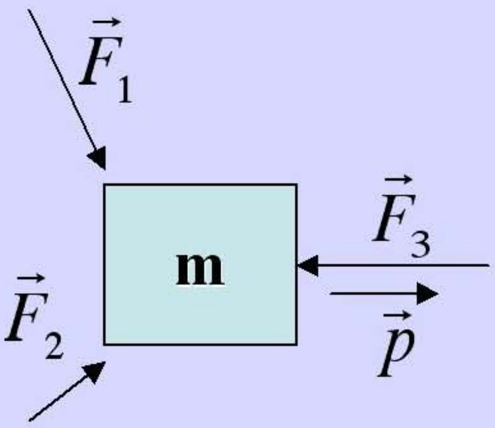
2. **The Law of Force, Mass and Acceleration:** The force on a body is equal to its change in momentum with time. For constant mass, this is the mass times the acceleration.

3. **Law of Action and Reaction:** For each force acting on body B from body A, there is an equal and opposite force acting on body A from body B.

Newton's First Law of Motion

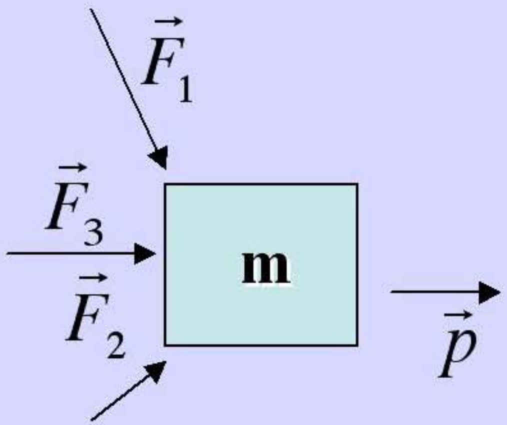
The Law of Inertia: **The velocity of a body will not change unless a net force acts on that body.**

$$\vec{F}_{net} = \sum \vec{F}_i = 0 \Rightarrow \vec{p} = const.$$



If $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

Then $\vec{p}$ **won't change.**



If $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_{net}$

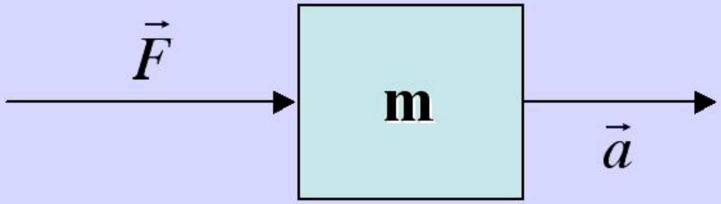
Then $\dfrac{d\vec{p}}{dt} = \vec{F}_{net}$

Newton's Second Law of Motion

The Law of Force, Mass and Acceleration: The force on a body is equal to its change in momentum with time. For constant mass, this is the mass times the acceleration.

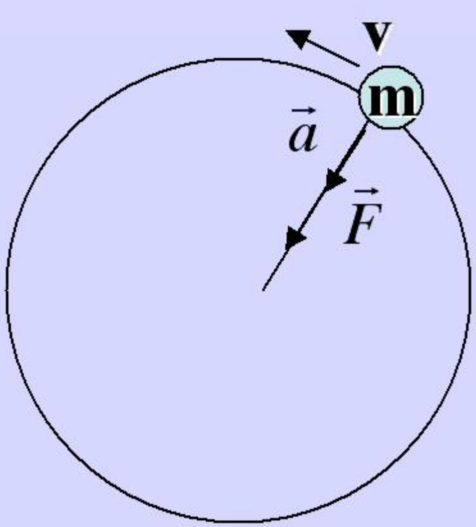
$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} \quad \text{or} \quad \vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

For m constant



$$\vec{F} = m\vec{a}$$

For a body in linear motion this accelerates the body.

