PHYSICS 4C FINAL EXAM FALL QUARTER 2007
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Formulas:

$\vec{F}_2 = \frac{q_1 q_2}{r_{21}^{\ 2}} \hat{r}_{21}$ Coulomb's law ; $\vec{E} = \vec{F}/q_0$ electric field ; $\vec{E}(x,y,z) = \int \frac{\rho(x',y',z')(\hat{r} - \hat{r}')}{|\vec{r} - \vec{r}'|^2} dx'dy'dz'$

$\oint \vec{E} \cdot d\vec{a} = 4\pi q_{enc} = 4\pi \int \rho dv$ Gauss' law 1 charge at the origin: $\vec{E}(\vec{r}) = \frac{q}{r^2}\hat{r}$

Linear, surface, volume charge density : $dq = \lambda\, ds$, $dq = \sigma\, dA$, $dq = \rho\, dV$

Electric field of : charge : $E = \frac{q}{r^2}$; line of charge : $E = \frac{2\lambda}{r}$; sheet of charge : $E = 2\pi\sigma$

Potential of single charge q : $\phi(\vec{r}) = \frac{q}{r}$; charge distribution : $\phi(\vec{r}) = \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dx'dy'dz'$

$\phi(x,y,z) - \phi(x_0,y_0,z_0) = -\int_{(x_0,y_0,z_0)}^{(x,y,z)} \vec{E} \cdot d\vec{s}$; $\vec{E} = -\nabla\phi$; $\nabla^2\phi = -4\pi\rho$; $\text{div}\vec{E} = 4\pi\rho$

$\text{div}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$; $u = \frac{E^2}{8\pi}$ electric energy density

energy of 3 charges: $U = \frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}$; energy of q in potential ϕ: $U = q\ \phi(x,y,z)$

Electric field right next to a conducting surface: $E=4\pi\sigma$

Capacitors: Q=CV ; Parallel plates: $C = \frac{A}{4\pi s}$ A=area, s=dist. betw. plates; $U = \frac{Q^2}{2C}$ energy

$I = \int \vec{J} \cdot d\vec{a}$, $I = \frac{dq}{dt}$, $\vec{J} = nq\vec{u}$; $div\vec{J} = -\frac{\partial\rho}{\partial t}$; Power: $P = I^2 R$; $P = \varepsilon I$

V=IR , $\vec{J} = \sigma\vec{E}$; $\vec{E} = \rho\vec{J}$; $R = \rho\frac{L}{A}$; $\sigma = \frac{ne^2\tau}{m_e}$; $Q(t) = C\varepsilon(1 - e^{-t/RC})$

Ampere's law: $\oint_C \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} I_{enc} = \frac{4\pi}{c}\int_S \vec{J} \cdot d\vec{a}$; Biot-Savart law: $d\vec{B} = \frac{I d\vec{\ell} \times \hat{r}}{cr^2}$

$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J}$; $\vec{\nabla} \cdot \vec{B} = 0$; $\vec{\nabla} \times \vec{E} = 0$ (electrostatics) ; $\vec{\nabla} \times \vec{A} = \vec{B}$; $\vec{\nabla} \cdot \vec{A} = 0$

Lorentz force : $\vec{F} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$; force on wire : $d\vec{F} = \frac{I}{c} d\vec{\ell} \times \vec{B}$; cyclotron : $\omega = \frac{qB}{mc}$

Field of: long wire: $B = \frac{2I}{cr}\hat{\varphi}$; ring: $\vec{B} = \frac{2\pi b^2 I}{c(b^2 + z^2)^{3/2}}\hat{z}$; solenoid: $\vec{B} = \frac{4\pi I n}{c}\hat{z}$

Faraday law: $\varepsilon = \oint_c \vec{E} \cdot d\vec{s} = -\frac{1}{c}\frac{\partial}{\partial t}\Phi_B = -\frac{1}{c}\frac{\partial}{\partial t}\int \vec{B} \cdot d\vec{a}$; $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$

