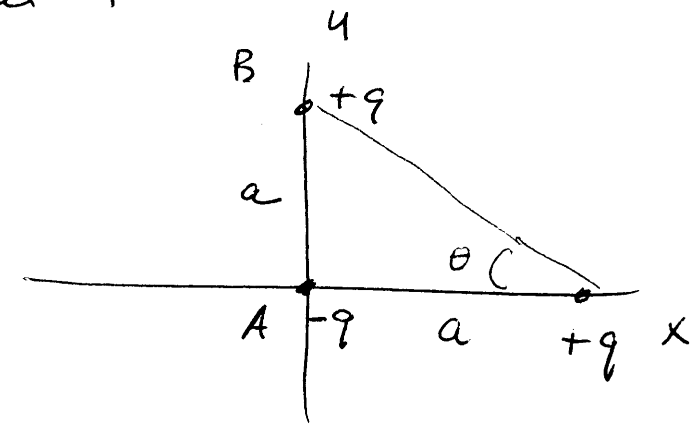
Final - 1

1)



$$\vec{F} = \frac{q^2}{4\pi\epsilon_0}\left[-\frac{\hat{x}}{a^2} + \frac{\hat{x}\cos\theta}{2a^2} - \frac{\hat{y}\sin\theta}{2a^2}\right]$$

$$= \frac{q^2}{4\pi\epsilon_0}\left[-\frac{\hat{x}}{a^2}\left(\frac{2\sqrt{2}-1}{2\sqrt{2}}\right) - \frac{\hat{y}}{2\sqrt{2}a^2}\right]$$

$$= \frac{q^2}{8\sqrt{2}\pi\epsilon_0}\left[-(2\sqrt{2}-1)\hat{x} - \hat{y}\right]$$

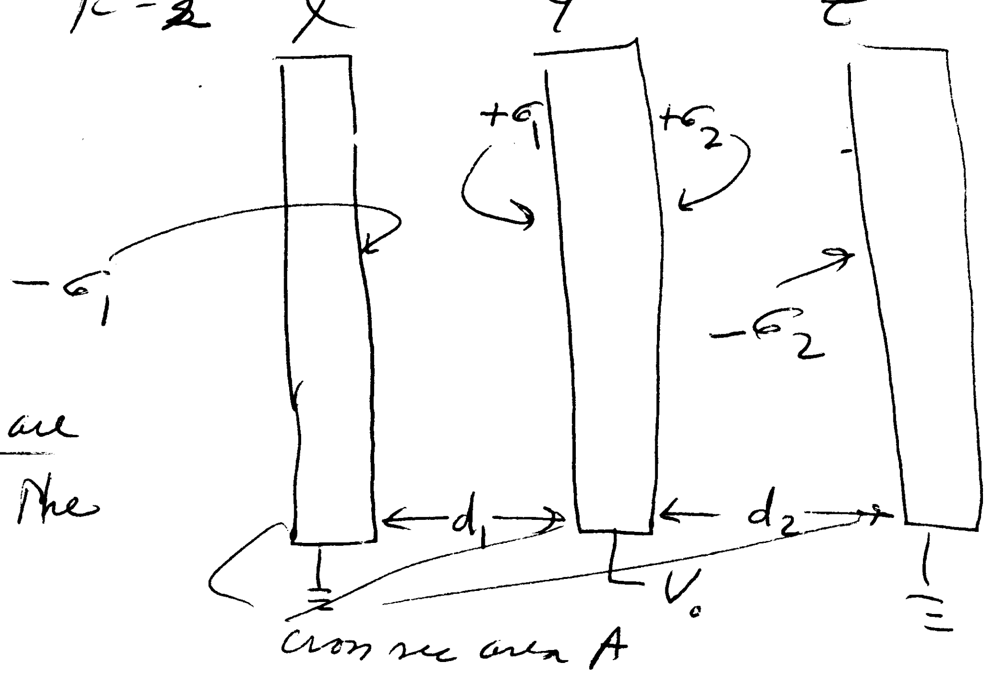
(b) $$qV_{AB} = \frac{-q^2}{4\pi\epsilon_0 a}$$

$$qV_{CA} + qV_{CB} = \frac{-q^2}{4\pi\epsilon_0 a} + \frac{q^2}{4\pi\epsilon_0\sqrt{2}a}$$

$$W = \frac{q^2}{4\pi\epsilon_0 a}\left[-2 + \frac{1}{\sqrt{2}}\right] = \frac{-q^2}{4\pi\epsilon_0 a}\left[\frac{-2\sqrt{2}+1}{\sqrt{2}}\right]$$