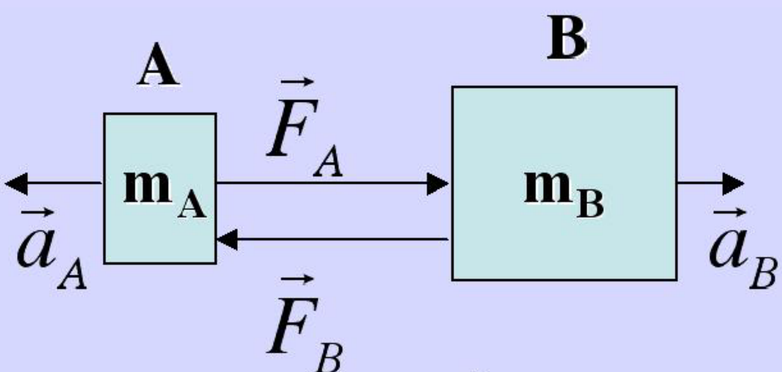


In uniform circular motion there is a radial force inward.

Newton's Third Law of Motion

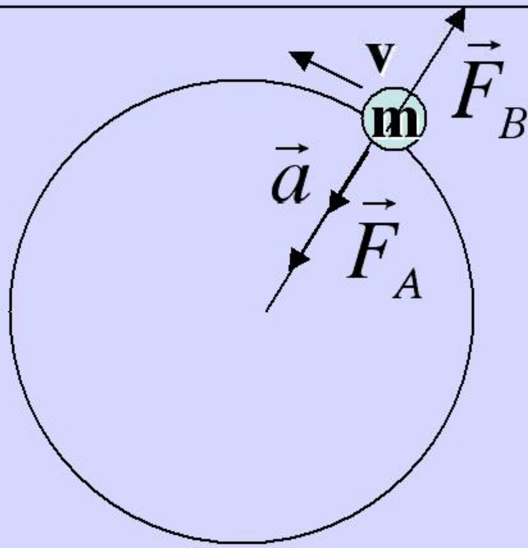
Law of Action and Reaction: **For each force acting on body B from body A, there is an equal and opposite force acting on body A from body B.**

$$\vec{F}_A = -\vec{F}_B$$



$$\vec{F} = m\vec{a}$$

Both bodies move, the lighter mass accelerates faster.



Uniform circular motion F_B is outward on the string.

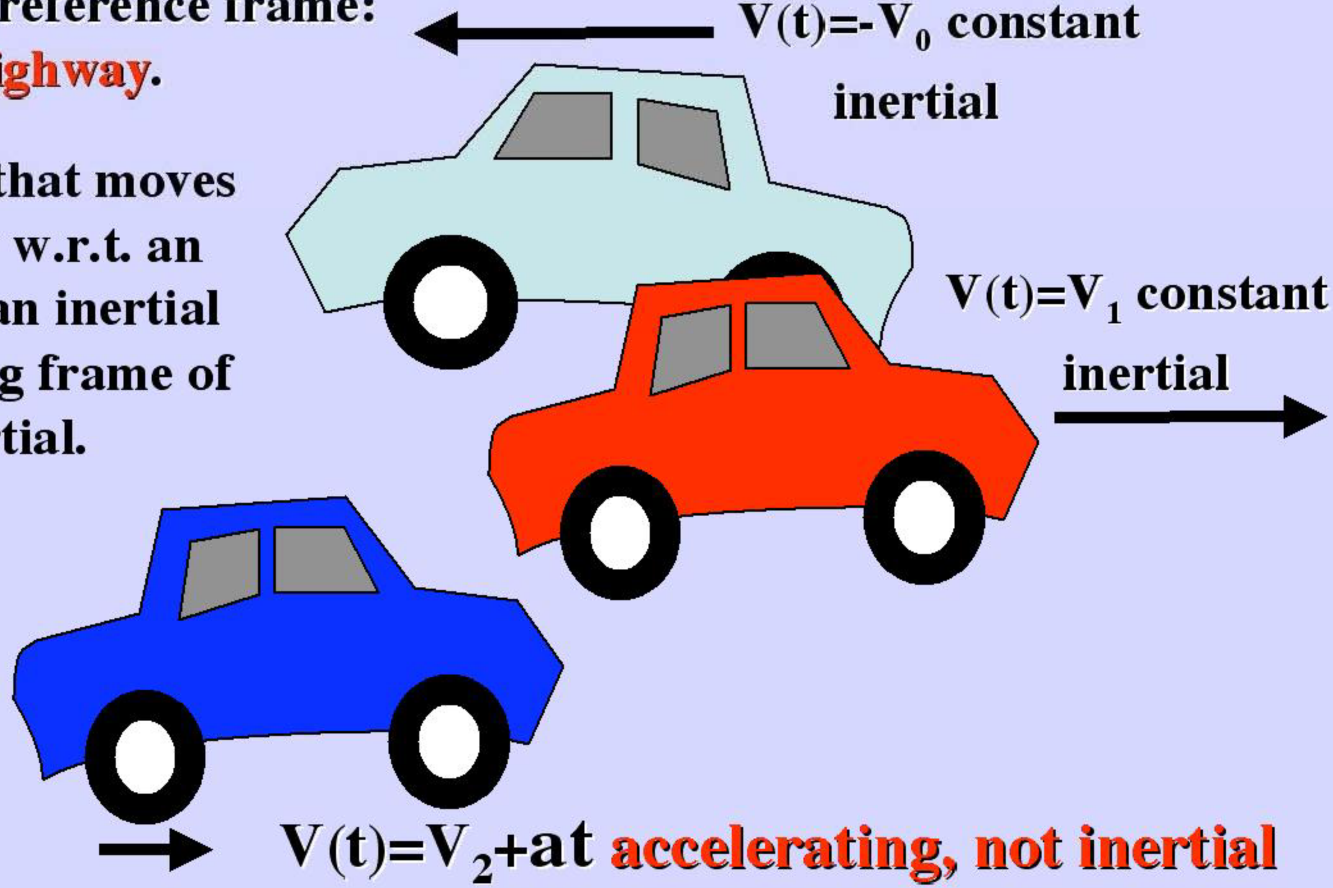
Where are Newton's Laws of Motion Valid? Any inertial reference frame.

