Forces and Motion

Newton’s Laws Review
Newton’s Third Law
Hook’s Law – Springs

Lecture 8

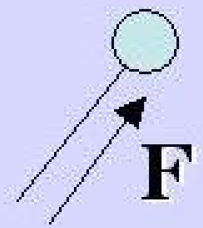
Newton's Laws of Motion

1. **The Law of Inertia:** The velocity of a body will not change unless a net force acts on that body.

2. **The Law of Force, Mass and Acceleration:** The force on a body is equal to its change in momentum with time. For constant mass, this is the mass times the acceleration.

3. **Law of Action and Reaction:** For each force acting on body B from body A, there is an equal and opposite force acting on body A from body B.

Example: Predicting Motion Using Newton's 2nd



A force $\vec{F} = 2.4\hat{i} + 1.7\hat{j}$ acts on a ***1.2 kg*** object which is initially at rest at the origin. (a) What is the object's acceleration? (b) What is its location after ***3.5s***? (c) How fast and in what direction is it moving after ***3.5s***?

(a) From Newton's 2nd the acceleration is:

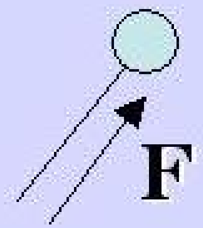
$$\vec{a} = \vec{F}/m = \left(2.4\hat{i} + 1.7\hat{j}\right)/1.2 = 2.0\hat{i} + 1.4\hat{j}\, m/s^2$$

(b) From the kinematics for an object with uniform acceleration the location is:

$$\vec{r}(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = 0 + \frac{1}{2}\left(2.0\hat{i} + 1.4\hat{j}\right)t^2$$

$$\vec{r}(3.5) = \frac{1}{2}\left(2.0\hat{i} + 1.4\hat{j}\right)3.5^2 = 12.25\hat{i} + 8.68\hat{j}\, m$$

Example: Predicting Motion Using Newton's 2nd



A force $\vec{F} = 2.4\hat{i} + 1.7\hat{j}$ acts on a ***1.2 kg*** object which is initially at rest at the origin. (a) What is the object's acceleration? (b) What is its location after ***3.5s***? (c) How fast and in what direction is it moving after ***3.5s***?

(c) From the kinematics for an object with uniform acceleration the velocity is:

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t = 0 + \left(2.0\hat{i} + 1.4\hat{j}\right)t$$
$$\vec{v}(3.5) = \left(2.0\hat{i} + 1.4\hat{j}\right)3.5 = 7\hat{i} + 5\hat{j}\,m/s$$

The object's speed after ***3.5s*** is: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{7^2 + 5^2} = 8.6m/s$

It's direction (as measured from the x axis) is: $\theta = \tan^{-1}\frac{5}{7} = 35.5^\circ$

Example: Hockey Player and that Puck

A hockey player strikes a ***170g*** puck accelerating it from rest to ***50 m/s***. If the hockey stick is in contact with the puck for ***2.5 ms***, what is the average force on the puck?

To find the average force we need the average acceleration:

$$\langle a \rangle = \frac{\Delta v}{\Delta t} = \frac{50 m/s}{2.5 \times 10^{-3} s} = 20 \times 10^{3} m/s^{2}$$

From Newton's 2nd the average force is:

$$\langle F \rangle = m\langle a \rangle = .17kg \times 20 \times 10^{3} m/s^{2} = 3.4kN$$

Example: A Truck and That Pole

A truck moving at ***70km/h*** collides with a pole. The front of the truck is compressed by ***.94m***. What average force must a seatbelt exert in order to restrain a ***75 kg*** passenger in this accident?

