PHYSICS 2B - Lecture Notes

Ch. 30: Sources of the Magnetic Field

Preliminaries

In the last section, we studied the force on a current from an existing magnetic field. Here we will examine how the magnetic fields are produced.

In 1820, Oersted made the first discovery of the connection between electricity and magnetism. Oersted gave public lectures about the science of the day illustrated by spectacular demonstrations. The highlight of his show was his proof that electricity and magnetism were NOT related. He placed a compass next to a current carrying wire such that the wire lay in the rotation plane of the compass needle. He reasoned that any magnetic field associated with the current carrying wire would cause the compass needle to align itself parallel to the wire. One evening, Oersted's assistant placed the compass so that its rotation plane did not contain the wire. When the current was turned on, the compass aligned itself perpendicular to the wire. Oersted changed his lecture and took credit for this discovery. The name of the assistant is lost to history.

The Biot-Savart Law

Within weeks of Oersted's discovery, Biot and Savart, working together, were able to determine the dependence of the magnetic field produced by a long, straight wire to be

$$B \propto \frac{I}{y},$$

where I is the current in the wire and y is the perpendicular distance to the wire. The direction of the field was found to be given by a second *right-hand rule:*

> *If a current carrying wire is grasped with the right hand, with the thumb pointing in the direction of the current, the fingers will point in the direction of the field produced.*

From these and other observations, the magnetic field contribution from a differential line segment carrying a current, I, at a field point, $\vec{r}$, relative to the line segment was inferred to be,

$$d\vec{B} = K\frac{Id\vec{\ell}\times\vec{r}}{r^3}.$$

This form is now called the Biot-Savart Law. The constant K will be determined by the choice of current units.

Applications

About an Infinitely Long Wire

We want the field at a distance y above a wire. The contribution from a length dx of the wire is

$$dB = KI\frac{\left|d\vec{\ell}\times\hat{r}\right|}{r^2} = KI\frac{dx}{(x^2+y^2)}\frac{y}{\sqrt{(x^2+y^2)}}.$$

Using the table in Appendix A to evaluate the integral, we find that the field due to the entire wire is

$$B = KIy\int_{-\infty}^{\infty}\frac{dx}{(x^2+y^2)^{3/2}} = KIy\frac{x}{y^2(x^2+y^2)^{1/2}}\Bigg|_{-\infty}^{\infty} = \frac{2KI}{y}.$$

On the Axis of a Current Loop

We use the Biot-Savart law to compute the field on the axis of a circular loop of radius a, a distance x above the plane of the loop. The magnitude of the field contribution on the axis is,

$$dB = K\frac{Id\ell}{r^2}$$

where $r^2 = a^2 + x^2$. The component of this contribution parallel to the axis is

$$dB_{//} = dB\cos\theta \quad \text{where} \quad \cos\theta = \frac{a}{r}.$$

Therefore,

$$B_{//} = KI\frac{a}{(x^2+a^2)^{3/2}}\int_{loop} d\ell = \frac{2\pi KIa^2}{(x^2+a^2)^{3/2}}.$$

An Infinite Solenoid

A solenoid is cylinder of radius a about which a wire, carrying a current I_0, is uniformly wound with n closely spaced turns per unit length of the solenoid. We can compute the field on the solenoid axis as a superposition of contributions from current loops each carrying a current

$$dI = nI_0dx.$$

Using the result above for the field due to a current loop, we have

$$B = 2\pi Ka^2nI_0\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^{3/2}} = 2\pi Ka^2nI_0\frac{x}{a^2\sqrt{(x^2+a^2)}}\Bigg|_{-\infty}^{\infty} = 4\pi KnI_0,$$

where the integral is the same one that appeared in the example of the infinitely long wire. It is interesting that this result does not depend on the radius of the solenoid.

Defining the Current Unit

Consider two long parallel wires each carrying the same current, I. The magnetic field produced by the first wire is given by,

$$B = \frac{2KI}{y}.$$

The force on a length { of the second wire is then,

$$F = BI\ell = \frac{2KI^2\ell}{y}.$$

We may now give a formal definition of current:

A current of one ampere, is that current which if two long, parallel wires are one meter apart, each carrying a current of one ampere, they will exert a force per unit length on one another of

$$2\times10^{-7}\,\frac{\text{newton}}{\text{meter}}.$$

The ampere is a *base unit*, in that it is not defined in terms of anything simpler. In this regard, it has the same status as the meter, second and kilogram. With the choice of the ampere as the unit of current in the SI system, then

$$K = 10^{-7}\,\frac{\text{newtons}}{\text{ampere}^2}.$$

Ampere's Law

We can easily show that for a circular path about a current-carrying wire,

$$\oint_{circ} \vec{B}\cdot d\vec{\ell} = 4\pi KI .$$

The combination, $4\pi K$, occurs so frequently that we define,

$$4\pi K \equiv \mu_0 = 4\pi\times10^{-7}\,\frac{\text{newtons}}{\text{ampere}^2}.$$

μ_0 is called the *permeability of free space.* Ampere's Law is then written,

$$\oint_{circ} \vec{B}\cdot d\vec{\ell} = \mu_0 I ,$$

where the direction of the path integral is that of the right-hand rule.

We generalize this to a general path by expressing any path as a succession of infinitesimal radial and circular segments. Then using the principle of superposition of fields and currents to any distribution of current so that,

$$\oint_{C} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{tot} .$$

In this form, Ampere's law holds for steady currents only. However, with the inclusion of an additional term devised by Maxwell, Ampere's law becomes a very fundamental statement and another of Maxwell's equations.

