Physics 100A — Midterm 1 — Fall 2008

Professor Clifford Surko — October 22, 2008

Useful formulas:

For a sphere

$$V = (4\pi/3)R^3$$

$$A = 4\pi R^2$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{\rho(\vec{r}')\,\hat{r}\,d\tau'}{r^2}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{\rho(\vec{r}')\,d\tau'}{r}$$

Fig. 1

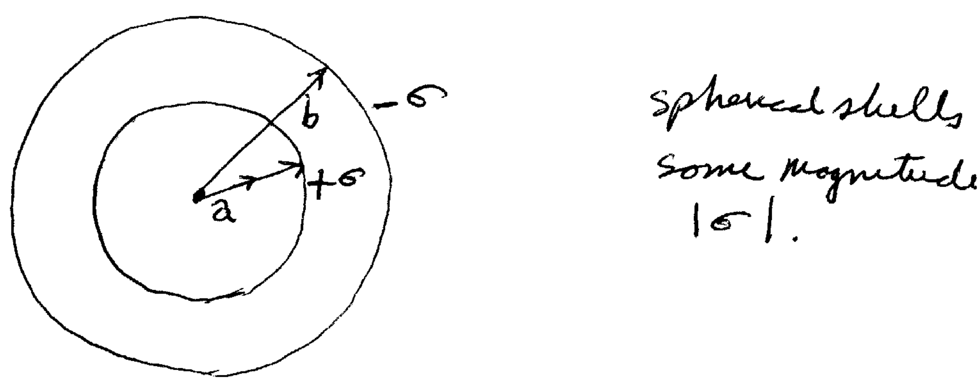
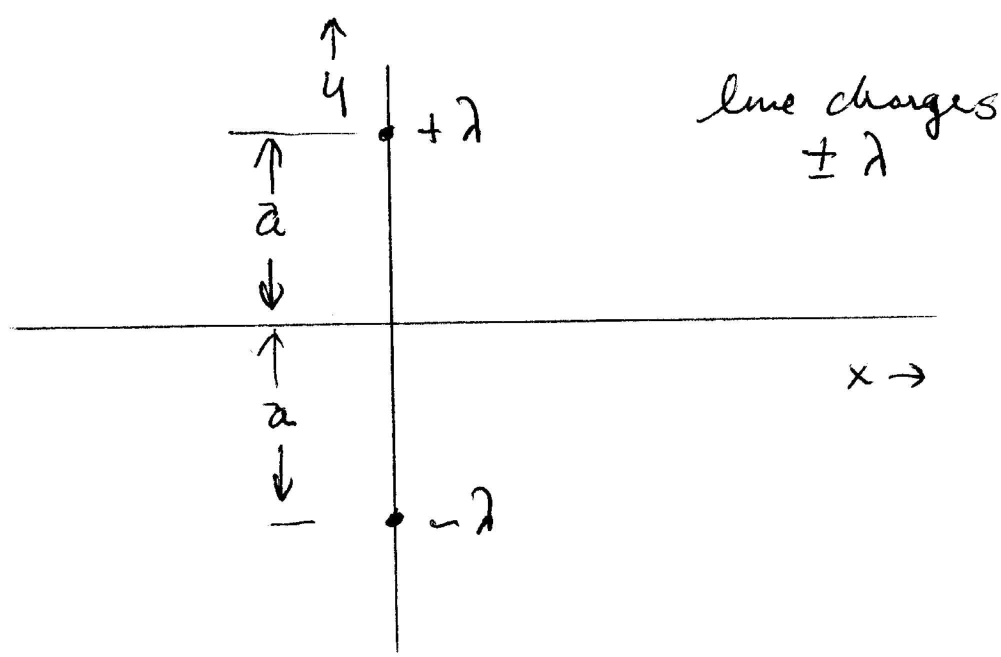


Fig. 2



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Please note: *Be sure to state clearly the reasoning behind your answers. Answers without explanation or supporting work will receive little or no credit.*

1. Consider the functions

$$\vec{F}_1 = (z^2 - ay)\hat{x} - ax\hat{y} + 2xz\hat{z}$$

and $$\vec{F}_2 = (z^2 - ay)\hat{x} - 2ax\hat{y} + 2az\hat{z},$$

where a is a constant and x, y and z are Cartesian coordinates.

(a) Calculate $\nabla \times \vec{F}_1$.

(b) Calculate $\nabla \bullet \vec{F}_1$.

(c) Can $\vec{F}_2$ be written as the gradient of some scalar function V(x, y, z)? If so, find V. If not, explain clearly why it cannot.

2. This problem relates to Fig. 1 on the formula page. Two concentric spherical shells of charge with radii a and b have <u>the same magnitude of charge density per unit area, σ, on each shell</u>, with + σ on the one with radius *a*, and – σ on the one with radius *b*, as shown.

(a) Find the electric field, $\vec{E}(\vec{r})$ for $0 \le r \le \infty$.

(b) Find the electrical potential, V(r) everywhere, assuming V = 0 at r = ∞.

(c) Make a careful sketch of V(r) labeling the magnitude of V at r = a, b, and 0, and labeling zero on each axis.

3. This problem relates to Fig. 2 on the formula page. Two line charges with charge per unit length +/- λ are located at x = 0, and y = + *a* and y = – *a*, respectively, and oriented in the direction perpendicular to the plane of the figure, as shown.

(a) Find the magnitude and direction of the electric field, $\vec{E}(x,y)$, as a function of position along the x axis [i.e., find $\vec{E}(x,0)$].

(b) Sketch the electric field, $\vec{E}(x,y)$, in the (x, y) plane, indicating the directions that the field points along these lines.

(c) Find an expression for the electrical potential V(x, y) everywhere in the (x,y) plane assuming V = 0 at x = y = 0. Use it to find the electrical potential along the x axis [i.e., find V(x, 0)].