Again, to find the average force we need the average acceleration. From the kinematic relation between velocity, distance, and acceleration:

$$v_f^2 - v_i^2 = 2\langle a \rangle \Delta x \rightarrow \langle a \rangle = \frac{v_f^2 - v_i^2}{2\Delta x}$$

$$\langle a \rangle = \frac{0 - (70/3.6)^2}{2(.94)} = -201 m/s^2$$

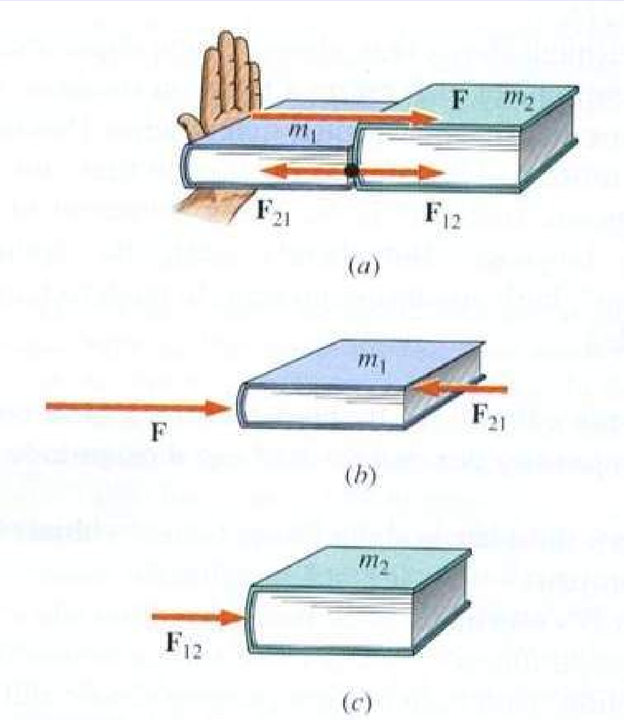
From Newton's 2nd the average force on the passenger is:

$$\langle F \rangle = m\langle a \rangle = 75kg \times (-201 m/s^2)$$

$$\langle F \rangle = -15kN$$

What's with the minus sign?

Example: Newton's 3rd - Pushing Those Books



On a surface with negligible friction, you push with a force $\boldsymbol{F}$ on a book of mass $\boldsymbol{m_1}$ that pushes on a book of mass $\boldsymbol{m_2}$. (a) What is the force, $\boldsymbol{F_{21}}$, exerted by the second book on the first?

From Newton's 2nd the acceleration of the books is:

$$a = \frac{F}{m_{tot}} = \frac{F}{m_1 + m_2}$$

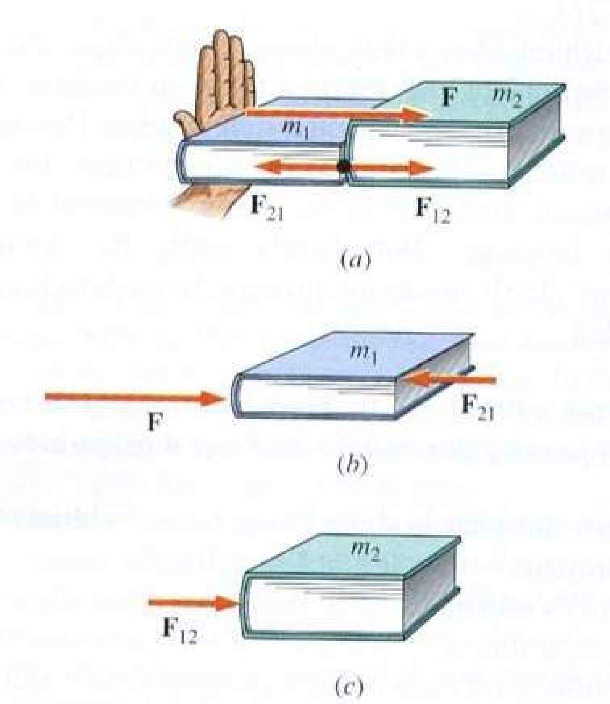
Both books are accelerating at the same rate. From Newton's 2nd the force of the first book on the second, $\boldsymbol{F_{12}}$, is:

$$F_{12} = m_2 a = \frac{m_2}{m_1 + m_2} F$$

From Newton's 3rd we know that the force of the second book on the first is:

$$F_{21} = -F_{12} = -\frac{m_2}{m_1 + m_2} F$$

Example: Newton's 3rd - Pushing Those Books



On a surface with negligible friction, you push with a force ***F*** on a book of mass $\boldsymbol{m_1}$ that pushes on a book of mass $\boldsymbol{m_2}$. (b) What is the net force, $\boldsymbol{F_1}$, exerted on the first book?

