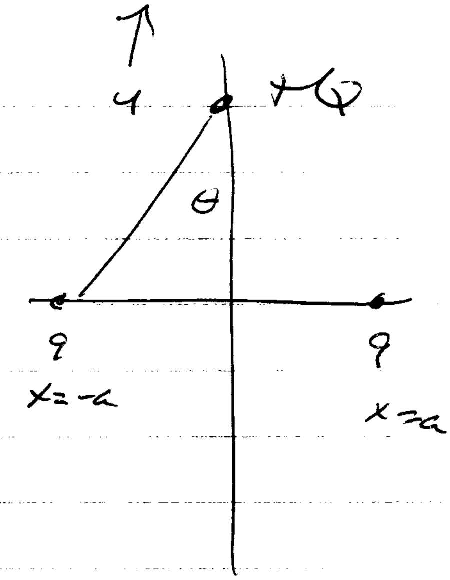
Final



1.

(a) $$\vec{F} = \frac{+2qQ\cos\theta}{4\pi\epsilon_0 (y^2+a^2)}\hat{y}$$

$$= \frac{qQy\,\hat{y}}{2\pi\epsilon_0 (y^2+a^2)^{3/2}}$$

(b) $$W = \frac{q^2}{4\pi\epsilon_0 (2a)} + \frac{2qQ}{4\pi\epsilon_0 (y^2+a^2)^{\frac{1}{2}}}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q^2}{2a} + \frac{2qQ}{(y^2+a^2)^{\frac{1}{2}}}\right]$$

2. $\rho = kr$

(a) $r < R$ $$4\pi r^2 E = \frac{1}{\epsilon_0}\int_0^r 4\pi r^2 kr\,dr = \frac{\pi k r^4}{\epsilon_0}$$

$r \le R$ $$\vec{E} = \frac{kr^2}{4\epsilon_0}\hat{r}$$

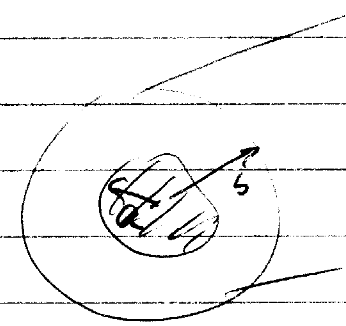
$r \geq R$ $$\vec{E} = \frac{kR^4}{4\epsilon_0 r^2}\hat{r}$$

2.

(b) $$V(0) = -\int E \cdot dr = \underbrace{\frac{kR^4}{4\epsilon_0 R}}_{-\int_\infty^R E\,dr} - \int_R^0 \frac{kr^2\,dr}{4\epsilon_0}$$

$$= \frac{k}{4\epsilon_0}\left[R^3 + \frac{R^3}{3}\right] = \frac{kR^3}{3\epsilon_0}$$

3.



(a)

$s \geq b \quad V = 0, \; E = 0$

$0 \leq s \leq a \qquad 2\pi s L E = \dfrac{\pi s^2 L\lambda}{\pi a^2 \epsilon_0}$

$$\vec{E} = \frac{\lambda s}{2\pi\epsilon_0 a^2}\hat{s}$$

$a \leq s \leq b \qquad 2\pi s L E = \dfrac{\lambda L}{\epsilon_0}$

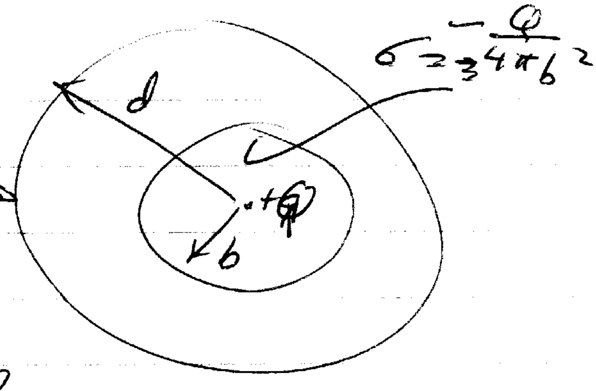
$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$$

(b) $$V = -\int_b^0 E \cdot dl = \frac{-\lambda}{2\pi\epsilon_0}\left[\frac{s^2}{2a^2}\Big|_a^0 + \ln s\Big|_b^a\right]$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[1 + 2\ln\left(\frac{b}{a}\right)\right]$$

4

(a) $\sigma = +\dfrac{Q}{4\pi d^2}$



At $r = b$ charge $-Q$

as uniform surface charge distribution

$$\sigma = \frac{-Q}{4\pi b^2}$$ so that $E = 0$ in

the conductor.