A reference frame where velocity does not change is called an inertial reference frame. Law of Inertia (1st Law).

Example of a inertial reference frame:
Steady drive on the highway.

Any reference frame that moves with constant velocity w.r.t. an inertial frame is also an inertial frame. An accelerating frame of reference is NOT inertial.

Effectively, we can place our origin in any inertial frame.

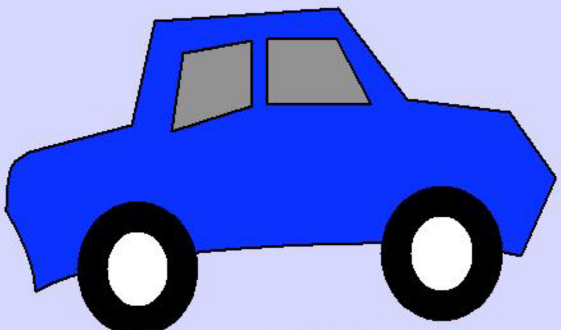


Where are Newton's Laws of Motion Valid? Any inertial reference frame.

For example, if we did place our frame of reference in the accelerating car, a parked car, and indeed any other car with constant velocity, would appear to be accelerating.

$V(t)=V_0 \rightarrow$

But no force acts on them!
Newton's laws not obeyed.

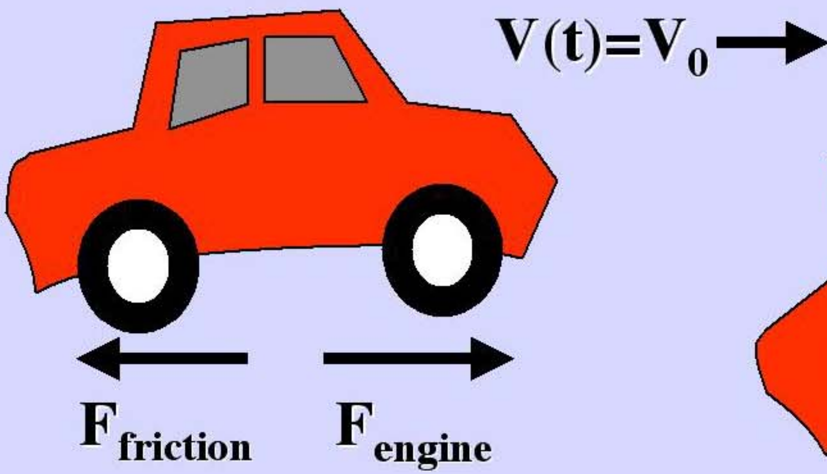


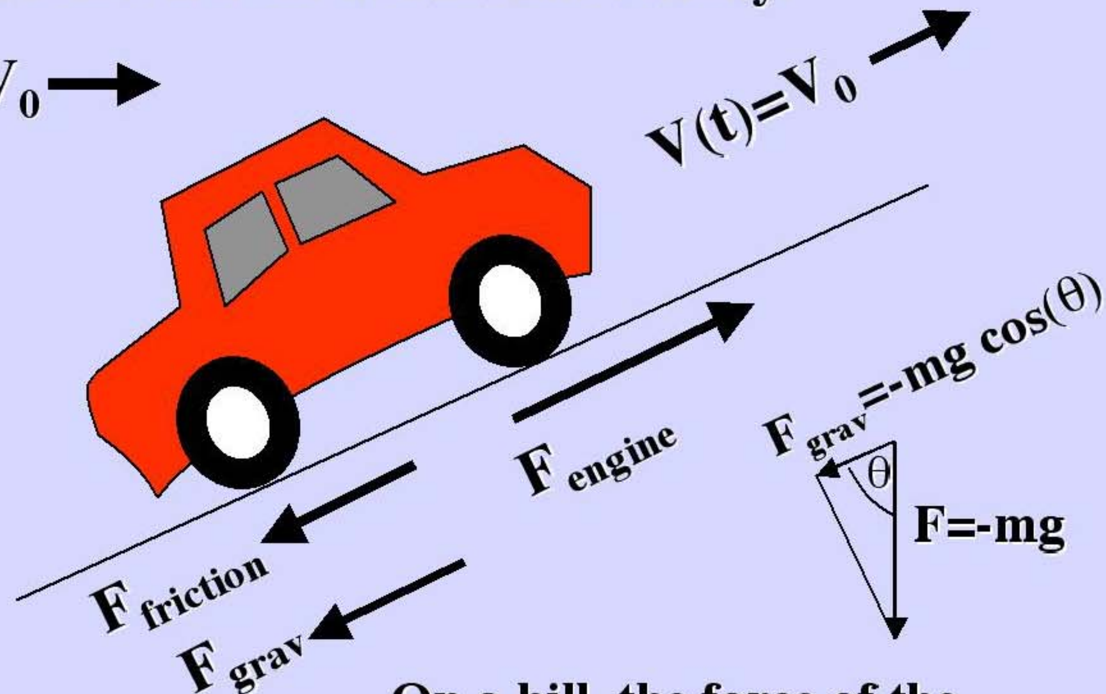
$\rightarrow$ $V(t)=V_2+at$ accelerating, not inertial

Examples of Newton's Laws of Motion

Law of Inertia (1st Law):The velocity of a body will not change unless a net force acts on that body.

A good example of Newton's first law is a car at constant velocity.





Both cars are at constant velocity.

On level ground the force of the engine counters the force of friction.

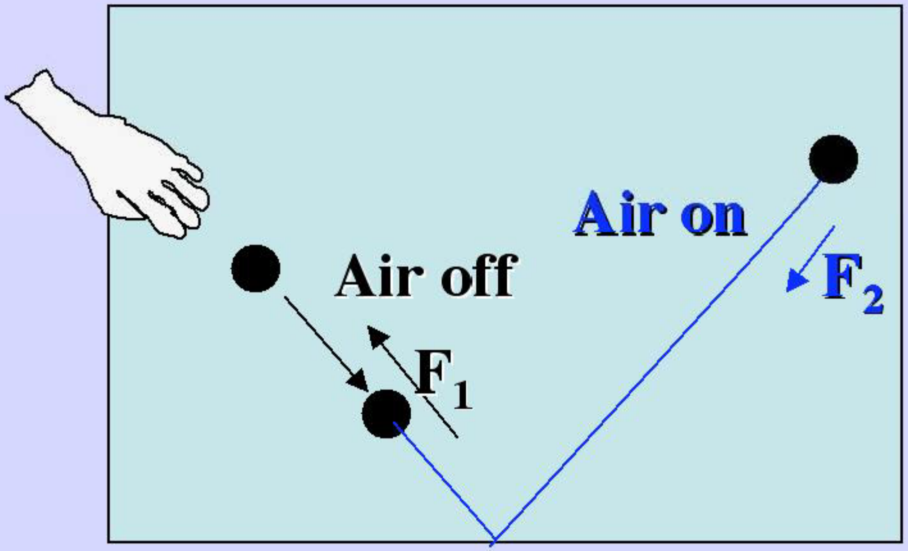
$$\boxed{\sum \vec{F} = 0}$$

On a hill, the force of the engine counters the force of friction and a component of gravity, working harder.

