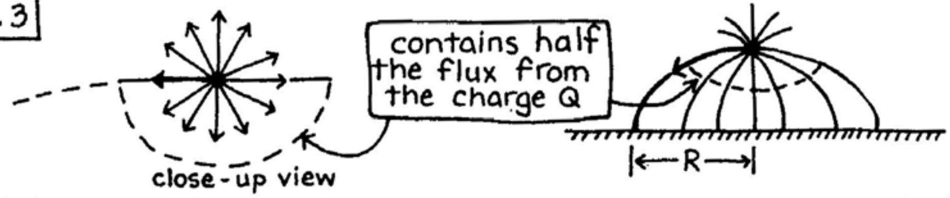
3.3

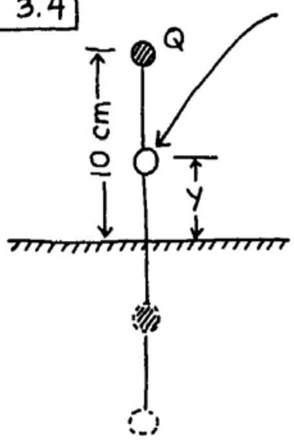


Using Eqs. 8 and 9, p. 100, we determine R so that half of $-Q$, the induced charge on the plane, is contained within the circle of radius R:

$$-\frac{Q}{2} = \int_0^R \sigma \cdot 2\pi r\,dr, \qquad \text{or} \quad \frac{1}{2} = \int_0^R \frac{h r\,dr}{(h^2+R^2)^{3/2}} = \left[\frac{-h}{\sqrt{h^2+R^2}}\right]_0^R = 1 - \frac{h}{\sqrt{h^2+R^2}}$$

Then $\frac{h}{\sqrt{h^2+R^2}} = \frac{1}{2}$, or $h^2 + R^2 = 4h^2$, or $R = \sqrt{3}\,h$

3.4



Calling upward force positive, the force on this charge due to the upper charge Q and the two image charges below the plane is

$$Q^2\left[\frac{1}{(10-y)^2} - \frac{1}{(2y)^2} + \frac{1}{(10+y)^2}\right]$$

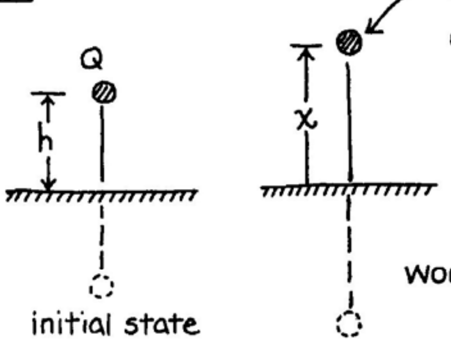
setting this equal to zero we get

$$\frac{1}{4y^2} = \frac{200 + 2y^2}{(100-y^2)^2}\,,$$ which is

a quadratic equation in y^2: $7y^4 + 1000y^2 - 10000 = 0$

with a positive root $y^2 = 9.38$, giving $y = 3.06$

3.5



Force required to move this charge upward $= Q^2/(2x)^2$

The second student calculates as follows:

$$\text{work} = \int F\,dx = \int_h^\infty \frac{Q^2}{(2x)^2}\,dx = \frac{Q^2}{4h}$$

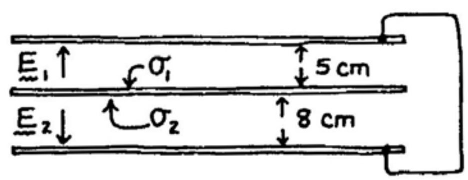
This is the correct answer.

Note that if two real charges Q and $-Q$ were being pulled apart symmetrically, the <u>total</u> work done would be $Q^2/2h$, but the agency moving Q would supply only half of it.

3.7 Consider a path that runs from conductor B across to conductor D, then through the interior of the wire that connects D to C, then across the gap to A, thence via the other wire, down to B. The line integral of $\mathbf{E}$ around any closed path must be zero, if $\mathbf{E}$ is a static electric field. But if the fields are as shown in (C) the line integral over the closed path just described is <u>not</u> zero. Each gap makes a positive contribution, while in the conductors, including the connecting wires, $\mathbf{E}$ is zero. So (C) cannot represent a static field or charge distribution. If it did, by the way, we could contrive to violate the Uniqueness Theorem too!

