Today's Lecture

Review of Newton's 3rd Law

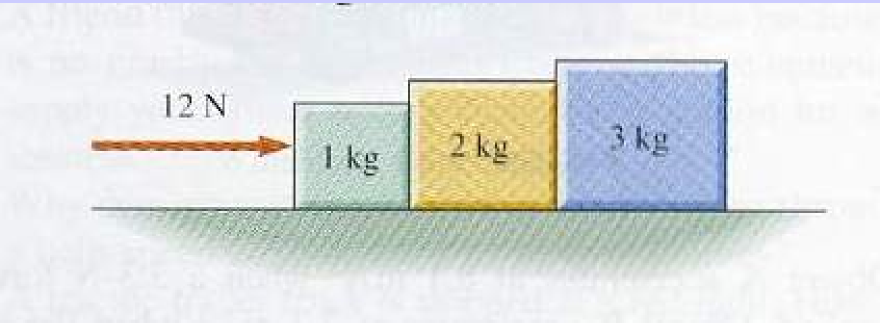
& Hook's Law

Application of Newton's Laws:

Friction

Lecture 9

Example: Using Newton's Third Law



A ***12N*** force is applied to left most of a ***1kg***, ***2kg***, and a ***3kg*** block as shown.
(a) What force does the middle block exert on the rightmost block, $\boldsymbol{F_{23}}$?

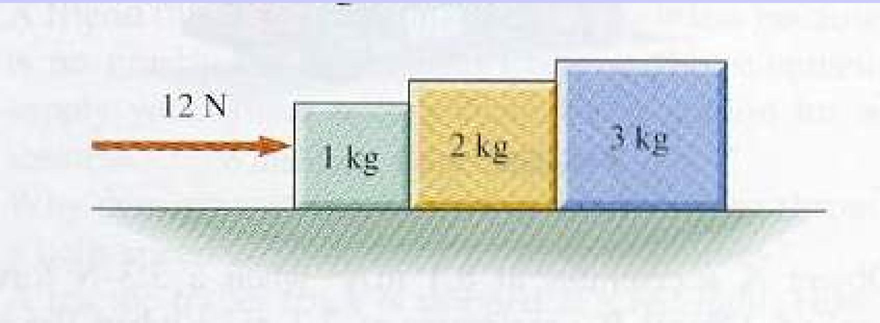
From Newton's 2nd, the acceleration of all of the blocks is:

$$a = F/m_{tot} = 12N/6kg = 2m/s^2$$

Hence the force from the middle block on the 3rd block is given by:

$$F_{23} = m_3 a = 3(2) = 6N$$

Example: Using Newton's Third Law



A ***12N*** force is applied to left most of a ***1kg***, ***2kg***, and a ***3kg*** block as shown.
(b) What force does the first block exert on the middle block?

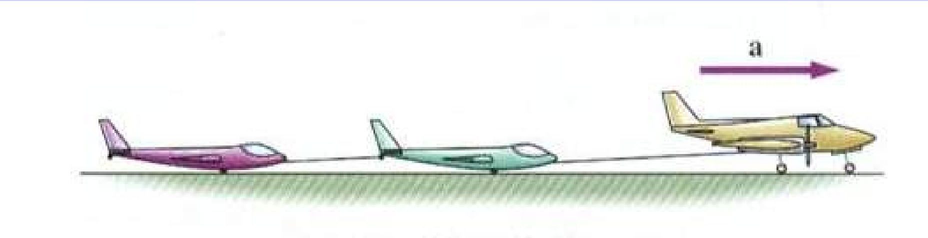
The acceleration of all of the blocks is $\boldsymbol{a = 2m/s^2}$. The net force of the middle block can also be determined from Newton's 3rd and Newton's 2nd:

$$F_{1net} = F_{12} + F_{32} = F_{12} - F_{23} = m_2 a = 2(2) = 4N$$

From Newton's 3rd $\boldsymbol{F_{23} = 6N}$. Hence $\boldsymbol{F_1}$ is given by:

$$F_{12} = F_{23} + F_{1net} = 6 + 4 = 10N$$

Example: Using Newton's Third Law



A ***2200kg*** plane is pulling two gliders down the runway with an acceleration of ***1.9m/s²***. The first glider has a mass of ***310kg*** and the second ***260kg***.

(a) Find the horizontal thrust of the plane's propeller.

From Newton's 2nd, the thrust is:

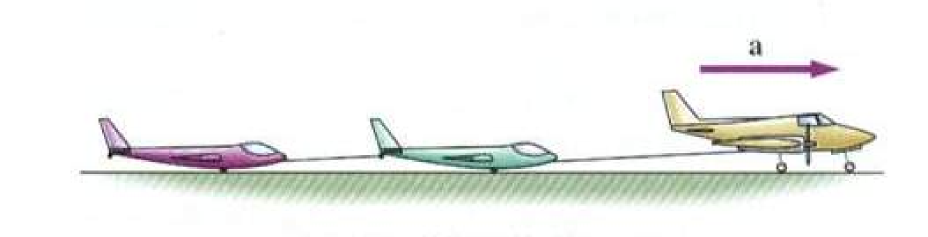
$$F_{thrust} = m_{tot}a = (310 + 260 + 2200) \times 1.9 = 5263N$$

(b) Find the tension in the first rope.

