$\boxed{7.1}$

$$\underset{\text{statvolt}}{\mathcal{E}_{max}} = \frac{1}{c}\left(\frac{d\Phi}{dt}\right)_{max} = \frac{\omega}{c} \times (\text{turns} \times \underset{\text{cm}^2}{\text{area}}) \times \underset{\text{gauss}}{B}$$

$$= \frac{2\pi \times 30}{3 \times 10^{10}} \times (4000 \times 144\pi) \times .5$$

$$= .0057 \text{ statvolt}$$

---

in S.I. : $$\underset{\text{volt}}{\mathcal{E}_{max}} = \left(\frac{d\Phi}{dt}\right)_{max} = \omega \times (\text{turns} \times \underset{\text{m}^2}{\text{area}}) \times \underset{\text{tesla}}{B}$$

$$= 2\pi \times 30 \times (4000 \times .0144\pi) \times .5 \times 10^{-4}$$

$$= 1.71 \text{ volt}$$

$\boxed{7.3}$ Within a circle of radius r the flux is $\pi r^2 B$.

$$\mathcal{E}_{max} = \frac{1}{c}\left(\frac{d\Phi}{dt}\right)_{max} = \frac{\omega \pi r^2 B}{c}$$

$$E = \frac{\mathcal{E}}{2\pi r} = \frac{\omega r B}{2c} = \frac{2\pi \times 2.5 \times 10^6 \times 3 \times 4}{2 \times 3 \times 10^{10}}$$

$$= 0.0031 \text{ statvolt}$$

$\boxed{7.5}$ $\mathcal{E} = \frac{vw}{c}(B_1 - B_2) \quad I = \frac{\mathcal{E}}{R} = \frac{vw}{cR}(B_1 - B_2)$

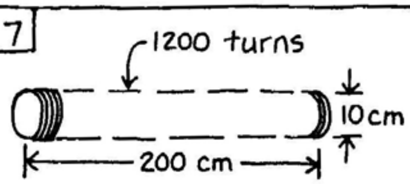
The force on the loop is : $F = \frac{IB_1 w}{c} - \frac{IB_2 w}{c} = \frac{Iw}{c}(B_1 - B_2)$

Rate at which work must be done to move the loop is

$$Fv = \frac{Iwv}{c}(B_1 - B_2) = I^2 R$$

In Fig. 7.14 the energy which is dissipated in the stationary loop has to be supplied by whatever agency is moving the coil. A force is required to move the coil because of the magnetic field arising from the induced current in the loop.

7.7



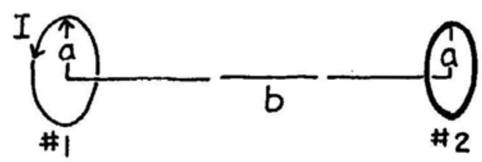
1 ampere in the coil causes a field inside the coil of magnitude $B = \frac{4\pi}{10} \times 1 \times \frac{1200}{200} = 7.55$ gauss

flux $\times$ turns $= \Phi N = \pi \times 5^2 \times 7.55 \times 1200 = 7.1 \times 10^5$

L (henrys) $= 10^{-8} \times 7.1 \times 10^5 = 7.1 \times 10^{-3}$ henrys

We have neglected the fact that the field inside the solenoid is not constant. It decreases near each end, so that the flux through the last turn is only about half that through a turn in the middle. This means that we have <u>over</u>- estimated the inductance. We might expect the error to be roughly $\frac{\text{diameter}}{\text{length}}$, or $\sim 5\%$ in this example. [In fact, the error is only 2% in this case, as one can discover by referring to tables which give exact values for the inductance of cylindrical coils.]