3.8 Given : $\sigma_1 + \sigma_2 = 10 \text{ esu/cm}^2$

$E_1 = 4\pi\sigma_1 \qquad E_2 = 4\pi\sigma_2$



Since top and bottom plates are at the same potential,
$E_1 \cdot 5\text{cm} = E_2 \cdot 8 \text{ cm}$, or $5\sigma_1 = 8\sigma_2$

$$\left.\begin{aligned} \sigma_1 + \sigma_2 &= 10 \\ 5\sigma_1 - 8\sigma_2 &= 0 \end{aligned}\right\} \quad \text{Solving for } \sigma_1 \text{ and } \sigma_2 : \quad \sigma_1 = 6.15, \quad \sigma_2 = 3.85 \ \frac{\text{esu}}{\text{cm}^2}$$

3.11 $Q = C_1V_1 = 10^{-10}$ farad $\times$ 100 volt $= 10^{-8}$ coulomb.
When the same charge Q is shared between C_1 and C_2 connected in parallel, $Q = (C_1 + C_2)V_2$

$$(C_1 + C_2)/C_1 = V_1/V_2 = 100/30 = 3.33$$

$$C_1 + C_2 = 333\ \text{pF} \qquad C_2 = 233\ \text{pF}$$

Energy stored was $\frac{1}{2}QV_1 = 0.5 \times 10^{-6}$ Joules
For same charge at 30 volts, energy is 0.15×10^{-6} J.
0.35×10^{-6} J of energy has been lost. That much energy has to go somewhere before the system can settle down to static equilibrium. If it is not stored anywhere else (for instance, in a weight lifted by a motor driven by the current from C_1 to C_2) it will eventually be dissipated in circuit resistance, no matter how small that resistance may be.

3.13 Assume the capacitance is that of a conducting sphere 1 meter in diameter.
Then $C \approx 50$ cm. 2 kilovolts $\approx$ 7 statvolts

$U = \frac{1}{2}CV^2 = \frac{50}{2} \times 7^2 = 1200$ erg. So 10^3 ergs would be a reasonable estimate.

3.14 Energy stored $= \dfrac{Q^2}{2C}$ · If capacitance of conducting disk is $2a/\pi$, $U = \dfrac{\pi}{4}\dfrac{Q^2}{a}$
For uniformly charged non-conducting disk we found, in Problem 2.27, $U = \dfrac{8}{3\pi}\dfrac{Q^2}{a}$
$(8/3\pi)/(\frac{\pi}{4}) = 32/3\pi^2 = 1.081$. The field of the uniform charge distribution has 8 percent more energy. On the conductor the charge has distributed itself so as to minimize the energy.

$\boxed{3.15}$ If λ is the charge per unit length on the inner cylinder the field between the cylinders (except close to the ends, a correction we shall ignore) is $2\lambda/r$. The potential difference between the cylinders is $\phi_2 - \phi_1 = \int_{r_1}^{r_2} \frac{2\lambda}{r}\,dr = 2\lambda \ln(r_2/r_1)$

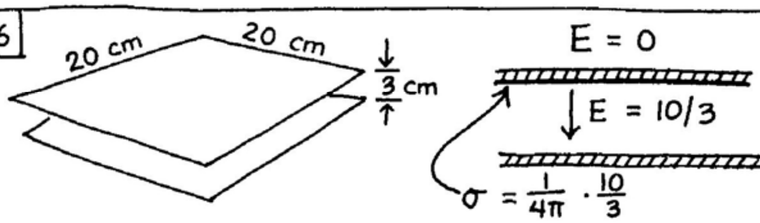
If L is the length of the cylinders the total charge Q is $L\lambda$. The capacitance C is

$$\frac{Q}{\phi_2 - \phi_1}, \text{ or } \frac{L}{2\ln(r_2/r_1)} = \frac{30}{2\ln(4/3)} = 52.1 \text{ cm}$$

45 volts = .15 statvolts

Energy stored $= \frac{1}{2}CV^2 = \frac{52.1}{2} \times (0.15)^2 = 0.59$ erg.

$\boxed{3.16}$



Force on unit area $= \sigma \cdot \frac{E}{2} = \frac{1}{8\pi}E^2 = 0.442$ dyne cm^{-2}

Force on entire plate $= 400 \times 0.442 = 177$ dynes

$F \times 3$ cm $= 530$ erg $=$ energy stored in field.

$\boxed{3.23}$ (a) Neglecting end effects, we assume charge Q is uniformly distributed along cylinder. Then field E is that of an axial line charge of density $\lambda = Q/L$. That is, $E = \frac{2\lambda}{r} = \frac{2Q}{rL}$. The potential difference ϕ_{ab} is:

$$\phi_{ab} = \int_a^b \frac{2Q}{L}\frac{dr}{r} = \frac{2Q}{L}\ln\frac{b}{a}.$$

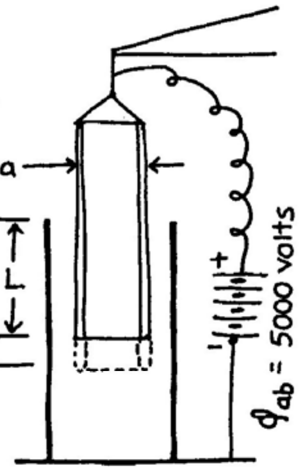
Since $Q = C\phi_{ab}$, the capacitance C is given by $C = \frac{L}{2\ln(\frac{b}{a})}$. If $b - a \ll a$,

$\ln(\frac{b}{a}) = \ln(1 + \frac{b-a}{a}) \approx \frac{b-a}{a}$ Then $C \simeq aL/2(b-a)$

But $2\pi aL$ is the area of the inner cylinder and $b-a$ is the plate separation, so this is just what we would get using the formula for the parallel plate capacitor: $C = \frac{1}{4\pi}\frac{\text{area}}{\text{separation}}$.