Inductance: $\varepsilon_{21} = -M_{21}\frac{\partial I_1}{\partial t}$; $M_{21} = \frac{\Phi_{21}}{I_1}$; $M_{21} = M_{12} = M$; $\varepsilon = -L\frac{\partial I}{\partial t}$; $L = \frac{\Phi}{I}$

L-R circuit: $I = \frac{\varepsilon_0}{R}(1 - e^{-(R/L)t})$; Energy: $U = \frac{1}{2}LI^2$; density $u = \frac{B^2}{8\pi}$

RLC circuit: $V(t) = e^{-(R/2L)t}(A\cos\omega t + B\sin\omega t)$; $\omega = \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$

Alternating current: $\varepsilon = \varepsilon_0 \cos\omega t$; $I = I_0 \cos(\omega t + \varphi)$; $\tan\varphi = \frac{1/(\omega C) - \omega L}{R}$

$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + (\omega L - 1/(\omega C))^2}}$; Power: $< P > = \frac{1}{2}\varepsilon_0 I_0 \cos\varphi$

Ampere-Maxwell law: $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$; $\oint_C \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} I_{enc} + \frac{1}{c}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{a}$

Displacement current: $\vec{J}_d = \frac{1}{4\pi}\frac{\partial \vec{E}}{\partial t}$; electromagnetic waves: v=c, $E_0=B_0$, c=3x10^{10}cm/s

Electric dipole: $\vec{p} = \int dv' \rho(\vec{r}')\vec{r}'$; $\varphi(\vec{r}) = \frac{\vec{p} \cdot \vec{r}}{r^3}$; $E_r = \frac{2p}{r^3}\cos\theta$; $E_\theta = \frac{p}{r^3}\sin\theta$

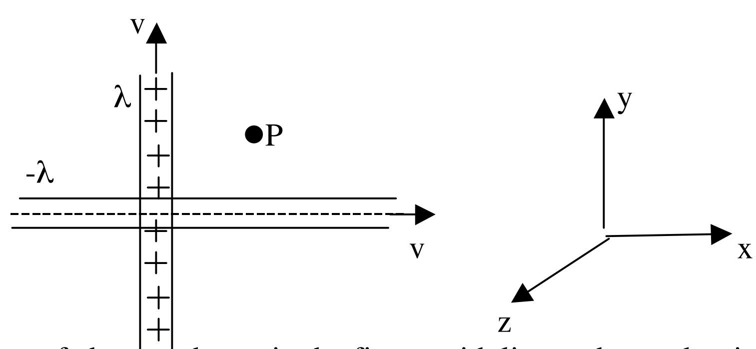
Energy and torque in external E field: $U = -\vec{p} \cdot \vec{E}$; $\vec{\tau} = \vec{p} \times \vec{E}$

Polarization: $E' = -4\pi P$; $\frac{P}{E} = \frac{\varepsilon - 1}{4\pi}$; capacitor w/dielectric: $C = \varepsilon C_0$

Magnetic dipole: $\vec{m} = \frac{I}{c}\vec{a}$; $U = -\vec{m} \cdot \vec{B}$; $\vec{\tau} = \vec{m} \times \vec{B}$

8 problems, 10 points each:

Problem 1 (10 pts)



Consider the perpendicular lines of charges shown in the figure with linear charge density λ and -λ, with λ=12esu/cm . The lines cross at the origin of the x,y,z coordinate system and the point P is at (x,y,z)=(4cm,4cm,0).

(a) Find the magnitude of the electric field at P, in statvolts/cm. In which direction does it point? (give either x-y-z components or the angle it makes with the axis).

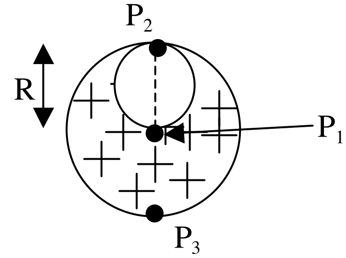
(b) Assume the lines of charge are moving with speed v=30,000cm/s, the horizontal line horizontally to the right and the vertical line vertically upwards. What is the electric current generated by each line of charge, in esu/s? Justify your answer.

(c) Find the magnitude of the magnetic field at point P, in Gauss. In which direction does it point?