7.9 With I amperes in ring #1, the field a distance b down the axis is:



$$B = \frac{2\pi a^2 I}{10(a^2+b^2)^{3/2}}$$

This is an application of Eq. 6.41, p. 227, with a replacing b, b replacing z, and $\frac{1}{10}$ replacing $\frac{1}{c}$ since I is in amperes. For $b \gg a$ this can be approximated:

$$B = \frac{2\pi a^2 I}{10 b^3}$$

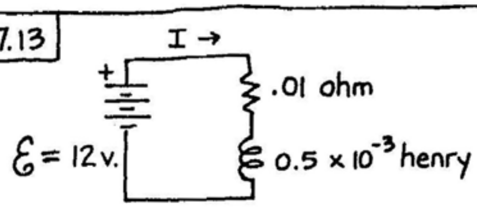
and also, for $b \gg a$, we can neglect the variation of B over the interior of ring #2. Then:

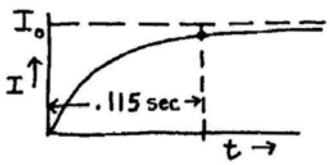
$$\Phi_{12} = B \cdot \pi a^2 = \frac{2\pi^2 a^4 I}{10\, b^3}$$

The mutual inductance in henrys is $10^{-8}\frac{\Phi}{I}$, or

$$M = \frac{2\pi^2 \times 10^{-9} a^4}{b^3} \text{ henrys}$$

7.13





$I_0 = \frac{\mathcal{E}}{R} = 1200$ amp.

$\frac{R}{L} = 20 \text{ sec}^{-1}$.

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right) = 1200\left(1 - e^{-20t}\right)$$

$$I = 0.9\,I_0 \text{ for } e^{-20t} = 0.1$$

$$20t = \ln_e 10 = 2.30 \qquad t = 0.115 \text{ sec}$$

$I = 1080$ amp at $t = 0.115$

Magnetic field energy $= \frac{1}{2} L I^2 = \frac{1}{2}(0.5 \times 10^{-3})(1080)^2 = 292$ joules

Energy supplied by battery between $t = 0$ and $t = .115$

$$= \int_0^{.115} \mathcal{E} I\, dt = \mathcal{E} I_0 \int_0^{.115}\left(1 - e^{-\frac{R}{L}t}\right)dt = \frac{\mathcal{E} I_0 L}{R}\int_0^{2.3}\left(1 - e^{-x}\right)dx$$

$$= \frac{\mathcal{E} I_0 L}{R}\left[x + e^{-x}\right]_0^{2.3} = \frac{\mathcal{E} I_0 L}{R}\left[2.3 + 0.1 - 1.0\right] = 1.4\,\frac{\mathcal{E} I_0 L}{R}$$

$$= \frac{1.4 \times 12 \times 1200}{20} = 1008 \text{ joules}$$

7.14 Let v be the instantaneous velocity of the bar.

$$|\mathcal{E}| = \left|\frac{1}{c}\frac{d\Phi}{dt}\right| = \frac{1}{c}Bbv \qquad I = \frac{|\mathcal{E}|}{R} = \frac{Bbv}{Rc}$$

The force on the bar : $F = \frac{IbB}{c} = \frac{b^2 B^2 v}{Rc^2}$

$F = -m\frac{dv}{dt}$ (minus because the force opposes the motion)

$$\frac{dv}{v} = -\frac{B^2 b^2}{Rmc^2}dt \qquad \text{Integrating both sides :}$$

$$\ln v = -\frac{B^2 b^2}{Rmc^2}t + \text{constant.} \qquad \text{If } v = v_0 \text{ at } t = 0,$$

we have : $v = v_0 e^{-t/T}$, where $T = \frac{Rmc^2}{B^2 b^2}$

The velocity decreases exponentially — in that sense, the rod never stops moving. But the distance it travels is finite : $x = \int_0^\infty v\,dt = \int_0^\infty v_0 e^{-t/T}dt = Tv_0$

The initial kinetic energy of the rod, $\frac{1}{2}mv_0^2$, is transferred to the resistor as heat :