At $r = d$ $\sigma = \dfrac{+Q}{4\pi d^2}$ to

conserve charge on the conductor

(b) $r \geq d$ $V = \dfrac{+Q}{4\pi \epsilon_0 r}$ $\quad E = \dfrac{+Q\,\hat{r}}{4\pi\epsilon_0 r^2}$

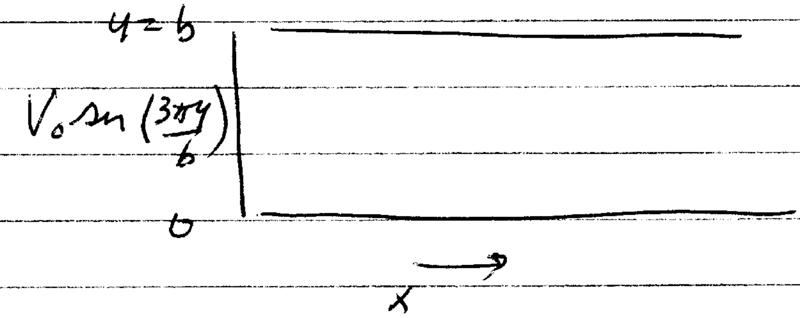
$V = -\int \vec{E}\cdot d\vec{r}$

$b \leq r \leq d$ $\vec{E} = 0$ $V = \dfrac{Q}{4\pi \epsilon_0 d}$

$r \leq b$ $V(r) = \dfrac{Q}{4\pi\epsilon_0}\left[\dfrac{1}{d} + \dfrac{1}{r} - \dfrac{1}{b}\right]$

$$E = \frac{+Q\,\hat{r}}{4\pi\epsilon_0 r^2}$$

5.



(a) Pick harmonic $\quad \underline{V(x,y) = V_0 \sin\left(\frac{3\pi y}{b}\right) e^{-\frac{3\pi x}{b}}}$

and it will satisfy $\quad \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$

(b) $$\underline{\vec{E} = -\nabla V = \frac{3\pi}{b} V_0 \left[-\hat{y} \cos\left(\frac{3\pi y}{b}\right) + \hat{x} \sin\left(\frac{3\pi y}{b}\right) \right] e^{-\frac{3\pi x}{b}}}$$

6. From formula sheet try

$$V = \left(Ar + \frac{B}{r^2}\right) \cos\theta$$

$A = 0$ outside $+$ $B = 0$ inside

So $r \le R$, $\quad \underline{V = V_0 \left(\frac{r}{R}\right) \cos\theta}$

$r \ge R \quad \underline{V = V_0 \left(\frac{R}{r}\right)^2 \cos\theta}$

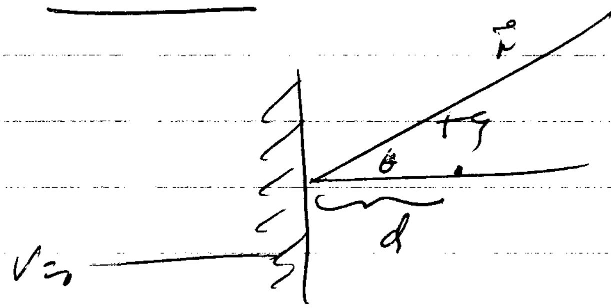
6 Gauss' law

(b) $\Delta E_r = \frac{\sigma}{\epsilon_0}$

$$\underline{\sigma} = -\epsilon_0\left(\nabla V_{out} - \nabla V_{in}\right) = \left(\frac{2V_0R^2}{R^3}\cos\theta + \frac{V_0}{R}\cos\theta\right)\epsilon_0$$

$$= \underline{\frac{3\epsilon_0 V_0}{R}\cos\theta}$$

7.