(d) Is there any speed v for which the magnetic field and the electric field at P have equal magnitude (in cgs units)? If yes give the value in cm/s. If not explain why not.

Speed of light: c=3x10^{10}cm/s

Problem 2 (10 pts)



A long cylinder of uniform positive charge density ρ and radius R has a smaller cylinder of radius R/2 carved out and left empty, as shown in the figure.

(a) Find the electric field (magnitude and direction) along the dashed line joining points P_1 and P_2 shown in the figure. P_1 is at the center of the cylinder and on the surface of the empty cavity, point P_2 is directly above it at the surface of the cylinder. The line joining P_1 and P_2 goes through the center of the cavity.

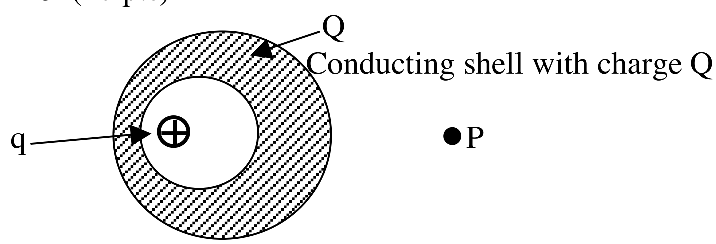
(b) Find the divergence of E at any point along the dashed line.

(c) Find the difference in electrostatic potential between points P_1 and P_2.

(d) Find the magnitude of the electric field at point P_3 shown in the figure, directly below P_1 at the surface of the cylinder.

<u>Hints</u>: use Gauss' law and superposition. First find the expressions for the field of a uniformly charged cylinder inside and outside.

Problem 3 (10 pts)



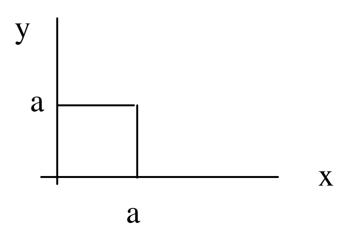
The conducting shell in the figure is bounded by two spherical surfaces that are non-concentric, so it is asymmetric as the figure shows. It has total charge Q. A point charge q is in the inner cavity, not at its center.

(a) Find the total charge on the inner and outer surfaces of the conducting shell.

(b) Give an expression for the electric field at point P outside the conducting shell. Does it depend on the distance from P to q, or on the distance from P to the center of the inner sphere, or to the center of the outer sphere, or on all, or it's impossible to tell? Explain.

(c) Give an expression for the surface charge density at the inner and outer surfaces of the conducting shell if you can, or explain why you can't.

Problem 5 (10 pts)



The electric field in the region of space 0<x<a, 0<y<a, any z, is given by

$$\vec{E}(x,y,z) = C(-y^2\hat{x} + x^2\hat{y}) = C(-x^2, y^2, 0)$$

where C is a constant. $\vec{E}$ is time-independent.

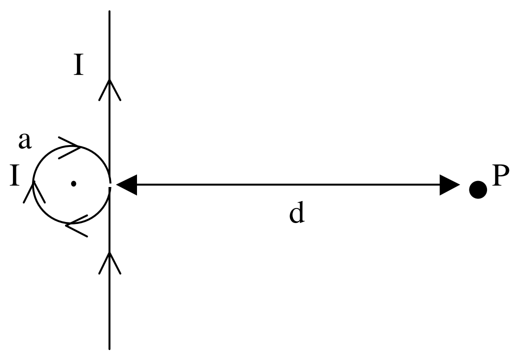
(a) Explain why this $\vec{E}$ cannot be an electrostatic field.

(b) Find the charge density *ρ(x,y,z=0)* at all points within the square shown in the figure (0<x<a, 0<y<a)

(c) Assuming the magnetic field at time t=0 is zero everywhere, find its value at time t_0 at all points 0<x<a, 0<y<a.

(d) Find the emf ε induced in the square loop of wire shown in the figure, of vertices (0,0), (a,0), (0,a), (a,a). <u>Hint:</u> there are 2 ways to do this, use either one.

