MT-1

1. (a) $E_q = \frac{1}{4\pi\epsilon_0}\frac{q}{\rho^2}$

$2\pi r L E_\lambda = \frac{\lambda L}{\epsilon_0} \qquad E_\lambda = \frac{\lambda}{2\pi\epsilon_0 \rho}$

so $\underbrace{\text{far from both}}_{\text{a distance } x}$ $E_\lambda \gg E_q$

$$\vec{E} \cong \frac{\lambda}{2\pi\epsilon_0 x}\hat{x}$$

(b) $\frac{q}{4\pi a^2\epsilon_0} = \frac{\lambda}{2\pi\epsilon_0 a}$ $\qquad \underline{q = 2\lambda a}$

(c) $2\left(\frac{\cancel{2}\lambda a}{\cancel{4\pi\epsilon_0}(x^2+a^2)}\right)\cancel{2} = \frac{\lambda}{\cancel{2\pi\epsilon_0}\sqrt{x^2+a^2}}$

$2a = \sqrt{x^2+a^2}$

$3a^2 = x^2 \qquad \underline{x = \sqrt{3}\,a}$

$\left\{ \frac{2\lambda a}{4a^2} = \frac{\lambda}{2a} \right\}$ ✓

MT 1

2.

(a) $\int E \cdot da = \frac{Q}{\epsilon_0}$

E only in $\hat{y}$ direction

$a^2 E(2a) - a^2 E(a) = \frac{Q}{\epsilon_0} = a^2 \cancel{k}[(2a)^2 - a^2] = 3ka^4$

$\underline{Q = 3\epsilon_0 k a^4}$

(b) $\epsilon_0 \nabla \cdot E = \rho$

$\cancel{\rho(y)} = \epsilon_0 \frac{\partial E}{\partial y} = \underline{2\epsilon_0 k} y$

Check answer

$\int \rho a^2 dy = \int_a^{2a} a^2 \, 2\cancel{\epsilon_0} k y \, dy = 2\epsilon_0 k a^2 \left[\frac{y^2}{2}\right]_a^{2a}$

$= 2\epsilon_0 k a^2 \frac{3a^2}{2} = \underline{3\epsilon_0 k a^4}$ ✓

3. All charge at same distance:

$$V = \frac{q}{4\pi\epsilon_0 r}$$

(a) $$V = \frac{2\pi b^2 \sigma}{4\pi\epsilon_0 b} = \underline{\frac{\sigma b}{2\epsilon_0}}$$

(b) $$\underline{\Delta V = -\frac{\sigma \, \delta A}{4\pi\epsilon_0 b}}$$ indep of position on the sphere.

(c) Use superposition. ~~For the~~ Outside the sphere, E is as if all charge is at center. Thus V is also

$$V = \frac{\sigma b}{2\epsilon_0} + \frac{\sigma \cancel{4\pi} b^2}{\cancel{4}\pi\epsilon_0 (2b)} = \frac{\sigma}{\epsilon_0}\left[\frac{b}{2} + \frac{b}{2}\right]$$

$$= \underline{\frac{\sigma b}{\epsilon_0}}$$