Examples of Newton's Laws of Motion

Law F=ma (2nd Law): The force on a body is equal to its change in momentum with time. For constant mass F=ma.

Air Hockey is a good demonstration of the first two laws.

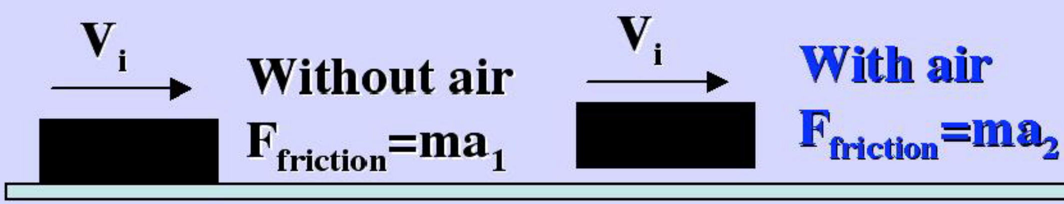


With the air off, the puck will stop shortly after it is released due to the friction force of the puck against the table.

With the air on the puck floats much much further because the friction force is reduced.

$F_1 > F_2$ therefore $a_1 > a_2$

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$



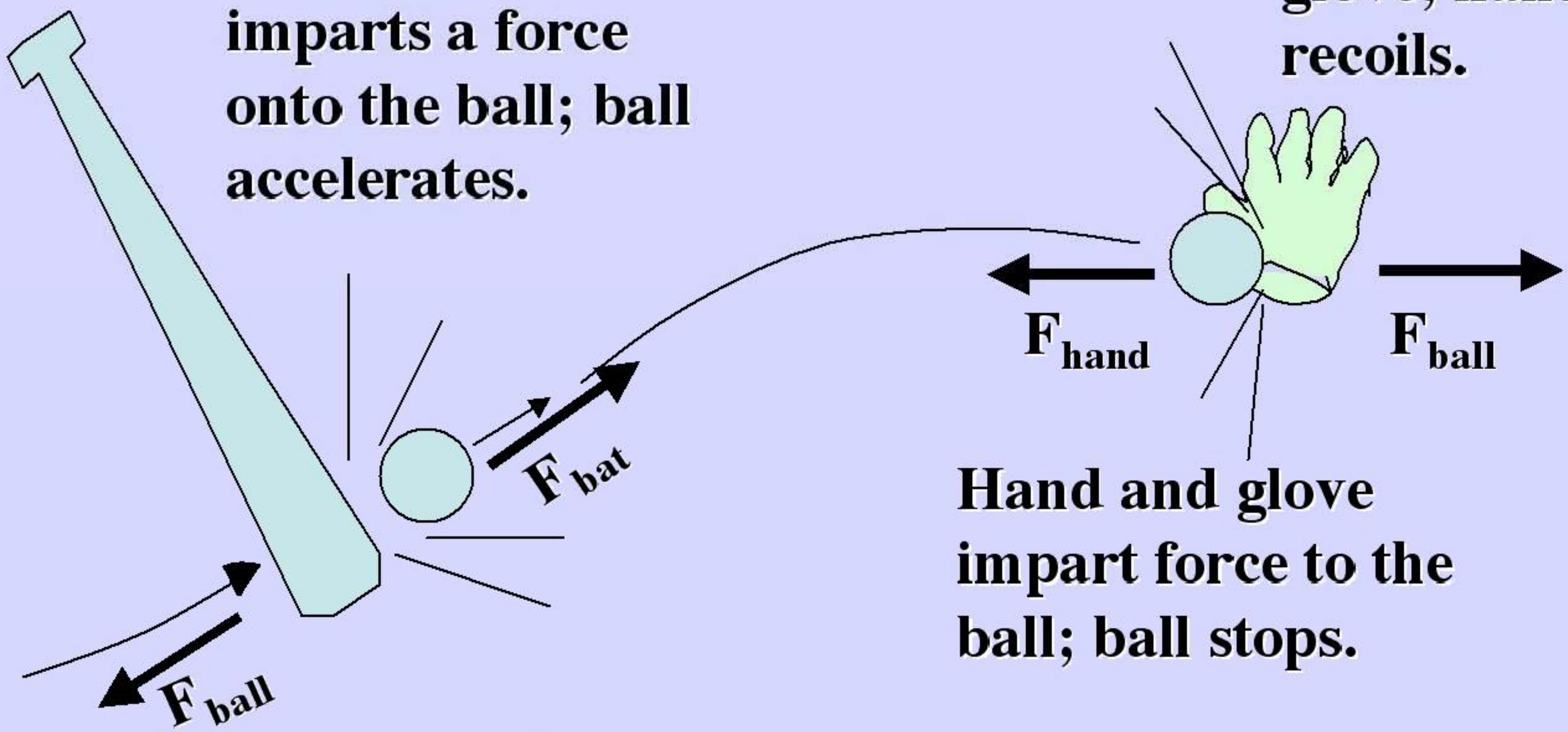
So, $x_2 > x_1$ (straight line)