Problem 4 (10 pts)

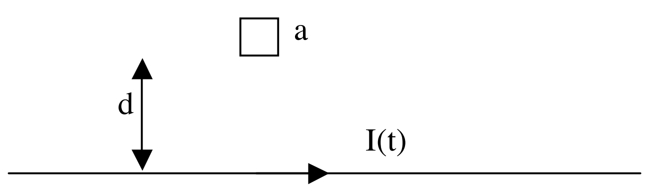


A long wire carrying current I is bent into the shape shown. The loop has radius a.

(a) Find the magnitude and direction of the magnetic field at the center of the loop.

(b) Find the magnitude and direction of the magnetic field at point P at distance d from the wire (P is on a line perpendicular to the wire that goes through the center of the loop). Assume d>>a, but don't ignore the contribution from the loop.

Problem 6 (10 pts)

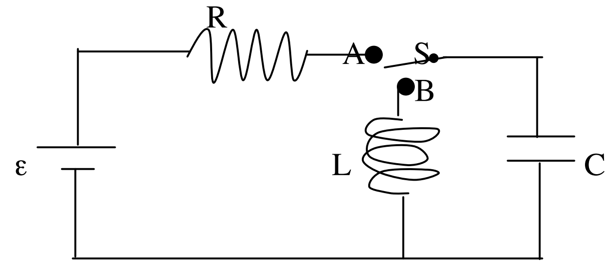


The current in the long wire is $I(t) = I_0 e^{-t/\tau}$, with τ a constant. The square loop of side length a is made of conducting wire, has two sides parallel to the long wire, and the side closest to the long wire is at distance d from the long wire, with d>>a. The resistance of the wire in the square loop is R.

(a) Find an expression for the current in the square loop, $I_l(t)$. Indicate in which direction it flows, and explain why.

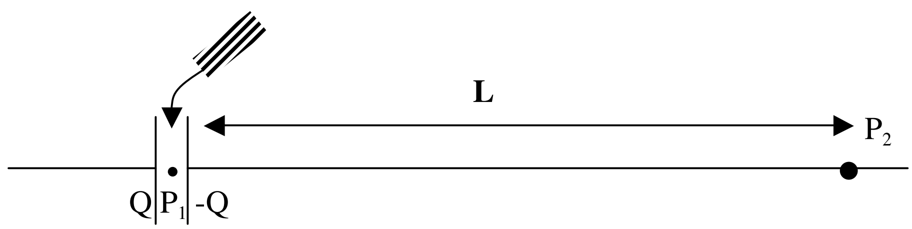
(b) Find an expression for the net force acting on the square loop. Express your answer in terms of I_1, I, d and a. Indicate in which direction the force points, and explain why.

Problem 7 (10 pts)



In the circuit in the figure, ε =5V, R=2Ω, C=3μF, L=4mH. Initially the capacitor C is uncharged and the circuits are open. At time t=0, the switch S is connected to point A.
(a) Find the current through the resistor $I_R(t)$ as function of time and plot it.
(b) What is the charge Q on the capacitor after a long time, in Coulombs?
(c) After a long time, at time t_0, the switch is shifted to position B. Make a plot of the current through L, I_L, as function of time for $t>t_0$.
(d) What is the maximum current that will go through L, in Amps?
(e) Describe in words what happens if you switch S from B to A at the instant when the current through L is maximum (there is more than one 'right' answer).

Problem 8 (10 pts)



The parallel plate capacitor shown in the figure has square plates of side length a and distance between plates d, with a>>d. It has charge Q and -Q on the left and right plate respectively and is not connected to any voltage source. The point P is at distance L from the capacitor as shown in the figure, with L>>a (and L>>d).
(a) Find the electric field at point P_1 at the center of the capacitor plates (magnitude and direction).
(b) Find the electric field at point P_2 (magnitude and direction).
<u>Hint:</u> treat the capacitor as a dipole.
Next, the region between the plates is filled with a dielectric material of dielectric constant ε. Find now
(c) the electric field at point P_1 (magnitude and direction).
(d) the electric field at point P_2 (magnitude and direction).
(e) The polarization of the dielectric, P (magnitude and direction).