There are two forces acting on the first book. The force that occurs from you pushing on book 1, ***F***, as well as the force of book 2 on book 1, $\boldsymbol{F_{21}}$. The net force on book 1, $\boldsymbol{F_1}$, is:

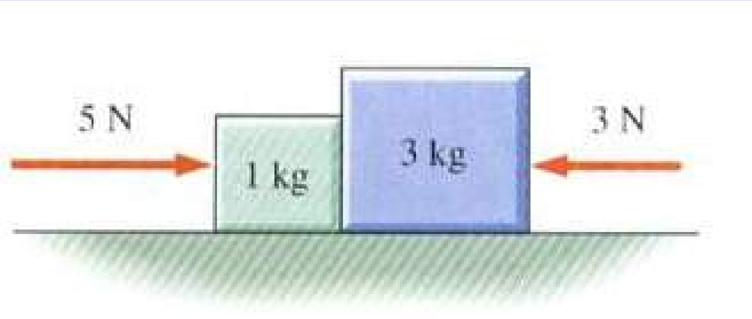
$$F_1 = F + F_{21} = F - \frac{m_2}{m_1 + m_2} F$$

$$F_1 = \frac{m_1}{m_1 + m_2} F$$

We should note that this force is consistent with Newton's 2nd as the acceleration of the first book is:

$$a_1 = \frac{F_1}{m_1} = \frac{F}{m_1 + m_2} = a!$$

Example: Newton's 3rd - Pushing Those Blocks



On a surface with negligible friction, there are two opposing forces, $\boldsymbol{F_1 = 5N}$ and $\boldsymbol{F_2 = -3N}$, acting on two blocks of mass $\boldsymbol{m_1 = 1kg}$ and $\boldsymbol{m_2 = 3kg}$. (a) What is the force, $\boldsymbol{F_{21}}$, of the second block acting on the first?

From Newton's 2nd the acceleration of both blocks is:

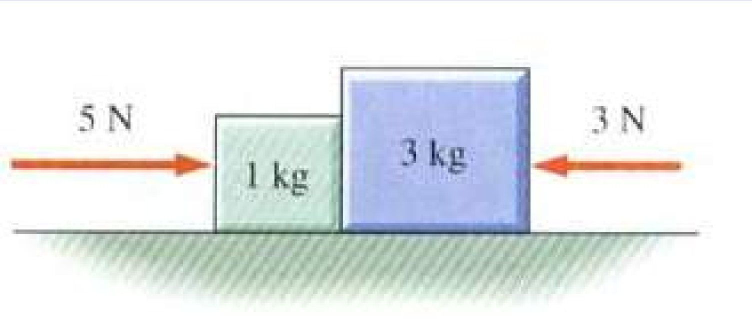
$$a = \frac{F_{net}}{m_{tot}} = \frac{F_1 + F_2}{m_1 + m_2} = \frac{5-3}{1+3} = \frac{1}{2} m/s^2$$

The net force acting on $\boldsymbol{m_2 = 3kg}$, is the force of the first block acting on the second, $\boldsymbol{F_{12}}$, and $\boldsymbol{F_2 = -3N}$. From Newton's 2nd:

$$F_{12} + F_2 = m_2 a \rightarrow F_{12} = m_2 a - F_2 = \frac{3}{2} + 3 = 4\frac{1}{2} N$$

From Newton's 3rd: $$F_{21} = -F_{12} = -4\frac{1}{2} N$$

Example: Newton's 3rd - Pushing Those Blocks



On a surface with negligible friction, there are two opposing forces, $\boldsymbol{F_1 = 5N}$ and $\boldsymbol{F_2 = -3N}$, acting on two blocks of mass $\boldsymbol{m_1 = 1kg}$ and $\boldsymbol{m_2 = 3kg}$. (b) What is the net force acting on the first block? Is this a consistent result?

