8.1 For a 60-watt, 120 volt bulb the normal current is $I = \frac{P}{V} = \frac{60}{120} = 0.5$ amp. The resistance of the filament is $R = \frac{120}{0.5} = 240$ ohms. We want to have the same current, 0.5 amp., when the bulb is connected in series with a reactance ωL, across 240 volts:

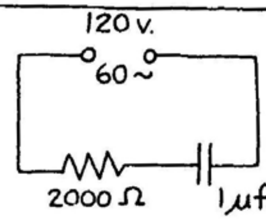
$$0.5 = |I| = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + (\omega L)^2}} \qquad 0.5 = \frac{240}{\sqrt{(240)^2 + (\omega L)^2}}$$

$(\omega L)^2 = 480^2 - 240^2 \quad \omega L = 240\sqrt{3} = 415$ ohms reactance.

$\omega = 2\pi f = 2\pi \times 60 = 377 \qquad L = \frac{415}{377} = 1.10$ henry

8.2 (a) $Z = R - \frac{i}{\omega C} = 2000 - \frac{i}{377 \times 10^{-6}}$

$= 2000 - 2650\,i$ ohms



The absolute value of the impedance $|Z|$ is: $|Z| = \sqrt{2000^2 + 2650^2} = 3320$ ohms

(b) The rms current is simply $\frac{V}{|Z|}$ where V is the rms voltage, in this case 120 volts: $I = \frac{120}{3320} = 0.036$ amp.

(c) The power dissipated in the circuit is $I^2 R$:

$P = I^2 R = (0.036)^2 \times 2000 = 2.59$ watts

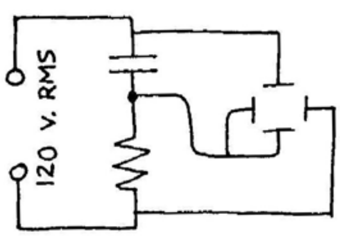
(d) A voltmeter connected across the resistor will read:

$V_R = IR = 0.036 \times 2000 = 72$ volts (rms)

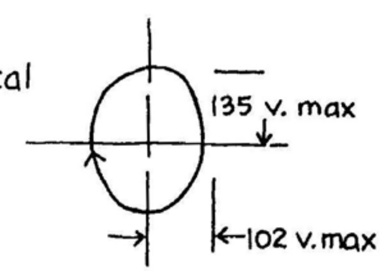
A voltmeter connected across the capacitor will read:

$V_c = \frac{I}{\omega C} = 0.036 \times 2650 = 95.5$ volts (rms)

If plates are connected as shown and if vertical and horizontal deflection sensitivities



are equal, beam will trace elliptical pattern, in the sense indicated.



$\boxed{8.6}$ $V(t) = Ae^{-\beta_1 t} + Be^{-\beta_2 t}$ was said to be the general solution in the over-damped case. To verify this, we try the solution $V = Ae^{-\beta t}$ for Eq. 2 :

$$\frac{dV}{dt} = -A\beta e^{-\beta t} \qquad \frac{d^2V}{dt^2} = A\beta^2 e^{-\beta t}$$

Substituting in $\frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV}{dt} + \frac{1}{LC}V = 0$ we have :

$$\cancel{A}\beta^2 \cancel{e^{-\beta t}} - \frac{R}{L}\cancel{A}\beta \cancel{e^{-\beta t}} + \frac{\cancel{A}}{LC}\cancel{e^{-\beta t}} = 0 \quad \text{or} \quad \beta^2 - \frac{R}{L}\beta + \frac{1}{LC} = 0$$

The roots of this quadratic equation are:

$$\beta = \frac{1}{2}\left(\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}\right) = \frac{R}{2L}\left(1 \pm \sqrt{1 - \frac{4L}{R^2C}}\right)$$

The roots are real if $R \geqslant 2\sqrt{\frac{L}{C}}$ Notice that both roots are positive. Call them β_1 and β_2. With $R = 600\,\Omega$, $L = 10^{-4}$ henrys $C = 10^{-8}$ farads,

$$\frac{4L}{R^2C} = \frac{4\times 10^{-4}}{36\times 10^4 \times 10^{-8}} = \frac{1}{9} \qquad \frac{R}{2L} = \frac{600}{2\times 10^{-4}} = 3\times 10^6$$

$$\beta_1 = 3\times 10^6\left(1 + \sqrt{\tfrac{8}{9}}\right) = 3\times 10^6 \times 1.943 = 5.84\times 10^6$$

$$\beta_2 = 3\times 10^6\left(1 - \sqrt{\tfrac{8}{9}}\right) = 3\times 10^6 \times 0.057 = 0.171\times 10^6$$

The initial condition is $\frac{dV}{dt} = 0$ at $t = 0$. Hence $-\beta_1 A - \beta_2 B = 0$, or $\frac{B}{A} = -\frac{\beta_1}{\beta_2} = 34$. The term $Be^{-\beta_2 t}$ is the <u>slowly</u> decaying term, and after a microsecond or so, it is practically all that is left.