Application

In common with Gauss's law, Ampere's law can make the evaluation of the field strength in cases where symmetry can simplify the path integral.

Solenoid

We take as our integration path, a rectangle with two sides of length ● parallel to the solenoid axis, one inside and the other outside the solenoid, and the other two sides perpendicular to the solenoid axis. The field is mainly confined to the interior of the solenoid and parallel to the axis. Therefore the only non-zero contribution to the Ampere integral is from the interior side that is parallel to the axis, and

$$\int_{rect} \vec{B} \cdot d\vec{\ell} = B\ell = \mu_0 I_{tot} .$$

The enclosed current will be

$$I_{tot} = nI_0\ell .$$

Therefore,

$$B\ell = \mu_0 n I_0 \ell \qquad \Rightarrow \qquad B = \mu_0 n I_0 .$$

This is the same result as obtained by direct calculation for the field on the axis of the solenoid. Only now this result is independent of the location of the rectangle side inside the solenoid. Therefore, the field inside the solenoid is *uniform.*

Toroid

A toroid may be thought of as a solenoid bent into a circle of radius r. We want the magnetic field on the axis of a toroid wound with a total of N turns, each carrying current I. We apply Ampere's law to a path along the axis, so that.

$$\oint_C \vec{B} \cdot d\vec{\ell} = 2\pi r B = \mu_0 I_{tot} = \mu_0 N I .$$

Magnetic Monopoles and Gauss's Law for Magnetism

The idea of a magnetic monopole, the analog of isolated electric charge, has been intriguing since Maxwell's formulation of electromagnetic theory. If they were to exist, the equations of electricity and magnetism would have a one-to-one correspondence with one another. Further, theoretical considerations suggest that magnetic monopoles should have been produced at the time of the *big bang.* Yet, despite extensive experimental investigations, no magnetic monopole has ever been observed.

In the electrical case, Gauss's law relates the flux of the electric field through a closed surface to electric charge inside the surface,

$$\phi_E = \oint_A \vec{E} \cdot \hat{n} dA = \frac{q_{tot}}{\varepsilon_0} .$$

In the magnetic case, with no isolated magnetic "charge", the corresponding relation for the magnetic flux through a closed surface is,

$$\phi_B = \oint_A \vec{B} \cdot \hat{n} dA = 0 ,$$

for any closed surface A. This is a fundamental statement about the magnetic and will be seen to be the third of Maxwell's equation.

Magnetic Materials

The magnetic properties of materials arise at the atomic level. In the simple planetary model of the atom, the orbiting electron constitutes a current and hence a magnetic dipole with its magnetic moment pointing opposite to its angular momentum. When an external magnetic field is applied, these magnetic moments will tend to align themselves with the applied field. The manner in which the material responds to this tendency will distinguish the three types of magnetic materials.

Paramagnetism

In these materials, the atomic moments tend to align with the field, increasing the field inside the material. The atoms, however, interact weakly with one another, so that collisions and thermal excitations prevent the overall alignment from getting too large. Because the magnetic dipoles align with the applied field, the potential energy of the dipole is negative and the material is weakly attracted to the source of the external field.

Ferromagnetism

In ferromagnetic materials the atoms interact strongly with one another creating *domains* in the material in which the alignment of the atomic moments is almost total, even in the absence of an external field. With the application of an external field, the domains align with one another, creating a very strong internal field.

Diamagnetism

In these materials the atomic moments cancel one another totally in the absence of an external field. When a field is applied, however, the atomic moments are altered so that the internal field induced by the external field *opposes* the applied field, so that the material is *repelled* by the external field. A simplified classical explanation of this phenomenon follows.

An applied magnetic field is directed into the page. The diamagnetic material consists of equal numbers of clockwise and counter-clockwise circular atomic currents. These currents have radius r and angular velocity, ω, so that the atomic magnetic moments are

$$\mu = IA = \frac{q\omega A}{2\pi}.$$

For the counter-clockwise currents, the force from the applied field is in the same direction as the atomic binding force and causes the angular velocity to increase, which in turn increases the magnetic moment in the direction opposing the applied field. For clockwise currents, the angular velocity is reduced, which reduces the magnetic moment in the direction of the applied field. The net effect is to induce a magnetic moment in the material, which is opposite to the applied field and leads to a repulsive force.

Magnetic Susceptibility

When an *external* magnetic field is applied to a material, it produces an *internal* field in the material. We represent the relation between these two fields by the introduction of a parameter called the *relative permeability*, where

$$B_{int} = K_M B_{ext}.$$

The permeability is usually expressed in terms of the *magnetic susceptibility*, given by

$$K_M = 1 + \chi_M.$$

A table of susceptibilities can be found on page 793 of your text. It may be seen that those for paramagnets and diamagnets are of the order $10^{-3 \text{ to } -6}$ and are negative for diamagnets. The susceptibilities for ferromagnets are many orders of magnitude greater. The entries for ferromagnets are at *saturation.* That is, as the external field is increased, the internal field rises also until all of the domains are aligned. This is the situation called saturation. If the external field is now reduced, the internal field will drop also but not proportionally. The material retains a "memory" of having been magnetized and a plot of the internal field versus the external field is the open loop called the *hysteresis curve,* an example of which is given on page 792 of your text. Finally, we note the entry in the table of susceptibilities for a superconductor which has a susceptibility of -1. This perfect diamagnetism will be discussed in the next chapter.