Summing the forces acting on block 1 yields $\boldsymbol{F_{net}}$:

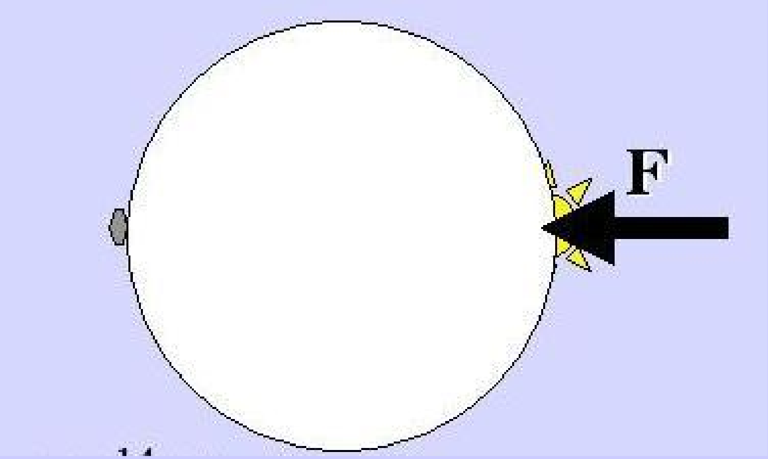
$$F_{net} = F_1 + F_{21} = 5 - 4\frac{1}{2} = \frac{1}{2}N$$

With this net force, from Newton's 2nd the acceleration of block 1 is:

$$a_1 = \frac{1/2N}{1kg} = \frac{1}{2}m/s^2 = a$$

Yes, this is a consistent result!

Example: Newton's 3rd - Pushing Those Blocks



A asteroid slams into the moon with a force of ***2x10⁹ N***. What is the force of a ***2kg*** rock on the other side of the moon acting on the moon?

From Newton's 2nd the acceleration of the moon is:

$$a = \frac{F}{M_{moon}} = \frac{2 \times 10^9}{7.34 \times 10^{22}} = 2.7 \times 10^{-14} m/s^2$$

To induce this acceleration the force, $\boldsymbol{F_{MR}}$, of the moon on the ***2kg*** rock is:

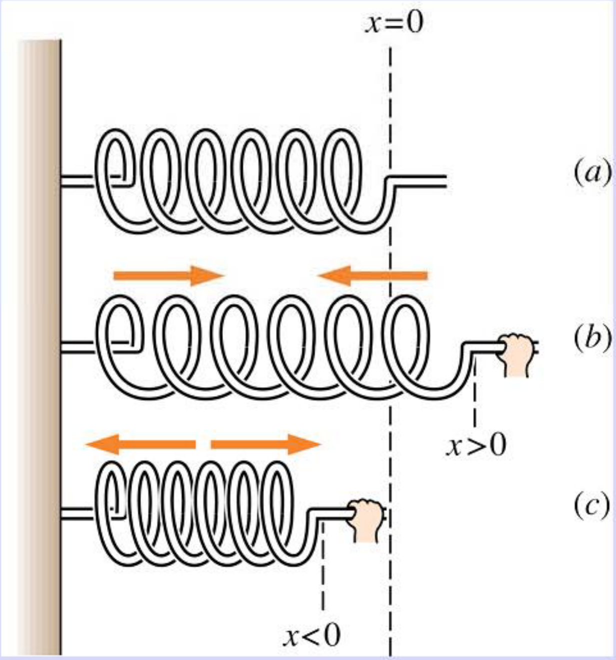
$$F_{MR} = ma = 2 \times 2.7 \times 10^{-14} = 5.4 \times 10^{-14} N$$

The force of the rock on the moon is the negative of this which is tiny! Again it is the acceleration of the total mass that is important. For a huge mass, the acceleration is small, hence this result!

Hook's Law and Newton's Laws

Hook's Law states that the elastic force of a spring is proportional to the displacement of the spring (for small displacements).

$$F = -kx$$



Where k is the "spring constant".

k has units of N/m.