Since the only force on the first glider is the tension in the first rope, from Newton's 2nd we have:

$$T_1 = m_1 a = 310 \times 1.9 = 589N$$

Example: Using Newton's Third Law



A ***2200kg*** plane is pulling two gliders down the runway with an acceleration of ***1.9m/s²***. The first glider has a mass of ***310kg*** and the second ***260kg***.

(c) Find the tension in the second rope.

From Newton's 3rd the tension in the first rope opposes the motion of the 2nd glider. The acceleration of the all gliders is ***1.9m/s²***. Newton's 2nd:

$$F_{net} = T_2 - T_1 = ma \rightarrow T_2 = T_1 + ma$$

$$T_2 = 589 + 260 \times 1.9 = 1083N$$

(d) Find the net force on the plane.

The net force on the first plane is the thrust minus the tension in the second rope:

$$F_{net} = F_{Thrust} - T_2$$

$$F_{net} = 5263 - 1083 = 4180N$$

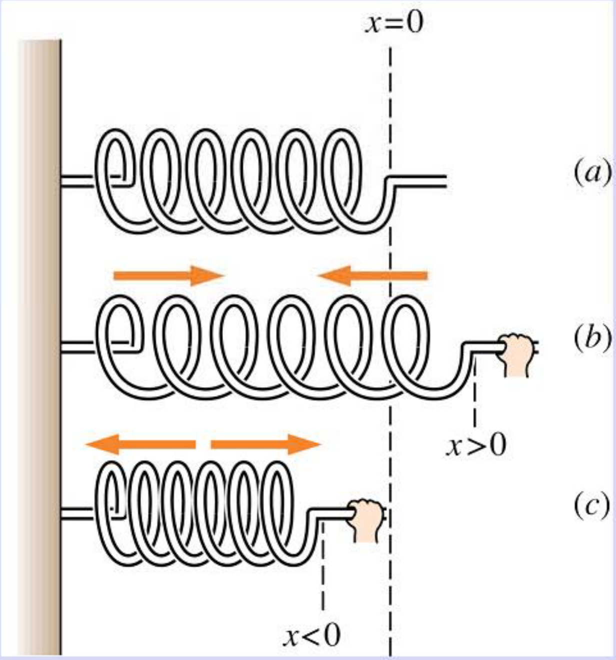
Is this consistent with ***a=1.9m/s²***?

$$a = 4180N/2200kg = 1.9m/s^2$$ Yes!

Hook's Law and Newton's Laws

Hook's Law states that the elastic force of a spring is proportional to the displacement of the spring (for small displacements).

$$F = -kx$$

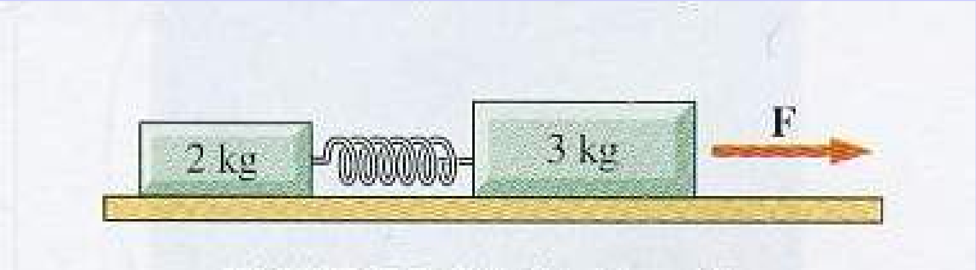


Where k is the "spring constant".

k has units of N/m.

Hooks law is routinely used to measure the forces on objects in our every day life, from weight scales to other indicator needle instruments such as pressure monitors.

Example: Springs in Series with an Additional Mass



Two masses of mass $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ are connected by a spring with spring constant $\boldsymbol{k}$. A force $\boldsymbol{F}$ is applied to the larger of the two masses. (a) How much does the spring stretch from its equilibrium length? (b) Find the net force on the larger mass.

(a) The acceleration of this system is determined by Newton's 2nd:

$$a = F/(m_1 + m_2)$$

From Hook's law and Newton's 2nd, the displacement of the spring is

$$x = \frac{m_1 a}{k} = \frac{m_1}{m_1 + m_2}\frac{F}{k}$$

(b) The net force on the larger mass is:

$$F_{net} = F - kx = \left(1 - \frac{m_1}{m_1 + m_2}\right)F = \frac{m_2}{m_1 + m_2}F$$

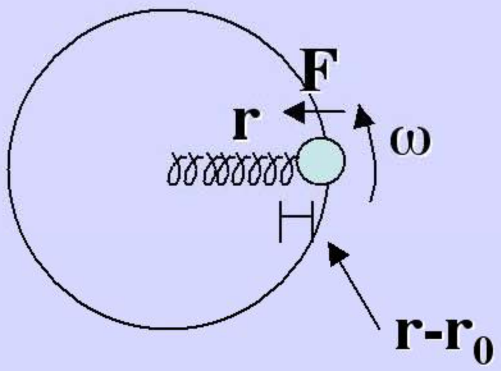
For a force of ***15N*** and a spring constant of ***140N/m***:

$$x = \frac{2}{5}\frac{15}{140} = 4.3cm$$

Example: Hook's Law

A mass m is in uniform circular motion at angular frequency ω on a spring, which displaces a distance $r-r_0$. What is the constant k of the spring?