$\boxed{8.8}$ $V = Ae^{-\alpha t}\cos\omega t \quad I = AC\omega e^{-\alpha t}(\sin\omega t + \frac{\alpha}{\omega}\cos\omega t)$

with $\alpha = \frac{R}{2L}$ and $\omega^2 = \frac{1}{LC} - \alpha^2$ Energy in circuit

$= \frac{1}{2}CV^2 + \frac{1}{2}LI^2 = \frac{1}{2}CA^2e^{-2\alpha t}\left[\cos^2\omega t + LC\omega^2(\sin\omega t + \frac{\alpha}{\omega}\cos\omega t)^2\right]$

$= \frac{1}{2}CA^2e^{-2\alpha t}\left[\cos^2\omega t + LC\omega^2(\sin^2\omega t + \frac{2\alpha}{\omega}\sin\omega t\cos\omega t + \frac{\alpha^2}{\omega^2}\cos^2\omega t)\right]$

$= \frac{1}{2}CA^2e^{-2\alpha t}\left[\cos^2\omega t + \sin^2\omega t + LC(-\alpha^2\sin^2\omega t + \alpha^2\cos^2\omega t + 2\alpha\omega\sin\omega t\cos\omega t)\right]$

$= \frac{1}{2}CA^2e^{-2\alpha t}\left[1 + LC(\alpha^2\cos 2\omega t + \alpha\omega\sin 2\omega t)\right]$

Except for the oscillating part of the factor in brackets, the energy decays exponentially. To dissipate the energy as quickly as possible, we want α to be as large as possible. But the limiting value of α, approaching from the under-damped side, corresponds to $R = 2\sqrt{\frac{L}{C}}$ or $\alpha = \frac{R}{2L} = \frac{1}{\sqrt{LC}}$. In that limit, $\omega = 0$ and the expression above reduces to $CA^2e^{-2\alpha t} = CA^2e^{-\frac{R}{L}t}$.

Look now at the over-damped case, using the results from Problem 8.6. We found $V = Ae^{-\beta_1 t} + Be^{-\beta_2 t}$ with $\beta_1 = \frac{R}{2L}\left(1 + \sqrt{1 - \frac{4L}{R^2C}}\right)$ and $\beta_2 = \frac{R}{2L}\left(1 - \sqrt{1 - \frac{4L}{R^2C}}\right)$

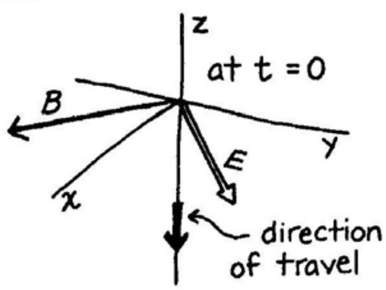
Evidently the root β_2 is necessarily less than $\frac{R}{2L}$, so there will be a term in the energy decaying like $e^{-2\beta_2 t}$ which will decay more slowly than $e^{-\frac{R}{L}t}$. Hence the over-damped case exhibits slower decay than the case of critical damping, as does the underdamped case.

$\boxed{9.10}$

If we neglect the edge fields, an approximation which is not very good unless $s \ll b$, the displacement current will be uniformly distributed in the gap, and the total displacement current in the gap will equal the conduction current I in the wire. The fraction of the current enclosed by a circle through P, centered on the axis, will be $\pi r^2/\pi b^2$.

Hence $2\pi r B = \frac{4\pi}{c}\frac{r^2}{b^2} I$ or $B = \frac{2Ir}{cb^2}$.

$\boxed{9.1}$



The wave is travelling in the $-\hat{z}$ direction, as shown by the sign in $(z + ct)$. Hence that is the direction of $\mathbf{E} \times \mathbf{B}$. $\mathbf{B}$ is perpendicular to $\mathbf{E}$ and equal in magnitude:

$$\mathbf{B} = 2(\hat{x} - \hat{y})\sin\left(\frac{2\pi}{\lambda}\right)(z + ct) \text{ gauss.}$$