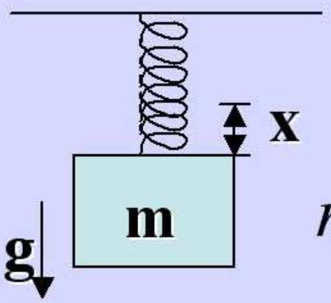
Hooks law is routinely used to measure the forces on objects in our every day life, from weight scales to other indicator needle instruments such as pressure monitors.

Measuring Mass with Hook's Law

Any time an accelerated object is connected to a spring and we know the displacement, spring constant, and acceleration of the spring and the object respectively, we can measure the mass.

$$F = -kx = ma$$

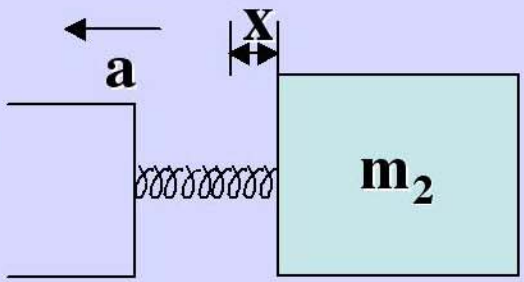
On Earth we compress or stretch the string against gravity to measure mass



$$m = \frac{kx}{g}$$

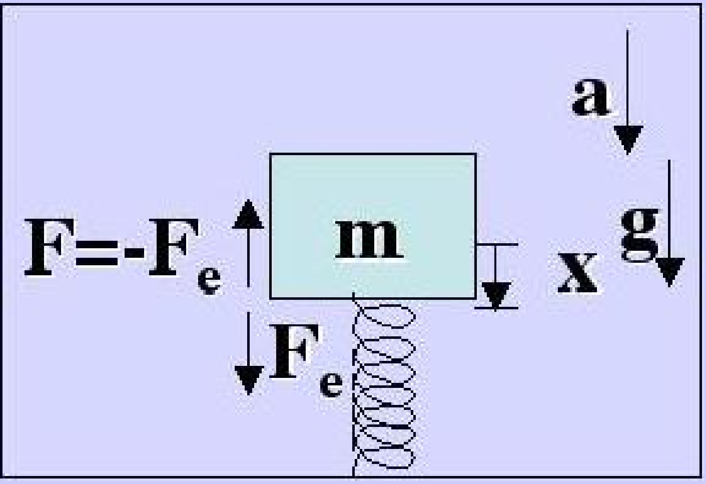
Note: on another planet, g and the object's weight will differ, but the mass will always stay the same.

We can also simply accelerate the object horizontally



$$m = \frac{kx}{a}$$

Example: Measuring Mass in an Elevator



An object in an elevator is on a spring which is accelerating downward at ***a = 1m/s²***. What is the mass of the object if the displacement of the spring is ***.02m*** (relative to having no mass) and the spring constant is ***k = 20N/m***?

There are two forces acting on the object, gravity and force of the spring in the elevator. Defining the positive direction to be upward, Newton's 2nd yields:

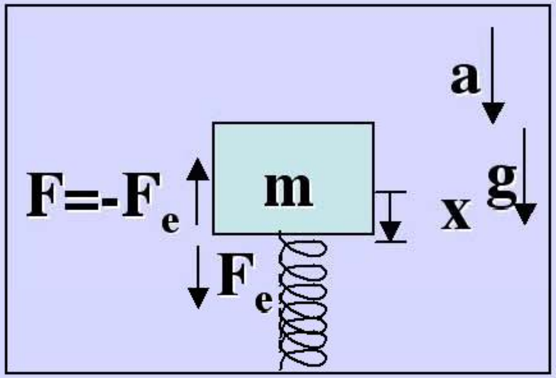
$$F_g + kx = -mg + kx = -ma \rightarrow kx = m(g - a)$$

$$m = \frac{kx}{g-a} = \frac{20(.02)}{9.8-1} = .045kg = 45g$$

Note that the displacement of the object is negative.

