2.13 $E_x = 6xy \quad E_y = 3x^2 - 3y^2 \quad E_z = 0$

$$(\text{curl } \underset{\sim}{E})_x = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0 \qquad (\text{curl } \underset{\sim}{E})_y = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0$$

$$(\text{curl } \underset{\sim}{E})_z = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 6x - 6x = 0$$

$$\text{div } \underset{\sim}{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 6y - 6y = 0$$

2.15 (a) $\underset{\sim}{F} = \hat{\underset{\sim}{x}}(x+y) + \hat{\underset{\sim}{y}}(-x+y) + \hat{\underset{\sim}{z}}(-2z)$

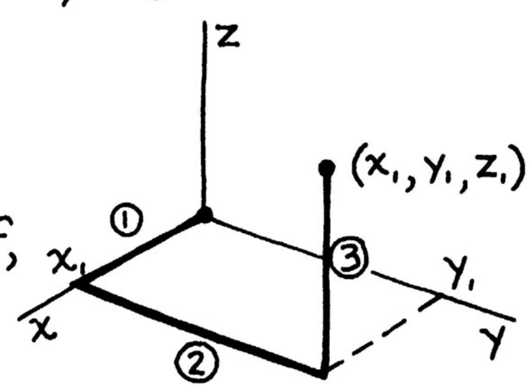
$$\nabla \times \underset{\sim}{F} = \hat{\underset{\sim}{x}}(0+0) + \hat{\underset{\sim}{y}}(0+0) + \hat{\underset{\sim}{z}}(-1-1) = -2\hat{\underset{\sim}{z}}$$

$$\nabla \cdot \underset{\sim}{F} = 1 + 1 - 2 = 0$$

(b) $\underset{\sim}{G} = \hat{\underset{\sim}{x}}(2y) + \hat{\underset{\sim}{y}}(2x+3z) + \hat{\underset{\sim}{z}}(3y)$

$$\nabla \times \underset{\sim}{G} = \hat{\underset{\sim}{x}}(3-3) + \hat{\underset{\sim}{y}}(0+0) + \hat{\underset{\sim}{z}}(2-2) = 0$$

$$\nabla \cdot \underset{\sim}{G} = 0 + 0 + 0 = 0$$



Since $\nabla \times \underset{\sim}{G} = 0$ there exists an f such that $\underset{\sim}{G} = \nabla f$. To determine f, compute the line integral of $\underset{\sim}{G}$ from a fixed point, say $(0,0,0)$, to a general point x_1, y_1, z_1, over any path. Using the path composed of lines ①, ② and ③ :

$$G\int_{(000)}^{(x_1,y_1,z_1)} \underset{\sim}{G}\cdot d\underset{\sim}{s} = \int_0^{x_1} G_x(x,0,0)\,dx + \int_0^{y_1} G_y(x_1,y_1,0)\,dy + \int_0^{z_1} G_z(x_1,y_1,z)\,dz$$

$$= \int_0^{x_1} 0\,dx + \int_0^{y_1} 2x_1\,dy + \int_0^{z_1} 3y_1\,dz = 2x_1y_1 + 3y_1z_1 = f$$

x_1, y_1, z_1 was a general point, so we can drop the subscripts and write : $f = 2xy + 3yz$.

To check that $\nabla f = \underset{\sim}{G}$:

$$\nabla f = 2y\hat{\underset{\sim}{x}} + (2x + 3z)\hat{\underset{\sim}{y}} + 3y\hat{\underset{\sim}{z}} = \underset{\sim}{G}$$

(c) $\underset{\sim}{H} = \hat{\underset{\sim}{x}}(x^2 - z^2) + \hat{\underset{\sim}{y}}(2) + \hat{\underset{\sim}{z}}(2xz)$

$$\nabla \times \underset{\sim}{H} = x(0+0) + \hat{\underset{\sim}{y}}(-2z - 2z) + \hat{\underset{\sim}{z}}(0+0) = -4z\hat{\underset{\sim}{y}}$$

$$\nabla \cdot \underset{\sim}{H} = 2x + 0 + 2x = 4x$$

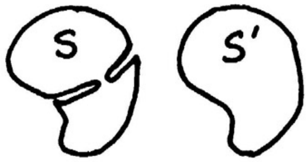
2.16 (a) $\nabla \cdot (\nabla \times \mathbf{A}) = \frac{\partial}{\partial x}(\nabla \times \mathbf{A})_x + \frac{\partial}{\partial y}(\nabla \times \mathbf{A})_y + \frac{\partial}{\partial z}(\nabla \times \mathbf{A})_z$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} + \frac{\partial^2 A_x}{\partial y \partial z} - \frac{\partial^2 A_z}{\partial y \partial x} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y} = 0,$$

because $\frac{\partial^2 A_z}{\partial x \partial y} = \frac{\partial^2 A_z}{\partial y \partial x}$ for any function A_z with continuous derivatives; likewise for A_x and A_y.

(b) If $\mathbf{A}$ is any vector field, $\int_C \mathbf{A} \cdot d\mathbf{s} \to 0$ over a curve such as C which consists of adjacent paths running in opposite directions. This path bounds the shaded surface S. Hence, by Stokes' theorem, $\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = 0$



But the slit is a negligible part of the whole closed surface S', so the same conclusion must apply to S': $\int_{S'} (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = 0$

S' could have been any surface, so we have concluded that the vector function $(\nabla \times \mathbf{A})$ has zero surface integral over any arbitrary closed surface. It follows from Gauss's theorem that div $(\nabla \times \mathbf{A}) = 0$ everywhere, for if the divergence were different from zero in any small region, a surface surrounding just that region would necessarily have a non-zero surface integral. (Of course, we can apply such an argument only to "well-behaved" functions.)