$\frac{1}{2}mv_0^2 = \int_0^\infty RI^2 dt$, as can be easily verified :

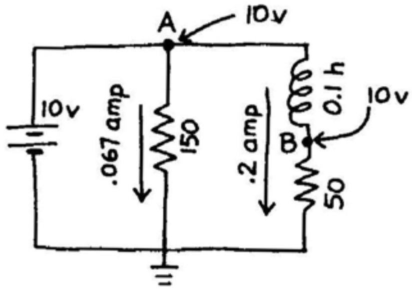
$$I = I_0 e^{-t/T} = \frac{Bbv_0}{Rc}e^{-t/T}$$

$$\int_0^\infty RI^2 dt = \frac{B^2 b^2 v_0^2}{Rc^2}\int_0^\infty e^{-2t/T}dt = \frac{B^2 b^2 v_0^2}{Rc^2}\cdot\left(\frac{T}{2}\right) = \frac{mv_0^2}{2}$$

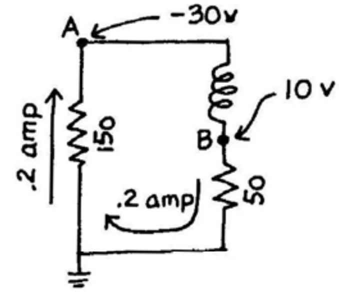
7.16 The current I in the moving frame is proportional to $\mathcal{E}/R$. The resistance R is proportional to resistivity/(rod diameter)^2. For given B, the electromotive force $\mathcal{E}$ is proportional to the frame's velocity v; and the force F which must be applied to maintain that velocity is proportional to I. Thus, other things being constant, $F \propto v/R \propto v \times \frac{(\text{rod diameter})^2}{\text{resistivity}}$

A force of 2 N will pull the frame out of the field in 0.5 sec. A brass frame of the same dimensions would be pulled out in 1 sec by 0.5 N. The aluminum frame of 1 cm diameter rod would be pulled out in 1 sec by a force of 4 N.

$\boxed{7.17}$

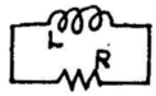


before switch opened

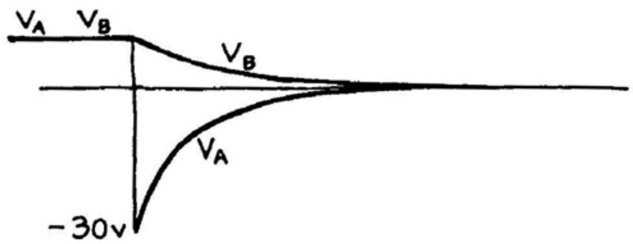


just after switch opened

Current through inductance cannot change abruptly!

Circuit on right is  with $L = 0.1$ h and $R = 200$ ohms

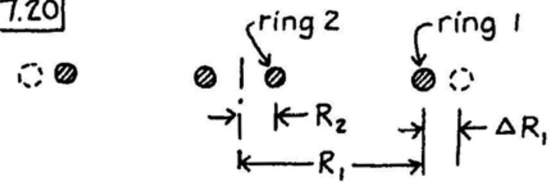
$I = I_0 e^{-\frac{R}{L}t}$; $I_0 = .2$ amp. $L/R = 0.5$ millisec.



$\boxed{7.18}$ $\Phi = N\pi a^2 B \quad \mathcal{E} = -\frac{1}{c}\frac{d\Phi}{dt} = -\frac{N\pi a^2}{c}\frac{dB}{dt}$

$I = \mathcal{E}/R \quad Q = \int I\,dt = -\frac{N\pi a^2}{cR}\int_{B_0}^{0} dB = \frac{N\pi a^2 B_0}{cR}$

$\boxed{7.20}$



With current I in the outer ring the flux through the inner ring, for $R_1 \gg R_2$, is:

$\Phi_{21} = \pi R_2^2 \cdot \frac{2\pi I}{cR_1}$ or $\Phi_{21} = \frac{2\pi^2 I}{c} \frac{R_2^2}{R_1}$. Suppose we change R_1 to $R_1 + \Delta R_1$, by expanding the outer ring while holding I constant. The resulting change in Φ_{21} is:

$$\Delta\Phi_{21} = \frac{\partial \Phi_{21}}{\partial R_1} \Delta R_1 = -\frac{2\pi^2 I}{c} \frac{R_2^2}{R_1^2} \Delta R_1$$

Now consider a current I in the inner ring, ring 2. Let B be the field strength at the radius of the outer ring, R_1. If we now expand the outer ring by ΔR_1 the flux Φ_{12} decreases by just the amount of flux between the circle of radius R_1 and the circle of radius $R_1 + \Delta R_1$. (Problem 7.19 explained why it is a <u>dec</u>rease.) The change in flux is $\Delta \Phi_{21} = -B \cdot 2\pi R_1 \Delta R_1$ since $2\pi R_1 \Delta R_1$ is the area between the circles.

Our theorem $\Phi_{12} = \Phi_{21}$ guarantees that $\Delta\Phi_{12} = \Delta\Phi_{21}$

Hence: $-\frac{2\pi^2 I}{c} \frac{R_2^2}{R_1^2} \Delta R_1 = -B \cdot 2\pi R_1 \Delta R_1$

Solving for B: $B = \frac{\pi R_2^2 I}{c R_1^3}$ or more generally, $B = \frac{\pi R_2^2 I}{c r^3}$ at any point in the plane of the ring where $r \gg R_2$.

7.21 Assume current I flows in the outer solenoid. The field inside, approximately uniform in the region occupied by the inner solenoid, is $B = \frac{4\pi I}{c}\frac{N_2}{b_2}$. We have assumed here that $\frac{b_2}{a_2}$ is so large that we can use the formula for an infinite solenoid. We can refine this by using Eq. 6.44, page 228, to calculate the field at the center of a finite solenoid of length b_2, radius a_2. The correction factor is simply $\cos\left(\tan^{-1}\frac{2a_2}{b^2}\right)$ or $b_2/\sqrt{b_2^2+4a_2^2}$. This will still not lead us to an exact result, for the inner coil includes a finite volume in which the field strength varies somewhat. But for the proportions shown in the Figure, the approximation will be pretty good. The flux linking the inner coil is

$$\Phi_{12} = \pi a_1^2 B N_1 \quad \text{or} \quad \frac{4\pi^2 I}{c}\,\frac{N_1 N_2 a_1^2}{b_2}\left(\frac{b_2}{\sqrt{b_2^2+4a_2^2}}\right)$$

Since $\mathcal{E}_{12} = -\frac{1}{c}\frac{d\Phi_{12}}{dt} = -M\frac{dI}{dt}$, we have, in CGS units, $M = \frac{4\pi^2 N_1 N_2 a_1^2}{c^2\, b_2}\left(\frac{b_2}{\sqrt{b_2^2+4a_2^2}}\right)$

To express M in henrys, replace the constant $\frac{1}{c^2}$ by 10^{-9}:

$$M = \frac{4\pi^2 \times 10^{-9} N_1 N_2 a_1^2}{\sqrt{b_2^2+4a_2^2}} \quad \text{henrys}$$

7.24 Energy density in $J m^{-3} = \frac{B^2 \leftarrow \text{tesla}}{2\mu_0 \leftarrow 4\pi\times 10^{-7}}$

For $B = 0.4$ tesla, $U = \frac{(0.4)^2}{2\times 4\pi\times 10^{-7}} = 6.4\times 10^4\, J\, m^{-3}$

To estimate $\int U\,dv$ we'll assume B is uniform through the interior of the solenoid and zero outside. The volume is $(\pi/4)\times .81\ m^2 \times 2.2\ m$ or $1.40\ m^3$, giving $9\times 10^4 J$ for the total stored energy. [A more accurate estimate could be made, if it were needed, by consulting a table of the inductance of finite solenoids, calculating the current required to produce the given central field, and then computing $LI^2/2$. The result, when that is carried out in this case, is 8.8×10^4 J, fortuitously close to the approximate estimate.]