The displacement is relative to the "unstretched or compressed" length of the spring. Thus the force:



$$F = -k(r - r_o)$$

But this force is equal and opposite to the centripetal force for uniform circular motion which points outward:

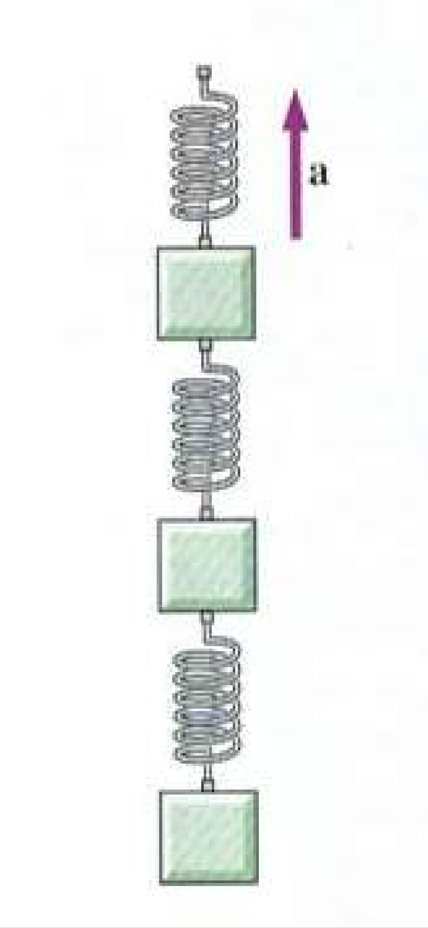
$$F = m\frac{v^2}{r} \quad \text{where} \quad v = r\omega$$

Equating them:

$$k = \frac{r}{r - r_0} m\omega^2$$

Remember that the change in a springs length is NOT the springs length!

Example: Three Springs



Three identical springs of equal unstretched length ***l*** and spring constant ***k*** are connected to equal masses *m* as shown. A force is applied to give the top of the upper spring that causes an acceleration ***a*** of the entire system. Determine the length of each spring.

From Newton's 2nd the force, tension, that induces the acceleration of the entire system is:

$$F = 3ma$$

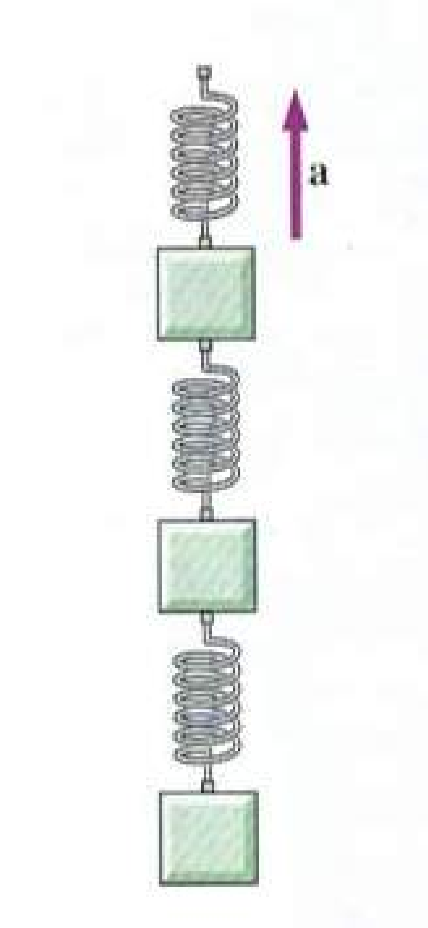
Hence the tension in the spring attached to the first (lowest) block is:

$$F_1 = ma = \frac{F}{3}$$

The length of this spring must be:

$$l_1 = l + x_1 = l + \frac{F_1}{k} = l + \frac{ma}{k}$$

Example: Three Springs



Three identical springs of equal unstretched length ***l*** and spring constant ***k*** are connected to equal masses *m* as shown. A force is applied to give the top of the upper spring that causes an acceleration ***a*** of the entire system. Determine the length of each spring.

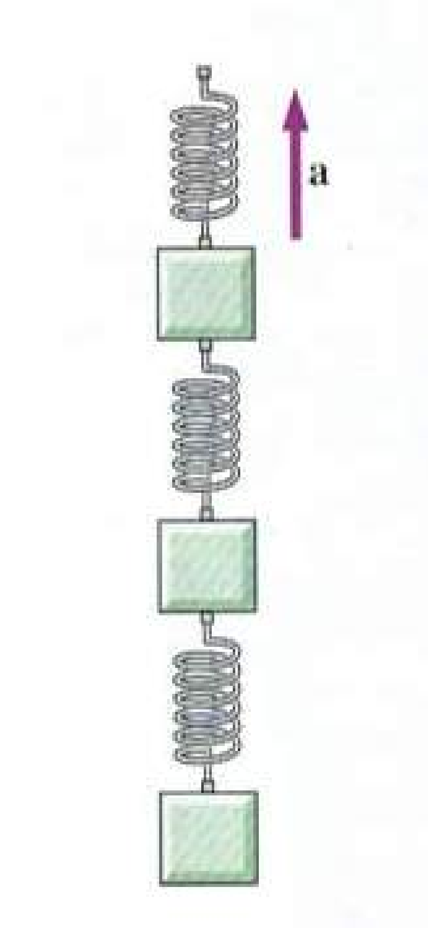
The net force on the second block is the tension in the second spring minus the tension in the first spring:

$$F_{2net} = F_2 - F_1 = ma \rightarrow F_2 = 2ma$$

The length of the second spring must be:

$$l_2 = l + x_2 = l + \frac{F_2}{k} = l + \frac{2ma}{k}$$

Example: Three Springs



Three identical springs of equal unstretched length ***l*** and spring constant ***k*** are connected to equal masses *m* as shown. A force is applied to give the top of the upper sppring that causes an acceleration ***a*** of the entire system. Determine the length of each spring.