Examples of Newton's Laws of Motion

Action-Reaction Pairs (3rd Law): Baseball!

Swinging bat imparts a force onto the ball; ball accelerates.

Ball imparts a force to the glove; hand recoils.



Hand and glove impart force to the ball; ball stops.

Ball imparts a force onto the bat; bat slows down.

A Guide to Problem Solving in NLM

Not always the case, but a good guide for typical cases.

1. Always identify all the forces acting on each body first.
2. Add all forces as vectors acting on each body.
3. Equate forces to ma for each body, typically determining a.
4. Use the kinematics you have learned in chapters 1-4 to solve motion.
5. Calculate any final quantities, such as momentum.

Example: Airport Runway

A fully loaded 747 jumbo jet has a mass of ***3.6x10^5kg***. Its four engines can exert a total thrust of ***F=7.7x10^5N***. How long a runway is required to obtain lift off speed of ***v=310km/h***?

From Newton's 2nd the acceleration is

$$a = F/m = 7.7 \times 10^5/3.6 \times 10^5 = 2.14 m/s^2$$

The liftoff velocity in *m/s* is ***v=310/3.6=86.1m/s.*** From the kinematic relation between velocity, distance, and acceleration we have:

$$v_f^2 - v_i^2 = 2a\Delta x \rightarrow \Delta x = v_f^2/2a = (86.1)^2/4.28 = 1730m$$

Example: Crate on a Truck

A ***280kg*** crate is secured in a truck with ropes as shown as shown in the figure. Assuming that initially the tension in the ropes is zero, what is tension in the when the deceleration of the truck is $\boldsymbol{a = 6.5m/s^2}$?

The horizontal force on the crate is the tension in the ropes. Under deceleration the tension in the rear rope provides the force required for the crate to decelerate. Newton's 2nd is then:

$$T = ma = 280(6.5) = 1820N$$

If the initial tension in the (inextensible) ropes is ***500N***, find the tension in both ropes?

$$\sum F = T_r - T_f = ma = 280(6.5) = 1820N$$

$$T_f = 500N \rightarrow T_r = (1820 + 500)N = 2320N$$

Mass and Weight
Force of Gravity

Weight is a force – the force that gravity exerts on a body.

Near the surface of the Earth all objects free fall with an acceleration of ***g = 9.8 m/s²***

From Newton's second law, ***F = ma***, the force of gravity is the body's weight:

$$W = mg$$

For example the weight of a ***80kg*** person on Earth is

$$W = 80(9.8)$$

Mass and Weight
Force of Gravity

The Pathfinder spacecraft that landed on Mars had a weight of ***2.70kN***.
(a) What was the Pathfinder's mass on Earth and Mars?