Side Note: Our Reference Frame

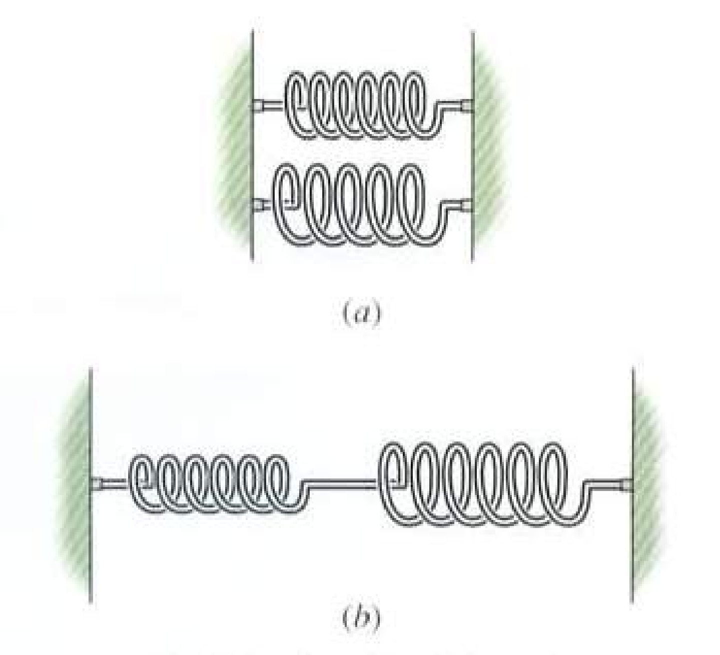
In this example, the reference frame has been that of being at rest and observing the elevator accelerating. This is often called the “Lab Frame”.



For a person inside the elevator, the displacement of the spring is measurable, however the acceleration is not. (they can’t see outside)

Reference frames have been on the side-lines so far, but should always be kept in mind. **They are important, know them!**

Example: Springs in (a) Parallel and (b) Series



(a) Two springs which have the same unstretched length but different spring constants, $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$**,** are connected side-by-side. Find the new effective spring constant.

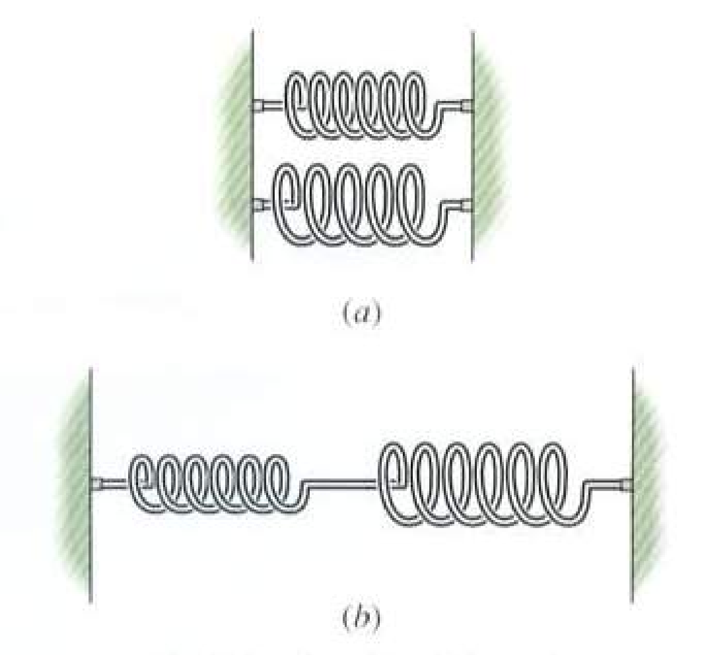
If the springs are compressed/stretched an equal distance $\boldsymbol{x}$ from equilibrium then the restoring force is simply:

$$F = -k_1 x - k_2 x = -(k_1 + k_2)x = -k_{eff}x$$

$$k_{eff} = k_1 + k_2$$

Thus if two springs are arranged in parallel (a) the effective spring constant is simply a sum of the two spring constants.