The net force on the third block is the force $F - F_2$. Hence the tension in the top spring, ***F***, is

$$F - F_2 = ma \rightarrow F = 3ma$$

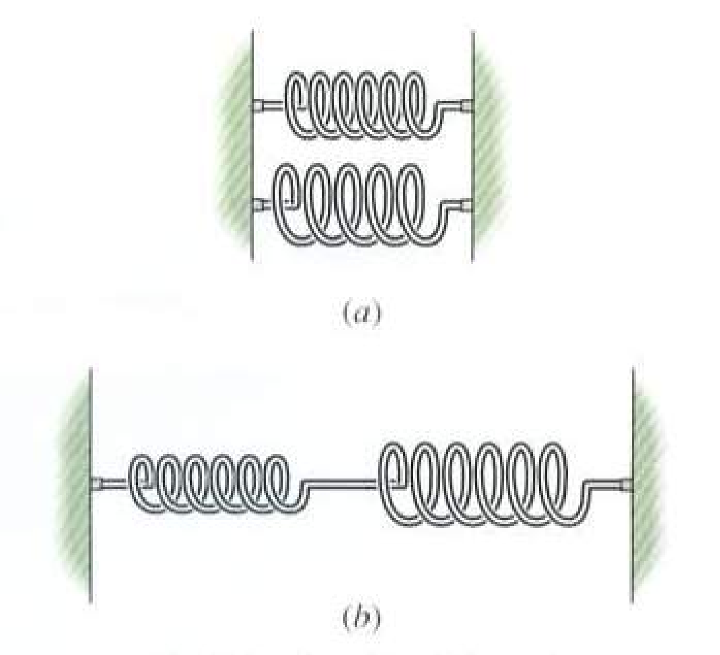
The length of the third spring must be:

$$l_3 = l + x_3 = l + \frac{F}{k} = l + \frac{3ma}{k}$$

What changes if there is a uniform gravitational field?

$$a \rightarrow g + a$$

Example: Springs in (a) Parallel and (b) Series



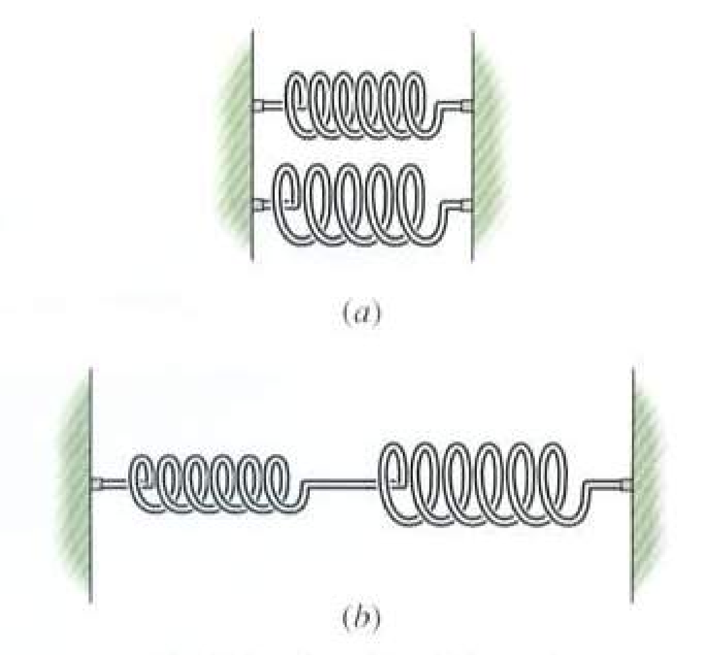
(a) Two springs which have the same unstretched length but different spring constants, $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$, are connected side-by-side. Find the new effective spring constant.

If the springs are compressed/stretched an equal distance $\boldsymbol{x}$ from equilibrium then the restoring force is simply:

$$F = -k_1 x - k_2 x = -(k_1 + k_2)x = -k_{eff}x$$

Thus if two springs are arranged in parallel (a) the effective spring constant is simply a sum of the two spring constants.

Example: Springs in (a) Parallel and (b) Series



(b) Two springs have different spring constants $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$ and are connected end-to-end. Find the new effective spring constant.

Now consider the forces acting on the springs in (b). From Newton's 3rd the springs are pulling/pushing on each other with equal strength. Hence the force, $\boldsymbol{F}$, of tension/compression in both springs is equal.

