PHYSICS 4C QUIZ 6 FALL QUARTER 2007
PROF. HIRSCH December 7, 2007

Formulas:

$\vec{F}_2 = \frac{q_1 q_2}{{r_{21}}^2}\hat{r}_{21}$ Coulomb's law ; $\vec{E} = \vec{F}/q_0$ electric field ; $\vec{E}(x,y,z) = \int \frac{\rho(x',y',z')(\hat{r} - \hat{r}')}{|\vec{r} - \vec{r}'|^2} dx'dy'dz'$

$\oint \vec{E} \cdot d\vec{a} = 4\pi q_{enc} = 4\pi \int \rho dv$ Gauss' law 1 charge at the origin: $\vec{E}(\vec{r}) = \frac{q}{r^2}\hat{r}$

Linear, surface, volume charge density : $dq = \lambda\, ds$, $dq = \sigma\, dA$, $dq = \rho\, dV$

Electric field of : charge : $E = \frac{q}{r^2}$; line of charge : $E = \frac{2\lambda}{r}$; sheet of charge : $E = 2\pi\sigma$

Potential of single charge q : $\phi(\vec{r}) = \frac{q}{r}$; charge distribution : $\phi(\vec{r}) = \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dx'dy'dz'$

$\phi(x,y,z) - \phi(x_0,y_0,z_0) = -\int_{(x_0,y_0,z_0)}^{(x,y,z)} \vec{E} \cdot d\vec{s}$; $\vec{E} = -\nabla\phi$; $\nabla^2\phi = -4\pi\rho$; $\mathrm{div}\vec{E} = 4\pi\rho$

$\mathrm{div}\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$; $u = \frac{E^2}{8\pi}$ electric energy density

energy of 3 charges: $U = \frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}$; energy of q in potential ϕ: $U = q\,\phi(x,y,z)$

Electric field right next to a conducting surface: $E=4\pi\sigma$

Capacitors: Q=CV ; Parallel plates: $C = \frac{A}{4\pi s}$ A=area, s=dist. betw. plates; $U = \frac{Q^2}{2C}$ energy

$I = \int \vec{J} \cdot d\vec{a}$, $I = \frac{dq}{dt}$, $\vec{J} = nq\vec{u}$; $div\vec{J} = -\frac{\partial\rho}{\partial t}$; Power: $P = I^2 R$; $P = \varepsilon I$

V=IR , $\vec{J} = \sigma\vec{E}$; $\vec{E} = \rho\vec{J}$; $R = \rho\frac{L}{A}$; $\sigma = \frac{ne^2\tau}{m_e}$; $Q(t) = C\varepsilon(1 - e^{-t/RC})$

Ampere's law: $\oint_C \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} I_{enc} = \frac{4\pi}{c}\int_S \vec{J} \cdot d\vec{a}$; Biot-Savart law: $d\vec{B} = \frac{I d\vec{\ell} \times \hat{r}}{cr^2}$

$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J}$; $\vec{\nabla} \cdot \vec{B} = 0$; $\vec{\nabla} \times \vec{E} = 0$ (electrostatics) ; $\vec{\nabla} \times \vec{A} = \vec{B}$; $\vec{\nabla} \cdot \vec{A} = 0$

Lorentz force : $\vec{F} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$; force on wire : $d\vec{F} = \frac{I}{c} d\vec{\ell} \times \vec{B}$; cyclotron : $\omega = \frac{qB}{mc}$

Field of: long wire: $B = \frac{2I}{cr}\hat{\varphi}$; ring: $\vec{B} = \frac{2\pi b^2 I}{c(b^2 + z^2)^{3/2}}\hat{z}$; solenoid: $\vec{B} = \frac{4\pi In}{c}\hat{z}$

Faraday law: $\varepsilon = \oint_c \vec{E} \cdot d\vec{s} = -\frac{1}{c}\frac{\partial}{\partial t}\Phi_B = -\frac{1}{c}\frac{\partial}{\partial t}\int \vec{B} \cdot d\vec{a}$; $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$

Inductance: $\varepsilon_{21} = -M_{21}\frac{\partial I_1}{\partial t}$; $M_{21} = \frac{\Phi_{21}}{I_1}$; $M_{21} = M_{12} = M$; $\varepsilon = -L\frac{\partial I}{\partial t}$; $L = \frac{\Phi}{I}$

L-R circuit: $I = \frac{\varepsilon_0}{R}(1 - e^{-(R/L)t})$; Energy: $U = \frac{1}{2}LI^2$; density $u = \frac{B^2}{8\pi}$

RLC circuit: $V(t) = e^{-(R/2L)t}(A\cos\omega t + B\sin\omega t)$; $\omega = \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$

Alternating current: $\varepsilon = \varepsilon_0 \cos\omega t$; $I = I_0 \cos(\omega t + \varphi)$; $\tan\varphi = \frac{1/(\omega C) - \omega L}{R}$