(b) Consider the energy changes involved in a downward displacement, by ΔL, of the inner cylinder. The capacitance increases from

$$C = \frac{L}{2 \ln \frac{b}{a}} \quad \text{to} \quad C + \Delta C = \frac{L + \Delta L}{2 \ln \frac{b}{a}} .$$



With constant potential difference φ_{ab}, the stored electrical energy $\frac{1}{2} C \varphi_{ab}^2$ increases by $\frac{\Delta C}{2} \varphi_{ab}^2$.

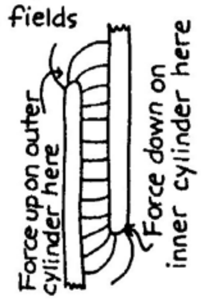
At the same time, an amount of charge $\Delta Q = \varphi_{ab} \Delta C$ flows into the capacitor. The battery thereby does work, in amount $\varphi_{ab} \Delta Q = \varphi_{ab}^2 \Delta C$. This is <u>twice</u> the increase in stored energy in the field. The difference is the work done against the external force F which balances the electrical attraction of the cylinders. That is :

$$\underset{\boxed{\text{work done by battery}}}{\varphi_{ab}^2 \Delta C} \begin{cases} \nearrow \frac{1}{2} \varphi_{ab}^2 \Delta C \ \boxed{\text{increase in field energy}} \\ \searrow F \Delta L \ \boxed{\text{work done against external force}} \end{cases}$$

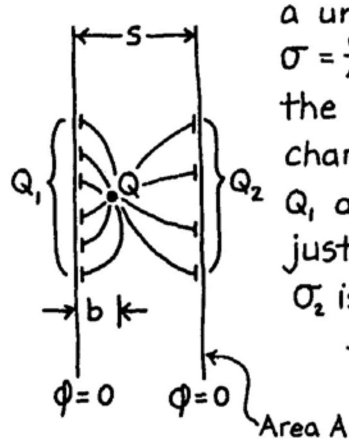
Hence $F \Delta L = \frac{1}{2} \varphi_{ab}^2 \Delta C$ from which we get $F = \frac{1}{2} \varphi_{ab}^2 \frac{\Delta C}{\Delta L}$. This is a quite general formula. In the case at hand $\frac{\Delta C}{\Delta L} = \frac{1}{2 \ln \frac{b}{a}}$. With $b/a = 3/2$ and $\varphi_{ab} = 5000$ volts $= 16.7$ stat-volts

we get $F = \frac{1}{2} (16.7)^2 \frac{1}{.812} = 171$ dynes

<u>Note</u>: End effects do not spoil the accuracy of this simple result, for the downward displacement of the inner cylinder leaves the end fields themselves unaltered; it simply lengthens the region where the field is nicely cylindrical. That is, we can safely ignore the end fields in calculating dC/dL, even when they would seriously affect C itself. Nevertheless, the <u>origin</u> of the force just calculated lies in the very end fields that our method permits us to ignore, for it is only at the ends that we find vertical components of the electric field.

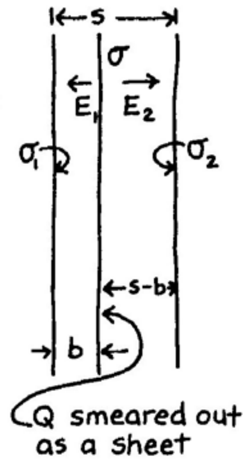


3.24



Imagine charge Q smeared out in a uniform sheet of density $\sigma = \frac{Q}{A}$. This cannot affect the total amount of induced charge on each plate, Q_1 and Q_2. In fact σ_1 is just Q_1 smeared out and σ_2 is Q_2 smeared out.

Thus : $\frac{Q_1}{Q_2} = \frac{\sigma_1}{\sigma_2}$



But from Gauss's law, $\sigma_1 = E_1/4\pi$ and $\sigma_2 = E_2/4\pi$. Also, because the two plates are at the same potential, $E_1 b = E_2(s-b)$. Hence :

$\frac{Q_1}{Q_2} = \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} = \frac{s-b}{b}$. Therefore, with due regard to sign, we have : $Q_1 = -Q\left(\frac{s-b}{s}\right)$ $\quad Q_2 = -Q\frac{b}{s}$.