Summing the displacements of the springs:

$$\Delta x_1 + \Delta x_2 = \frac{F}{k_1} + \frac{F}{k_2} = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

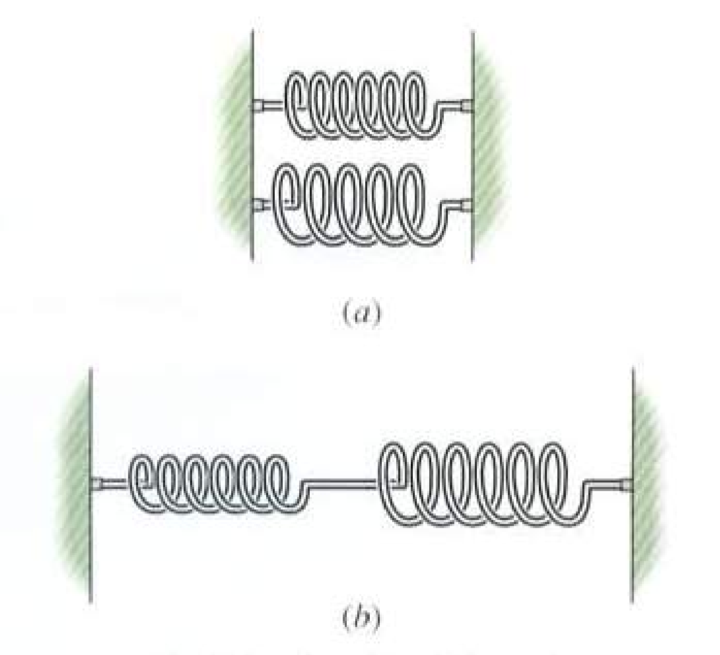
$\boldsymbol{\Delta x_1 + \Delta x_2}$ is the total displacement of the springs connected in series, (b).

Hence the effective spring constant is:

$$k_{eff} = \frac{F}{\Delta x} = \frac{F}{\Delta x_1 + \Delta x_2}$$

$$\frac{1}{k_{eff}} = \frac{\Delta x_1 + \Delta x_2}{F} = \frac{1}{k_1} + \frac{1}{k_2}$$

Example: Springs in (a) Parallel and (b) Series



To summarize, two springs connected in parallel each have the same **displacement**. This means that their restoring forces add, and the effective spring constant is:

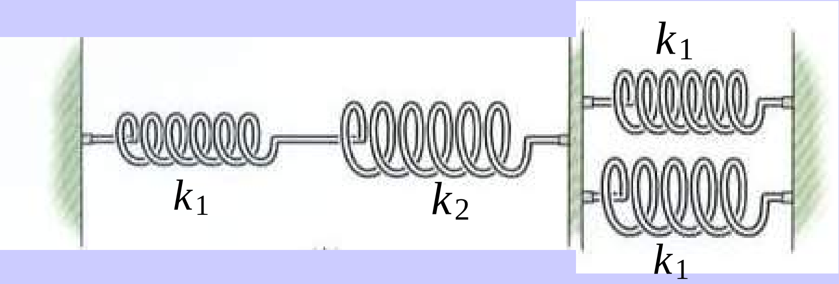
$$k_{eff} = k_1 + k_2$$

If the spring are connected in series, then from Newton's 3rd they each experience the same **restoring force** and the displacement is the sum of the individual displacements. The effective spring constant for this case is:

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} \rightarrow k_{eff} = \frac{k_1 k_2}{k_1 + k_2}$$

Which configuration is stiffer?

Example: Springs in Parallel Plus Series



Consider the combination of springs shown in the figure with $\boldsymbol{k_1 = 10N/cm}$ and $\boldsymbol{k_2 = 20N/cm}$. Find the effective spring constant for this combination.

The effective spring constant for the first two springs in series is:

$$\frac{1}{k_{1eff}} = \frac{1}{k_1} + \frac{1}{k_2}$$

The effective spring constant for the last two springs in parallel is:

$$k_{2eff} = 2k_1$$

The effective spring constant for these two effective spring constants is:

$$\frac{1}{k_{eff}} = \frac{1}{k_{1eff}} + \frac{1}{k_{2eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{2k_1} = \frac{3k_2 + 2k_1}{2k_1 k_2}$$

For the given spring constants:

$$k_{eff} = \frac{2k_1 k_2}{3k_2 + 2k_1} = \frac{400}{80}\frac{N}{cm} = 5N/cm$$

Chapter 6

Using Newton's Laws

Note, we have already begun to introduce some of the material in Chapter 6, such as problems including forces in two dimensions.

We will focus today on:

- Formal definition of Free Body Diagrams
- Definition and inclusion of Friction force.
- Tension in a string.

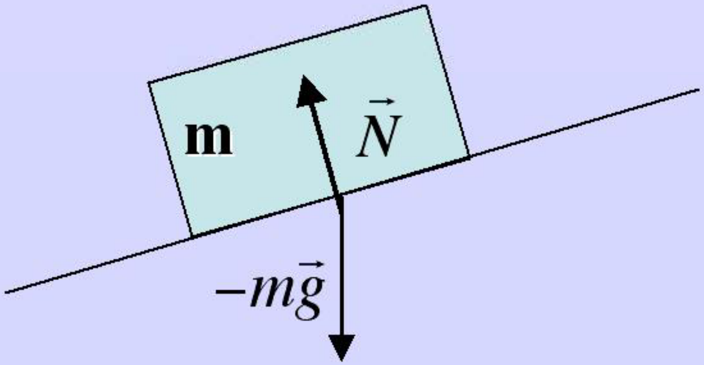
Free Body Diagrams

These are diagrams of all forces acting on an object at a single time. You've already done this occasionally, but here are a few points which reinforce their use.

Show vectors of every force that acts on the body from other bodies only.

Never include two counter forces (Newton's 3rd) in the same diagram.

Do not include forces this body imparts to others.

