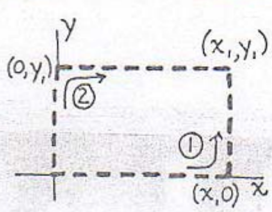
2.1

Path ①: $\displaystyle\int_{(0,0)}^{(x_1,y_1)} \underset{\sim}{E}\cdot d\underset{\sim}{s} = \int_0^{x_1} E_x(x,0)\,dx + \int_0^{y_1} E_y(x_1,y)\,dy$



$\displaystyle = 0 + \int_0^{y_1} (3x_1^2 - 3y^2)\,dy = 3x_1^2 y_1 - y_1^3$

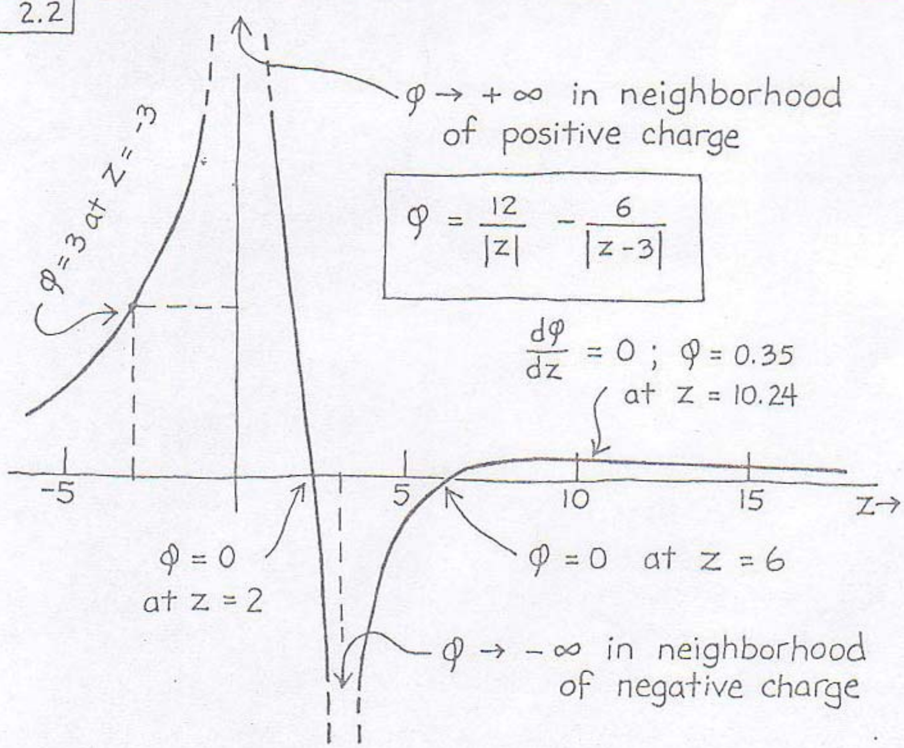
Path ②: $\displaystyle\int_{0,0}^{x_1,y_1} \underset{\sim}{E}\cdot d\underset{\sim}{s} = \int_0^{y_1} E_y(0,y)\,dy + \int_0^{x_1} E_x(x,y_1)\,dx$

$\displaystyle = \int_0^{y_1} -3y^2\,dy + \int_0^{x_1} 6xy_1\,dx = -y_1^3 + 3x_1^2 y_1$

same result

The electric potential φ, if taken as zero at (0,0), is just the negative of this, since we define φ by $-\int \underset{\sim}{E}\cdot d\underset{\sim}{s}$, or $\underset{\sim}{E} = -\nabla\varphi$. That is, $\varphi = y^3 - 3x^2 y$