Example: Springs in (a) Parallel and (b) Series



(b) Two springs have different spring constants $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$ and are connected end-to-end. Find the new effective spring constant.

Now consider the forces acting on the springs in (b). From Newton's 3rd the springs are pulling/pushing on each other with equal strength. Hence the tension/compression, $\boldsymbol{F}$, in both springs is equal.

Summing the displacements of the springs:

$$\Delta x_1 + \Delta x_2 = \frac{F}{k_1} + \frac{F}{k_2} = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

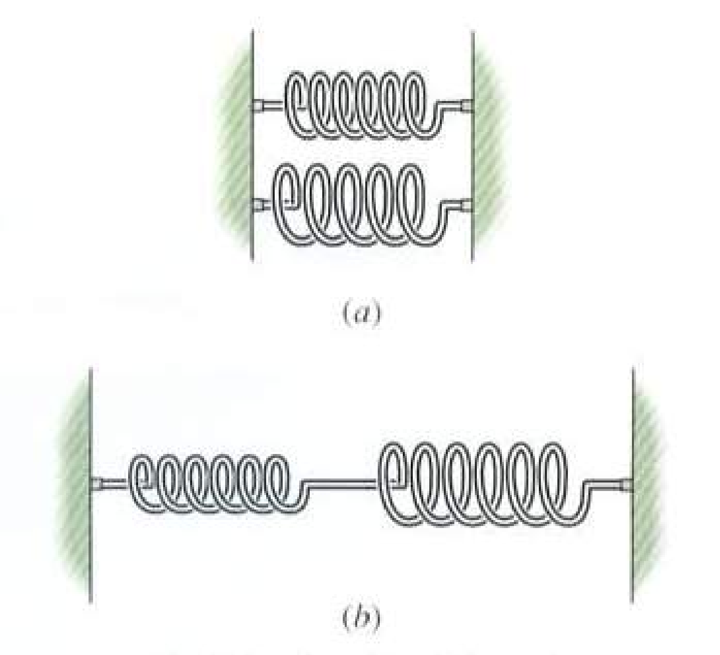
$\boldsymbol{\Delta x_1 + \Delta x_2}$ is the total displacement of the springs connected in series, (b).

Hence the effective spring constant is:

$$k_{eff} = \frac{F}{\Delta x} = \frac{F}{\Delta x_1 + \Delta x_2}$$

$$\frac{1}{k_{eff}} = \frac{\Delta x_1 + \Delta x_2}{F} = \frac{1}{k_1} + \frac{1}{k_2}$$

Example: Springs in (a) Parallel and (b) Series



To summarize, two springs connected in parallel each have the same **displacement**. This means that their restoring forces add, and the effective spring constant is:

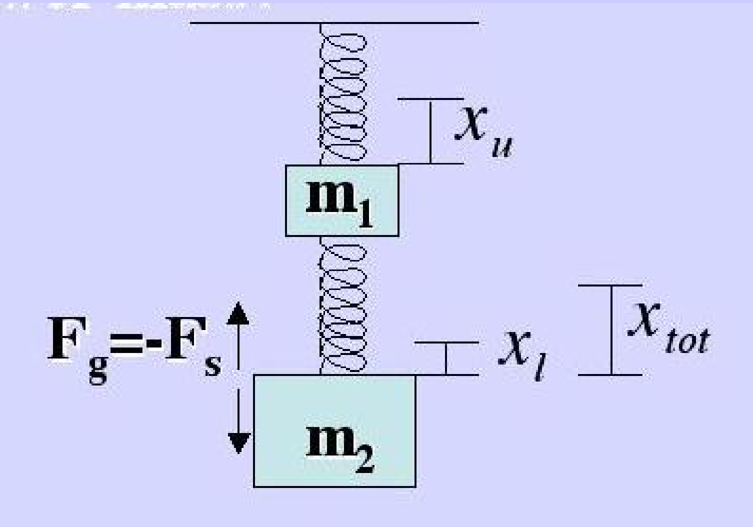
$$k_{eff} = k_1 + k_2$$

If the spring are connected in series, then from Newton's 3rd they each experience the same **restoring force** and the displacement is the sum of the individual displacements. The effective spring constant for this case is:

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} \rightarrow k_{eff} = \frac{k_1 k_2}{k_1 + k_2}$$

Which configuration has the larger spring constant?