$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + (\omega L - 1/(\omega C))^2}}$; Power: $< P >= \frac{1}{2}\varepsilon_0 I_0 \cos\varphi$

Ampere-Maxwell law: $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$; $\oint_C \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} I_{enc} + \frac{1}{c}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{a}$

Displacement current: $\vec{J}_d = \frac{1}{4\pi}\frac{\partial \vec{E}}{\partial t}$; electromagnetic waves: v=c, E_0=B_0, c=3x10^{10}cm/s

Electric dipole: $\vec{p} = \int dv' \rho(\vec{r}')\vec{r}'$; $\varphi(\vec{r}) = \frac{\vec{p} \cdot \vec{r}}{r^3}$; $E_r = \frac{2p}{r^3}\cos\theta$; $E_\theta = \frac{p}{r^3}\sin\theta$

Energy and torque in external E field: $U = -\vec{p} \cdot \vec{E}$; $\vec{\tau} = \vec{p} \times \vec{E}$

Polarization: $E' = -4\pi P$; $\frac{P}{E} = \frac{\varepsilon - 1}{4\pi}$; capacitor w/dielectric: $C = \varepsilon C_0$

Magnetic dipole: $\vec{m} = \frac{I}{c}\vec{a}$; $U = -\vec{m} \cdot \vec{B}$; $\vec{\tau} = \vec{m} \times \vec{B}$

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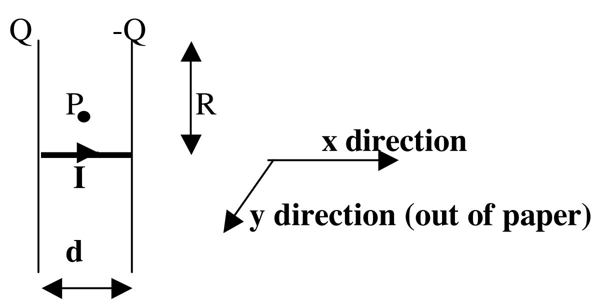
3 problems, 10 points each:

Problem 1 (10 pts)



A capacitor with circular plates of radius R and distance between the plates d is discharged by connecting the centers of the plates with a straight conducting wire as shown in the figure. The point P is at distance R/10 from the axis in the center of the gap between the capacitor plates. Q>0. I>0 denotes the <u>magnitude</u> of the current.

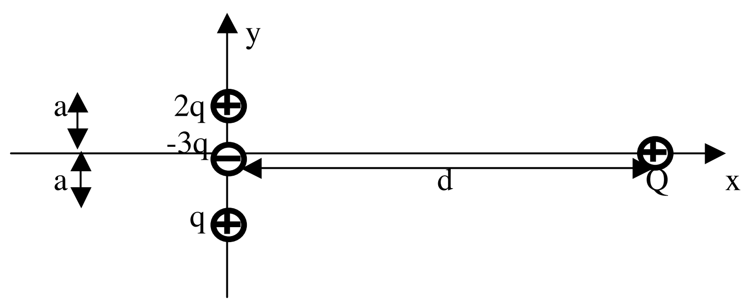
(a) Give an expression for the electric field at point P in terms of the charge on the capacitor plates Q, R, and d.

(b) Give expressions for $\vec{\nabla} \times \vec{B}$ at point P and for the displacement current $\vec{J}_d$ at point P, in terms of the current I in the wire, R, and d. Do they point it the +x or in the -x direction? Explain.

(c) Give an expression for the magnetic field at point P in terms of I, R and d. Assume d>>R/10. <u>Hint:</u> you need to include two separate contributions, watch their sign.

Problem 2 (10 pts)
An electromagnetic wave in free space at time t=0 is described by the electric field
$\vec{E}(x,y,z,t=0) = E_0 e^{-z^2/a^2}\hat{y}$
with a=10^8m.
(a) Find the magnitude of the electric field at x=y=z=0 at time t=1s, in terms of E_0.
(b) What are the possible directions that the associated magnetic field $\vec{B}$ points to?
(c) What are the possible directions of propagation of this wave?
(d) Find an expression for $\frac{\partial \vec{B}}{\partial t}(x,y,z,t=0)$. The answer should be in terms of E_0, a, c, and z.

Problem 3 (10 pts)



In the figure, charge 2q is at y=a, charge q is at y=-a and charge -3q is at y=0. All are at x=0. Charge Q is at y=0, x=d. The distance d>>a.
(a) Find the dipole moment of the 3 charges near the origin, $\vec{p}$ (magnitude and direction).
(b) Using the result of (a), find an expression for the net force exerted on the charge Q by the 3 charges near the origin, $\vec{F}_Q$. Give both magnitude and direction of the force. You may assume the dipole $\vec{p}$ is at the origin.
(c) Using a general principle and the result of (b), give an expression for the force exerted on the dipole $\vec{p}$ by the charge Q, $\vec{F}_p$. Explain how you would verify this result by a calculation if you had enough time.