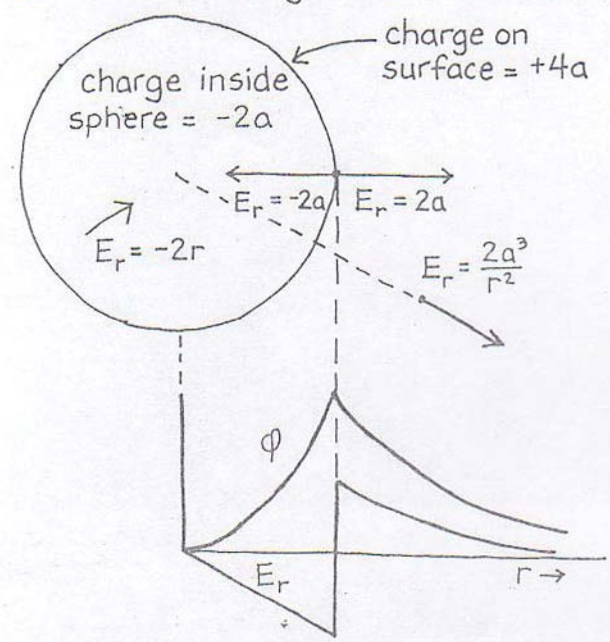
2.2



$\boxed{2.4}$ This is the potential of a spherical charge distribution, more briefly described by

$$\varphi = r^2 \text{ for } r \leqslant a\,; \quad \varphi = -a^2 + 2a^3/r \text{ for } r > a.$$

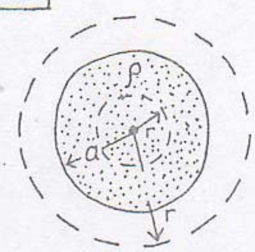
For $r = (x^2+y^2+z^2)^{1/2} < a$, $\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}$ $= 2+2+2 = 6$. Using $\nabla^2\varphi = -4\pi\rho$, we find $\rho = -3/2\pi$ for $r < a$. [In spherical polar coordinates $\nabla^2\varphi = \frac{1}{r}\frac{d}{dr}r\frac{d\varphi}{dr}$ when φ is a function of r only. This gives the same answer.] Outside the sphere of uniform charge density ρ is zero, as we find by computing $\nabla^2(1/r)$, or just by recognizing $2a^3/r$ as the potential of point charge $2a^3$ at the origin. The electric field E_r at radius $r < a$ is that of a charge $(4\pi/3)r^3\rho$ divided by r^2: $E_r = (4\pi/3)\rho r = -2r$, $r<a$. This tells us there is a surface charge σ on the sphere: $4\pi\sigma = 2a - (-2a)$. $\sigma = a/\pi$. The total surface charge is $4\pi a^2\sigma$, or $4a^3$. This is positive and twice as large as the negative charge $-2a^3$ distributed through the interior of the sphere. Thus the external field of the sphere as a whole is that of a positive charge $2a^3$.



$\boxed{2.5}$ Assume the diameter is about 1 foot, 30 cm. 1000 volts is 3.3 statvolts. This is Q/r, where $r = 15$ cm. The charge Q is therefore 15×3.3 or -50 esu. The number of extra electrons per cm^2 is

$$\frac{50}{4\pi \times 15^2 \times 4.8 \times 10^{-10}} = 3.7 \times 10^7$$

$\boxed{2.8}$



(a) Consider a cylinder of unit length, of radius $r < a$. Charge contained is $\pi r^2 \rho$. Area of surface is $2\pi r$; flux through surface is $2\pi r E$. Gauss's law says:

$$\underbrace{2\pi r E}_{\text{flux}} = 4\pi \underbrace{(\pi r^2 \rho)}_{\text{charge}}, \text{ from which } E = 2\pi\rho r.$$

Considering a cylinder of radius $r > a$, which contains an amount of charge $\pi a^2 \rho$, we find

$$2\pi r E = 4\pi(\pi a^2 \rho), \text{ or } E = \frac{2\pi\rho a^2}{r}$$

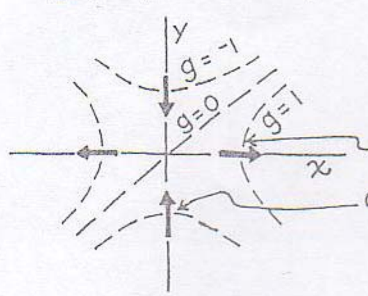
(b) Take $\varphi = 0$ at $r = 0$:

for $r < a$, $\varphi = \int_0^r -2\pi\rho r' dr' = -\pi\rho r^2$

for $r > a$, $\varphi = -\pi\rho a^2 - \int_a^r \frac{2\pi\rho a^2 dr'}{r'} = -\pi\rho a^2 - 2\pi\rho a^2 \ln\frac{r}{a}$