Example: Springs in Series with an Additional Mass



Two springs each with a spring constant of ***k = 20N/m*** support two mass, ***m_1 = .2kg*** and ***m_2 = .4kg*** as shown. Find the displacement from equilibrium of each spring.

For this configuration the lower spring is supporting ***m_2***. From Hook's law its displacement is:

$$x_l = \frac{m_2 g}{k} = \frac{.4 \times 9.8}{20} = .196m = 19.6cm$$

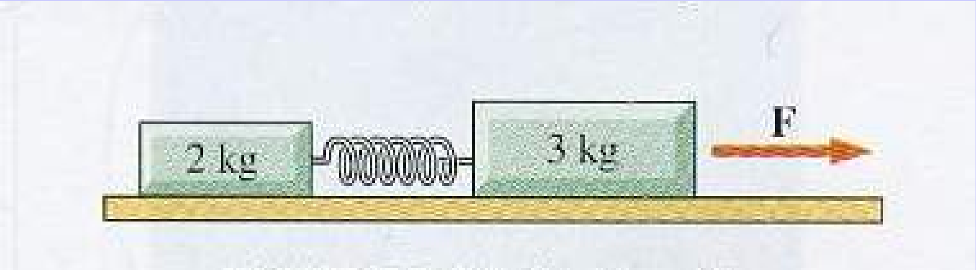
From Newton's 3rd, the upper spring is supporting both masses. From Hook's law:

$$x_u = \frac{(m_2 + m_1)g}{k} = \frac{.6 \times 9.8}{20} = .294m = 29.4cm$$

The total displacement is:

$$x_{tot} = x_u + x_l = 29.4 + 19.6 = 49cm$$

Example: Springs in Series with an Additional Mass



Two masses of mass $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ are connected by a spring with spring constant $\boldsymbol{k}$. A force $\boldsymbol{F}$ is applied to the larger of the two masses. (a) How much does the spring stretch from its equilibrium length? (b) Find the net force on the larger mass.

(a) This acceleration of this combination is determined by Newton's 2nd:

$$a = F/(m_1 + m_2)$$

From Hook's law and Newton's 2nd, the displacement of the spring is

$$x = \frac{m_1 a}{k} = \frac{m_1}{m_1 + m_2}\frac{F}{k}$$

(b) The net force on the larger mass is:

$$F_{net} = F - kx = \left(1 - \frac{m_1}{m_1 + m_2}\right)F = \frac{m_2}{m_1 + m_2}F$$

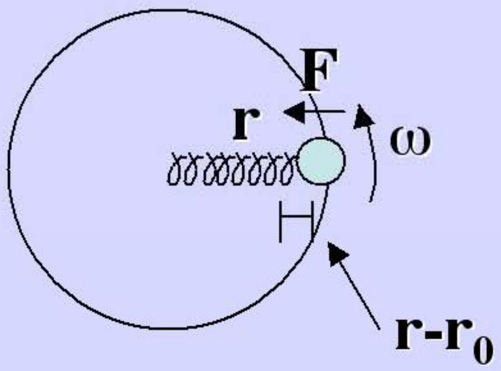
For a force of ***15N*** and a spring constant of ***140N/m***:

$$x = \frac{2}{5}\frac{15}{140} = 4.3cm$$

Example: Hook's Law

A mass m is in uniform circular motion at angular frequency ω on a spring, which displaces a distance $r-r_0$. What is the constant k of the spring?

The displacement is relative to the "unstretched or compressed" length of the spring. Thus the force:



$$F = -k(r - r_o)$$

But this force is equal and opposite to the centripetal force for uniform circular motion which points outward:

$$F = m\frac{v^2}{r} \quad \text{where} \quad v = r\omega$$

Equating them:

$$\boxed{k = \frac{r}{r - r_0} m\omega^2}$$

Remember that the change in a springs length is NOT the springs length!