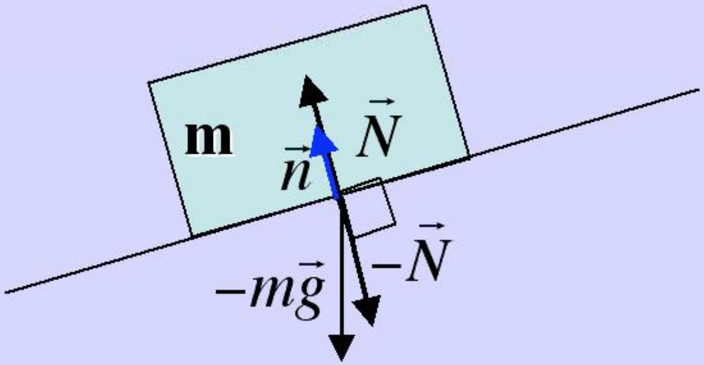


A "raw" free body diagram of a block on an incline plane.

The Normal Force

When a body pushes against a surface, a component of force points directly into that surface at a right angle to the surface. The unit vector along the direction outward from the surface is called the "normal" and is written $\vec{n}$.

By Newton's 3rd Law we know that a corresponding force acts equal and opposite, outward from the surface. This force is called the **Normal Force.** We call this force $\vec{N}$.



The normal force determines the magnitude of the Friction Force between the body and the surface.

What is Friction?

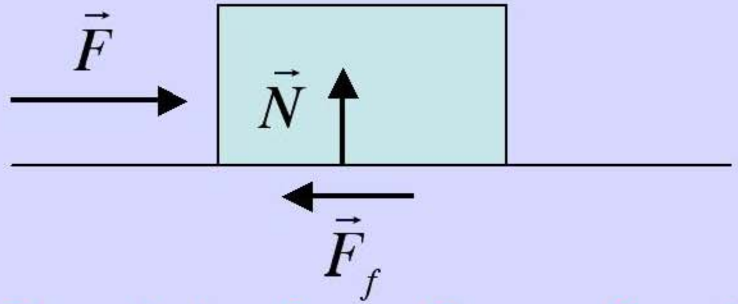
When a body is in contact with a surface, bonding and roughness between the surface and the contact point of the body cause a resistance to sliding along that surface.

This resistance can be represented by a net friction force.

The friction force is always opposite to the velocity of the body. For a sliding body it is along the surface perpendicular to *N*.

The magnitude of the friction force is proportional to the normal force.

$$F_f = \mu_f N$$



μ is dimensionless.

Remember F_f points along the surface!

The Two Types of Friction

When a body is in contact with a surface and does not move, the friction force is different for the same pushing force than when the body is sliding along the surface. These two magnitudes of friction are called Static and Kinetic Friction, respectively. Typically, $\mu_{fS} > \mu_{fK}$.

Static

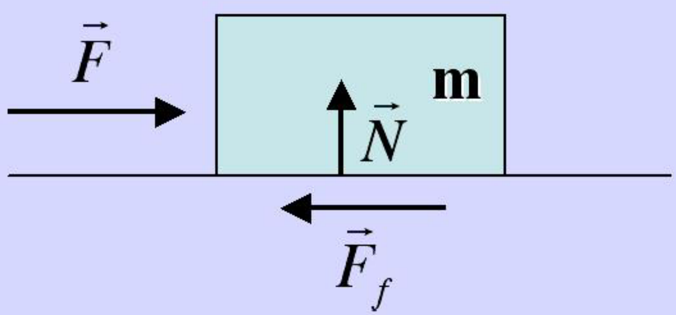
$$\vec{V} = \vec{0}$$

$$\sum \vec{F} = \vec{0}$$

$$\vec{a} = \vec{0}$$

$$F_{fS} \leq \mu_{fS} N$$

NOTE!