$\boxed{2.14}$ $f(x,y) = x^2 + y^2 \quad \nabla^2 f = 2 + 2 \neq 0$

$g(x,y) = x^2 - y^2 \quad \nabla^2 g = 2 - 2 = 0$

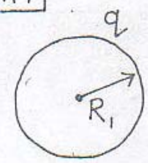


$\nabla g = 2x\hat{x} - 2y\hat{y}$

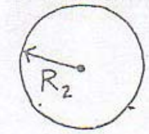
at $(1,0)$ $\nabla g = +2\hat{x}$

at $(0,-1)$ $\nabla g = +2\hat{y}$

2.19



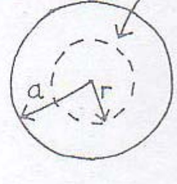
$$U = \frac{q^2}{2R_1} + \frac{(Q-q)^2}{2R_2}$$



$$\frac{dU}{dq} = \frac{q}{R_1} - \frac{(Q-q)}{R_2}$$

This must vanish for an extremum in U. But q/R_1 is just the <u>potential</u> φ_1 of that sphere and $(Q-q)/R_2$ is the potential φ_2 of the other sphere. So the condition can be expressed as equality of potential. It is easy to see that the extremum is a <u>minimum</u> in U, not a maximum: if $R_1 = R_2$, equal division of charge involves half as much energy as piling all of Q on one sphere.

2.20



total charge Q uniformly distributed.

charge inside radius $r = Q\frac{r^3}{a^3}$

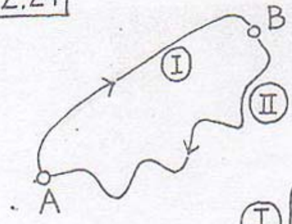
$E_r = Qr/a^3, \ r < a \qquad E_r = \frac{Q}{r^2}, \ r > a$

$$\left.\begin{aligned} \int_0^a E_r\,dr &= \varphi(0) - \varphi(a) = \frac{Q}{a^3}\int_0^a r\,dr = \frac{Q}{2a} \\ \int_a^\infty E_r\,dr &= \varphi(a) - \varphi(\infty) = Q\int_a^\infty \frac{dr}{r^2} = \frac{Q}{a} \end{aligned}\right\} \varphi(0) - \varphi(\infty) = \frac{3Q}{2a}$$

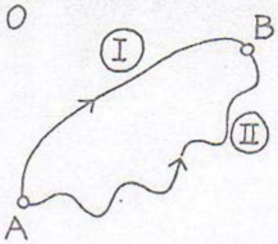
$$\frac{3Q}{2a} = \frac{3 \times 79 \times 4.8 \times 10^{-10}}{2 \times 6 \times 10^{-13}} = 9.5 \times 10^4 \text{ statvolts}$$

$$= 28.5 \text{ megavolts}$$

2.24



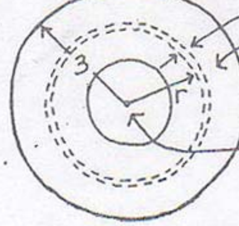
Given that $\oint_A^A \underset{\sim}{E} \cdot d\underset{\sim}{s} = 0$ for any path, consider the path ① + ②:



$$\text{(I)}\int_A^B \underset{\sim}{E} \cdot d\underset{\sim}{s} + \text{(II)}\int_B^A \underset{\sim}{E} \cdot d\underset{\sim}{s} = 0$$

It follows that $\text{(II)}\int_A^B \underset{\sim}{E} \cdot d\underset{\sim}{s} = -\text{(II)}\int_B^A \underset{\sim}{E} \cdot d\underset{\sim}{s} = \text{(I)}\int_A^B \underset{\sim}{E} \cdot d\underset{\sim}{s}$ QED