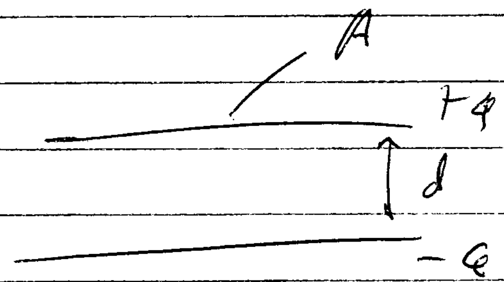
(a) Use images. Put $-q$ at $y=0$ & $x=-d$

(b) Use dipole expansion

$$V(r,\theta) = \frac{2qd\cos\theta}{4\pi\epsilon_0 r^2} = \underline{\frac{qd}{2\pi\epsilon_0 r^2}\cos\theta}$$

8.

(a)



Inside

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \qquad \Delta V = -\int E \cdot dl = \underline{\frac{Qd}{\epsilon_0 A}}$$

$Q/\Delta V$

(b) $$C = \frac{\epsilon_0 A}{d}$$

(c) $$W = \frac{\epsilon_0}{2}\int E^2 d\tau = \frac{\epsilon_0 Q^2}{2\epsilon_0^2 A^2} A d$$

$$= \underline{\frac{Q^2 d}{2\epsilon_0 A}}$$

9. (a) $$D = \sigma = \epsilon E \qquad \Delta V = -\int E \cdot dl = \frac{Qd}{\epsilon A}$$

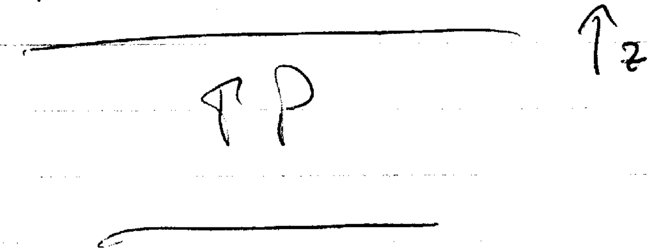
$$= \underline{\frac{Qd}{\epsilon_0 (1+\chi_e) A}}$$

(b) $$W = \frac{\epsilon}{2}\int E^2 d\tau = \frac{\epsilon}{2}\left[\frac{Q^2}{A^2 \epsilon^2}\right] A d$$

$$= \frac{Q^2 d}{2\epsilon A} = \underline{\frac{Q^2 d}{2(1+\chi_e) A}}$$

(c) $$C = \frac{Q}{\Delta V} = \underline{\frac{\epsilon_0 (1+\chi_e) A}{d}}$$

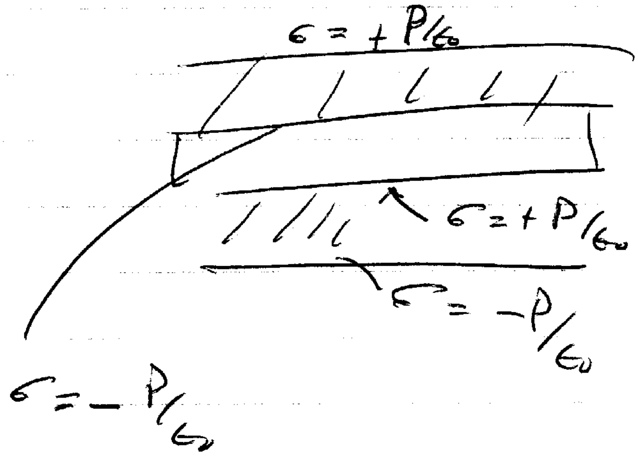
10.



(a) $\sigma_f = 0$ no $\Delta P_\parallel$ so $\vec{D} = 0$ inside

$$\vec{E} = -\frac{\sigma}{\epsilon_0}\hat{z} = -\frac{\vec{P}\cdot\hat{n}}{\epsilon_0} = -\frac{P}{\epsilon_0}\hat{z}$$

(b)



now the E fields from the charge layers cancel.

$$\vec{E}_{cav} - \vec{E}_{mat} = \sigma_B/\epsilon_0 = -P/\epsilon_0$$

$$\vec{E}_{cavity} = 0 \qquad \vec{D}_{cavity} = \epsilon_0 E + P = 0$$