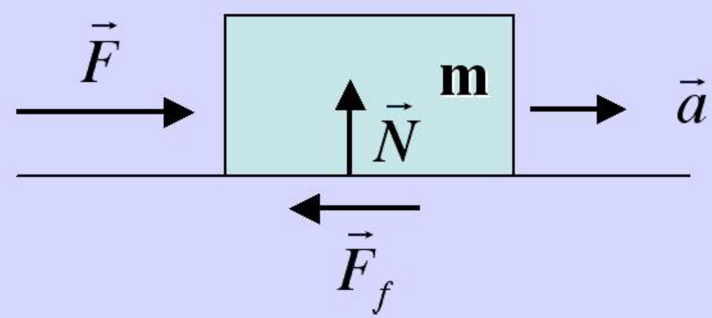
Kinetic

$$\vec{V} = \vec{V}_0 + \vec{a}t$$

$$\sum \vec{F} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

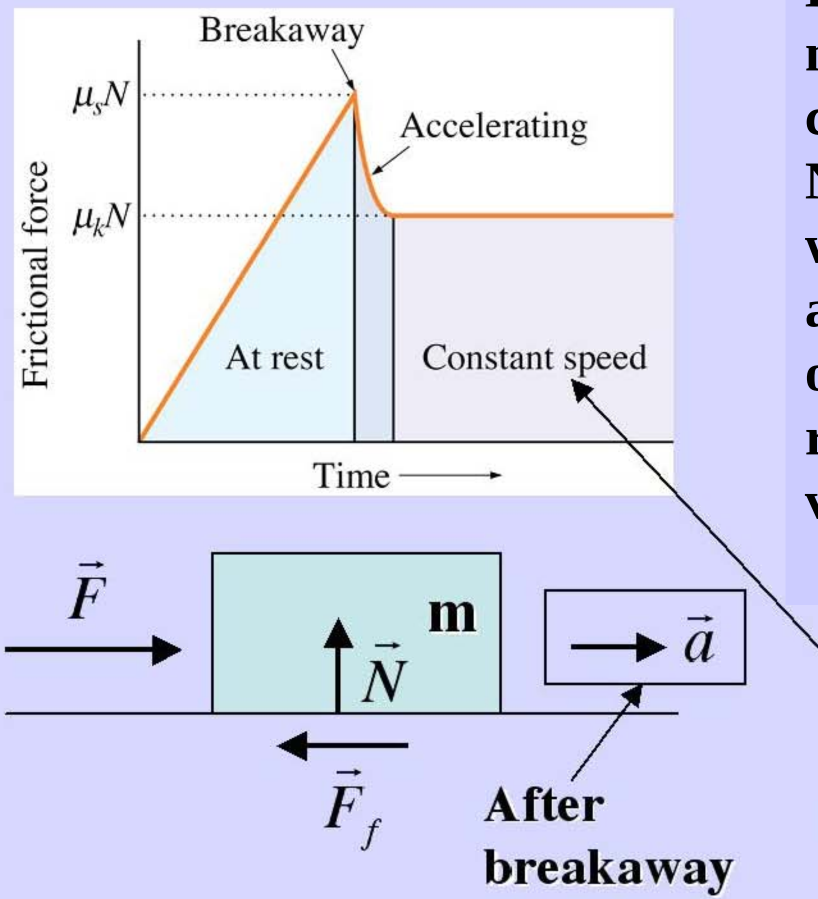
$$F_{fK} = \mu_{fK} N$$



The Two Types of Friction

The fact that $\mu_{fS} > \mu_{fK}$ is apparent in transition from a stationary object to a moving one with an increasing pushing force.

Before the object breaks away and begins moving, the applied force is exactly countered by a static frictional force by Newton's 3rd. After break away, the object will accelerate. The frictional force will adjust to the lower kinetic value once the object is in motion. If the applied force is reduced to match the frictional force, the velocity will be constant.

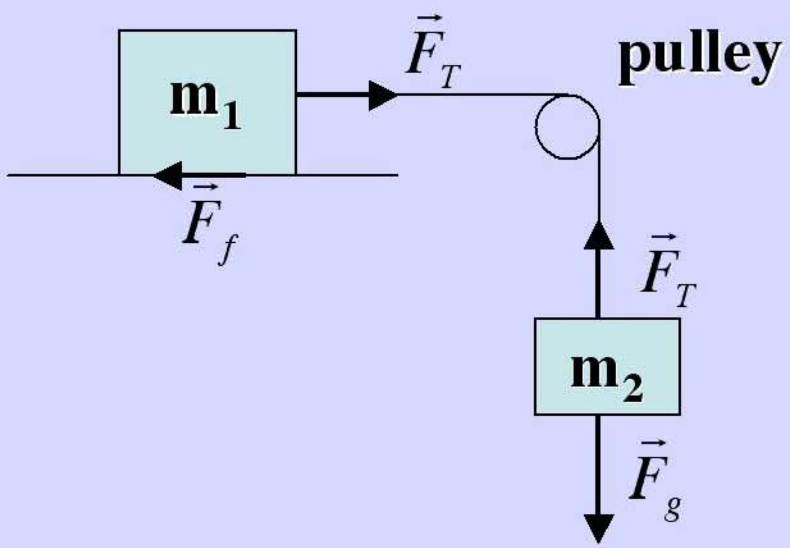


(Assumes applied force adjusted down)

If the applied force remains greater than the friction force, the object will accelerate.

Tension in a String

When a string is attached to a body and pulled on one side by a force, the string equally pulls on the body in the opposite direction (3rd Law). This force is called the tension force.

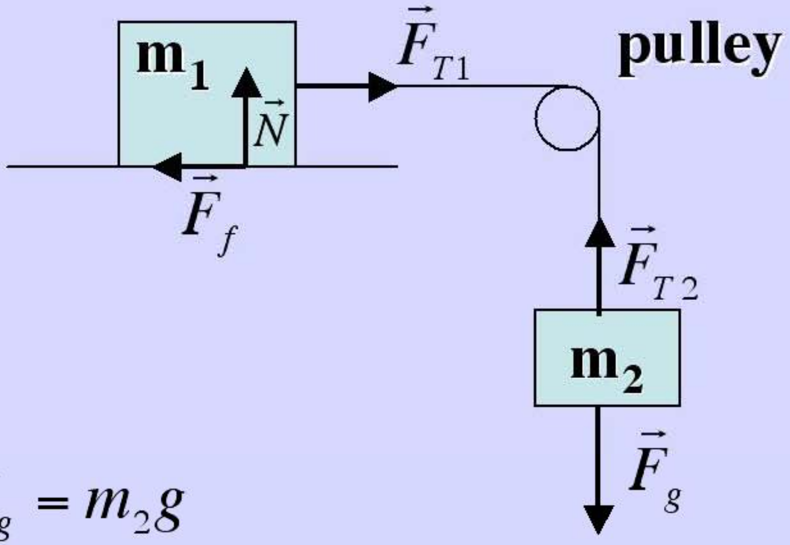


Two action-reaction pairs exist, one at each end. The tension force (always) points inward to the string at the two ends of the string, while imparted the forces point out.

If the string wraps around a frictionless and massless pulley, the force is transferred along the string. Thus the "positive direction" must follow along the string.

Example: Tension in a String

A mass of 5kg hangs from a string attached via a frictionless massless pulley to a mass of 10kg which rests on a table. How heavy does the hanging mass have to be to cause the other mass to move if the coefficient of static friction is 0.3? Does it move?