2.28



dr

$\sigma = -4$ esu/cm²

$$dq = \sigma \times 2\pi r\, dr$$

$$\varphi = \int \frac{dq}{r} = 2\pi\sigma \int_1^3 dr = -16\pi \text{ statvolt}$$

Electron's final K.E. $= e\varphi = 4.8 \times 10^{-10} \times 16\pi = 2.41 \times 10^{-8}$ erg
Electron rest energy $mc^2 = 81 \times 10^{-8}$ erg. Since K.E./$mc^2 \approx 0.03$ a non-relativistic calculation should be good enough :

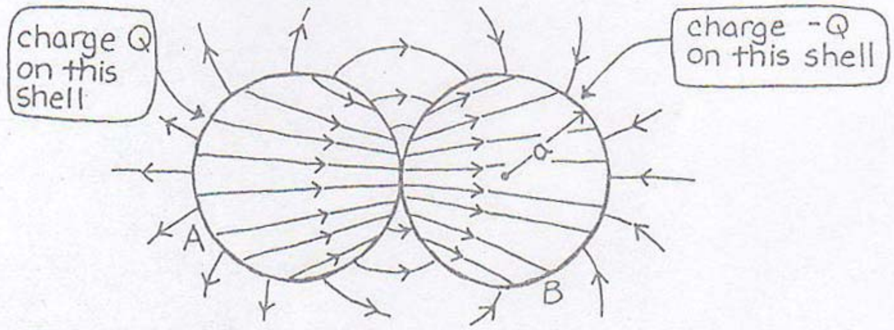
$$v = \left(\frac{2\text{ K.E.}}{m}\right)^{1/2} = \frac{2 \times 2.41 \times 10^{-8}}{9 \times 10^{-28}} = 7.32 \times 10^9 \text{ cm/sec}$$

[A relativistic calculation using the same constants :

$$\gamma = 1 + \frac{\text{K.E.}}{mc^2} = 1 + \frac{2.41 \times 10^{-8}}{8.1 \times 10^{-7}} = 1.0298$$

$$\beta = (1 - 1/\gamma^2)^{1/2} = 0.2388 \qquad v = \beta c = 7.16 \times 10^9 \text{ cm/sec}$$]

$\boxed{2.29}$ Outside both shells the electric field is that of two point charges. Inside each shell the field is that of a point charge at the center of the other shell.



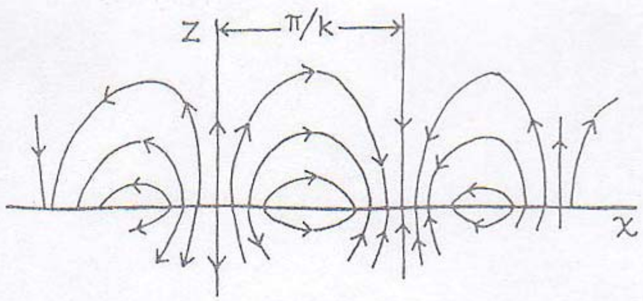
The external field of A alone is that of point charge Q. To move shell B to infinity takes the same amount of work as moving the point charge Q to infinity with B stationary. But that takes just $Q^2/2a$ for that point charge Q is initially a distance of 2a from the center of shell B.

$\boxed{2.31}$ $\phi = \phi_0 \cos kx e^{-kz}$ $\quad \frac{\partial \phi}{\partial x} = -k\phi_0 \sin kx e^{-kz} = -E_x$

$$\frac{\partial^2 \phi}{\partial x^2} = -k^2\phi_0 \cos kx\, e^{-kz} \qquad \frac{\partial \phi}{\partial z} = -k\phi_0 \cos kx\, e^{-kz} = -E_z$$

$$\frac{\partial^2 \phi}{\partial z^2} = k^2\phi_0 \cos kx\, e^{-kz}$$

$$\nabla^2\phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$



$\sigma = \frac{1}{2\pi} E_z$ at $z = 0$ $\qquad \sigma = \frac{k}{2\pi} \phi_0 \cos kx$