$\boxed{7.28}$ $R = \frac{\pi}{a\sigma}$ $B = \frac{2\pi I}{c\,(a/2)} = \frac{4\pi I}{ca}$

$\frac{B^2}{8\pi} = \frac{2\pi I^2}{c^2 a^2}$ volume $= 2\pi \times \frac{a}{2} \times a^2 = \pi a^3$

$$\left.\begin{array}{l} \text{stored energy} = \dfrac{2\pi^2 I^2 a}{c^2} \\[2ex] \text{ohmic dissipation} = I^2 R = \dfrac{\pi I^2}{a\sigma} \end{array}\right\} \text{decay time} = \frac{2\pi a^2 \sigma}{c^2}$$

$\frac{a}{c} = \frac{3 \times 10^8 \text{ cm}}{3 \times 10^{10} \text{ cm sec}^{-1}} = 10^{-2}$ sec $\sigma = 10^{16}$ sec^{-1}

time $= 2\pi \times 10^{12}$ sec $= 2000$ centuries

$\boxed{7.29}$ Neglecting the inductance and resistance of the two rings and the leads, the charge in the capacitor C_2 at any time t is $Q = \mathcal{E}_0 (\cos 2\pi ft) C_2$.
Since $I = \frac{dQ}{dt}$, $I = -2\pi f \mathcal{E}_0 C_2 \sin 2\pi ft$. The two rings are in series and this current flows in each. Assume $h \ll b$ so that we are justified in computing the force between the rings as if they were parallel straight wires, with force per unit length $= \frac{2I^2}{c^2 h}$. The length is $2\pi b$, so the force pulling the upper ring down (note currents are in same direction) is $F_m = \frac{4\pi b}{c^2 h} I^2$ or

$$F_m = \frac{4\pi b}{c^2 h}\left(2\pi f \mathcal{E}_0 C_2\right)^2 \sin^2 2\pi ft$$

The time-average of $\sin^2 2\pi ft$ is simply $\frac{1}{2}$
Hence the average force is

$$\overline{F}_m = \frac{8\pi^3 b f^2 \mathcal{E}_0^2 C_2^2}{hc^2}$$

In the capacitor at the left the electric field strength is: $E = \frac{\mathcal{E}_0 \cos 2\pi ft}{s}$. The downward force on the upper plate is $\frac{E^2}{8\pi} \times$ area,

$$\text{or } F_e = \frac{E^2}{8\pi}\cdot \pi a^2 = \frac{\mathcal{E}_0^2 a^2}{8 s^2}\cos^2 2\pi ft$$

The time average of $\cos^2 2\pi ft$ is $\frac{1}{2}$, so

$$\overline{F}_e = \frac{a^2 \mathcal{E}_0^2}{16 s^2}$$

This can be expressed in terms of the capacitance C_1, which is $\frac{\pi a^2}{4\pi s}$ or $\frac{a^2}{4s}$. Substituting for s:

$$\overline{F}_e = \frac{\mathcal{E}_0^2 C_1^2}{a^2}$$

When the forces are balanced (which might be brought about by varying C_2), we have $\overline{F}_e = \overline{F}_m$:

$$\frac{\cancel{\mathcal{E}_0^2} C_1^2}{a^2} = \frac{8\pi^3 b f^2 \cancel{\mathcal{E}_0^2} C_2^2}{hc^2}$$

Solving for c:

$$c = (2\pi)^{\frac{3}{2}}\, a\left(\frac{b}{h}\right)^{\frac{1}{2}} f\left(\frac{C_2}{C_1}\right)$$