The tension force is $F_{T1} = -F_{T2} = F_g = m_2 g$

This force is imparted to the mass on the table, and points against the friction force:

$$F_f \leq \mu_S N = \mu_S m_1 g$$

Thus, for the mass to overcome the static friction

$$F_{T1} = F_f \geq \mu_S m_1 g$$

$$m_2 g \geq \mu_S m_1 g$$

thus

$$m_2 \geq \mu_S m_1$$

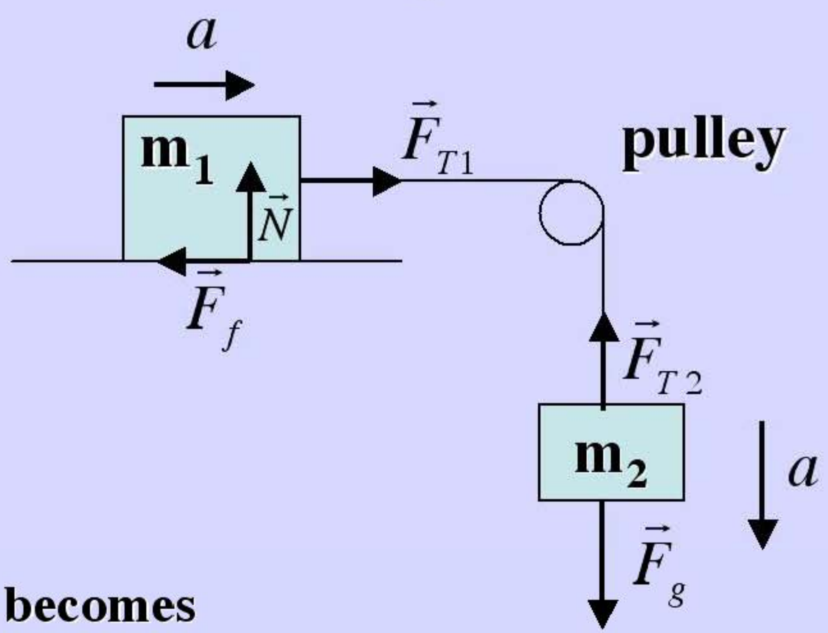
$$m_2 \geq (0.3)(10kg) = 3.0kg$$

Yes it moves!

Regardless of g!

Example: Tension in a String

A mass of 5kg hangs from a string attached via a frictionless massless pulley to a mass of 10kg which rests on a table. What is the acceleration of the mass and string system if the coefficient of kinetic friction between the mass and table is 0.2?



For kinetic friction, the friction force becomes

$$F_f = \mu_K N = \mu_K m_1 g$$

The sum of the forces on body 1 along the string are:

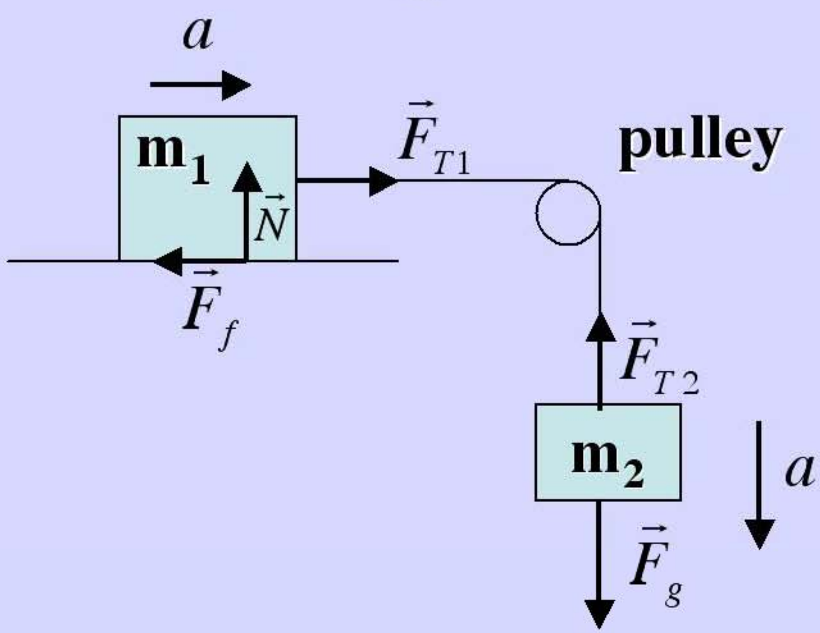
$$F_1 = F_{T1} - F_f = F_{T1} - \mu_K m_1 g = m_1 a$$

The sum of the forces on body 2 along the string are:

$$F_2 = F_g - F_{T2} = m_2 g - F_{T2} = m_2 a$$

Example: Tension in a String

A mass of 5kg hangs from a string attached via a frictionless massless pulley to a mass of 10kg which rests on a table. What is the acceleration of the mass and string system if the coefficient of kinetic friction between the mass and table is 0.2?



However, $F_{T1} = F_{T2}$

Which gives $m_2 g - m_1 a - \mu_K m_1 g = m_2 a$

And the acceleration is

$$a = \frac{m_2 - \mu_K m_1}{m_2 + m_1} g = \frac{(5) - (0.2)(10)}{(15)} (9.8) m/s^2$$

$$a = 1.96 m/s^2$$

This is just like example 6-11 in the text, and is a standard friction problem.