On Earth the mass is given by

$$m_E = \frac{W}{g} = \frac{2700}{9.8} = 275.5kg$$

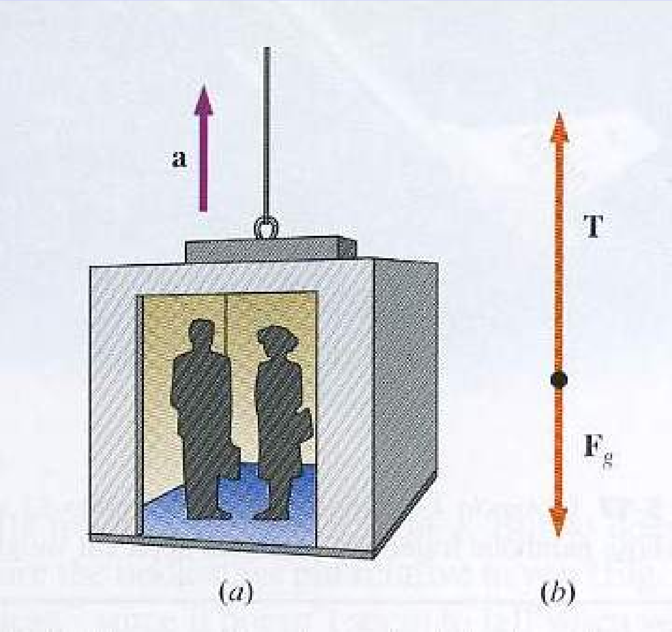
And its mass on Mars?

$$m_M = m_E \quad \text{doh!}$$

(b) If the free fall gravitational acceleration on Mars is ***g=3.74m/s²*** then the weight of the Pathfinder on Mars is?

$$W_M = mg_M = W_E \frac{g_M}{g_E} = 1.03kN$$

Example: Weight of an Elevator



A ***740kg*** elevator (including passengers) is accelerating upward at $\boldsymbol{a = 1.1 m/s^2}$. What is the tension force in the elevator cable?

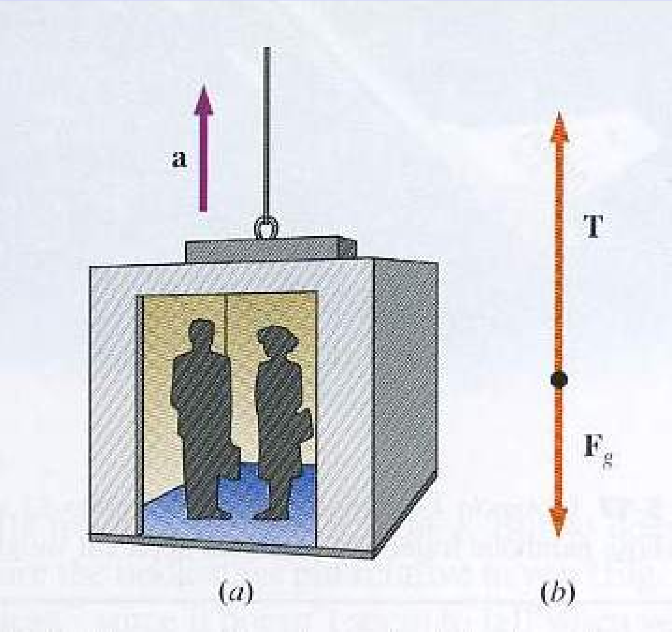
There are now two forces acting on the elevator, the force of gravity, ***W=Fg***, and the tension in the elevator cable.

Newton's second law states that it is the net force that induces the acceleration.

$$\vec{F}_{net} = \vec{T} - \vec{F}_g = m\vec{a}$$

For this problem we are only interested in the ***y*** (vertical) components of the forces and accelerations, and our equation only concerns itself with the those components (now a scalar equation).

Example: Weight of an Elevator



A *740kg* elevator (including passengers) is accelerating upward at $\boldsymbol{a = 1.1 m/s^2}$. What is the tension force in the elevator cable?

This scalar equation is:

$$T - mg = ma_y \rightarrow T = m(g + a_y)$$

Note that if $\boldsymbol{a_y = 0}$ then the tension is simply the weight. Also note that if $\boldsymbol{a_y = -g}$ (free fall) then the tension vanishes. For this acceleration, $\boldsymbol{a = 1.1 m/s^2}$, the tension is:

$$T = 740(9.8 + 1.1) = 8.07kN$$

What is the apparent change in the weight of the passengers?