$\boxed{9.2}$

The power density in electromagnetic waves is c times the energy density U. The power density given, 10^3 joule m^{-2} sec^{-1}, is equivalent to 10^6 erg cm^{-2} sec^{-1}. The energy density U is $10^6/3 \times 10^{10}$ erg cm^{-3}. Half of this is in magnetic field, with density $B_{rms}^2/8\pi$. Hence

$$\frac{B_{rms}^2}{8\pi} = \frac{1}{2} \times \frac{10^6}{3 \times 10^{10}}, \text{ or } B_{rms} = \left(\frac{4\pi \times 10^6}{3 \times 10^{10}}\right)^{\frac{1}{2}} = 0.02 \text{ gauss}$$

In SI, the energy density in J m^{-3} is $\frac{B_{rms}^2}{2\mu_0}$, with B_{rms} in tesla. Hence:

$$B_{rms}^2 = 2\mu_0 \times \frac{1}{2} \times \left(\frac{10^3}{3 \times 10^8}\right) \text{ or } B_{rms} = \left(\frac{4\pi \times 10^{-7} \times 10^3}{3 \times 10^8}\right)$$

$$= 2 \times 10^{-6} \text{ tesla}$$

9.3 The proton at the origin in Fig. 9.8 will experience the maximum electric field of 5 statvolt/cm at $t = 0$. The field falls to half value in the time, 1 nanosecond, it takes the wave to travel 30 cm (1 foot). The time variation of the field E at the origin is:

$E_y = \frac{5}{1 + (10^9 t)^2}$. The momentum acquired by the proton during the passage of the pulse will be

$$p_y = \int_{-\infty}^{\infty} eE_y\, dt = e \int_{-\infty}^{\infty} \frac{5\, dt}{1 + (10^9 t)^2} = e \times 5 \times 10^{-9}\pi$$

$p_y = 7.5 \times 10^{-18}$ gm cm sec^{-1} The proton's final speed is p_y/m or 4.7×10^6 cm/sec. Its displacement during the few nanoseconds of acceleration is negligible. One microsecond later it will be close to $y = 4.7$ cm.

9.4 Before the pulse has completely passed, the proton has acquired some velocity in the y direction, and will therefore experience a force $e\mathbf{v} \times \mathbf{B}$ in the magnetic field of the wave. This force will be in the direction $-\hat{\mathbf{x}}$, which is the direction in which the wave is travelling. And it would be in that direction for a negative particle also. The wave tends to knock the particle along. In order of magnitude, if τ is the duration of the pulse of amplitude E:

$$p_y = Ee\tau \qquad v_y = \frac{eE\tau}{m}$$

$$p_x \approx e\frac{v_y}{c}B\tau = e\frac{v_y}{c}E\tau \quad , \text{ since } B = E.$$

Then $p_x/p_y \approx v_y/c$ The "knock-on" is a second order effect.

$\boxed{9.5}$ $\mathbf{E} = \hat{\mathbf{y}} E_0 \sin(kx + \omega t) \qquad \mathbf{B} = -\hat{\mathbf{z}} E_0 \sin(kx + \omega t)$

$\nabla \cdot \mathbf{E} = 0\ ;\ \nabla \times \mathbf{E} = \hat{\mathbf{z}} k E_0 \cos(kx + \omega t)\ ;\ \dfrac{\partial \mathbf{E}}{\partial t} = \hat{\mathbf{y}} E_0 \omega \cos(kx + \omega t)$

$\nabla \cdot \mathbf{B} = 0\ ;\ \nabla \times \mathbf{B} = \hat{\mathbf{y}} k E_0 \cos(kx + \omega t)\ ;\ \dfrac{\partial \mathbf{B}}{\partial t} = -\hat{\mathbf{z}} \omega E_0 \cos(kx + \omega t)$

$\nabla \times \mathbf{E} = -\dfrac{1}{c} \dfrac{\partial \mathbf{B}}{\partial t}$ requires $k = \omega/c$

$\nabla \times \mathbf{B} = \dfrac{1}{c} \dfrac{\partial \mathbf{E}}{\partial t}$ also requires $k = \omega/c$

For $\omega = 10^{10}\ \text{sec}^{-1} \qquad \lambda = 2\pi c/\omega = 18.84$ cm

Energy density $= \left(\underbrace{\dfrac{E_0^2}{8\pi}}_{\text{electric field}} + \underbrace{\dfrac{E_0^2}{8\pi}}_{\text{magnetic field}}\right) \underbrace{\dfrac{1}{2}}_{\text{average of } \sin^2(kx+\omega t)} = \dfrac{E_0^2}{8\pi}$

For $E_0 = .05$ statvolt/cm $\quad E_0^2/8\pi = 0.99 \times 10^{-4}\ \text{erg cm}^{-3}$

Power density $= (E_0^2/8\pi)c = 3.0 \times 10^6\ \text{erg cm}^2\ \text{sec}^{-1}$