F-2

2)



Answer: both σ_1 & σ_2 are constant over the areas A

(a) $V_0 = d_1 E_1 = d_2 E_2$

$$E = \frac{\sigma}{\epsilon_0} \qquad \sigma_1 = \frac{\epsilon_0 V_0}{d_1} \qquad \sigma_2 = \frac{\epsilon_0 V_0}{d_2}$$

No volume charge charge on surfaces as indicated

(b) $W = \int V\,dq$

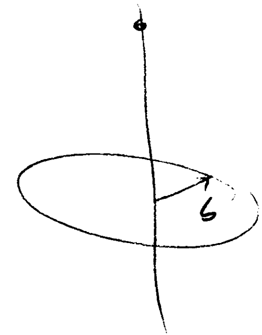
$$= \int V A\,d\sigma = \frac{d_1 A}{\epsilon_0}\int \sigma\,d\sigma \; + \text{ same for \#2}$$

$$= \frac{d_1 A}{2\epsilon_0}\,\frac{\epsilon_0^2 V_0^2}{d_1^2} \; + \text{ same for \#2}$$

$$= \frac{\epsilon_0 A V_0^2}{2}\left[\frac{1}{d_1} + \frac{1}{d_2}\right]$$

(c) $C = \frac{Q}{V} = \frac{(\sigma_1 + \sigma_2)A}{V}$

$= \epsilon_0 A\left(\frac{1}{d_1} + \frac{1}{d_2}\right)$



3.)

(a) $V = \frac{1}{4\pi\epsilon_0}\frac{2\pi b\lambda}{\sqrt{b^2+z^2}} = \frac{b\lambda}{2\epsilon_0\sqrt{b^2+z^2}}$

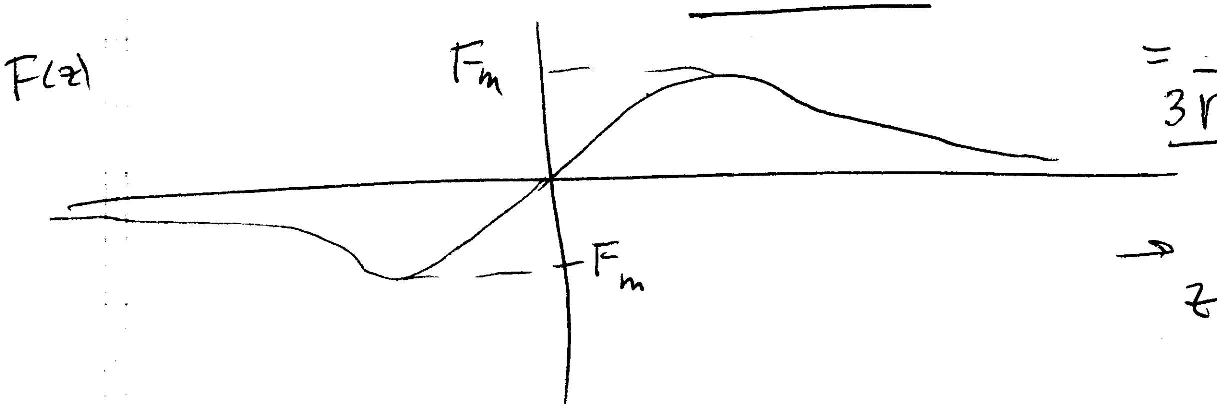
(b) $\vec{F} = q\vec{E} = -q\nabla V = +\underline{\frac{b\lambda q z}{2\epsilon_0(b^2+z^2)^{3/2}}\hat{z}}$

(c) minimum is $\vec{F} = 0$

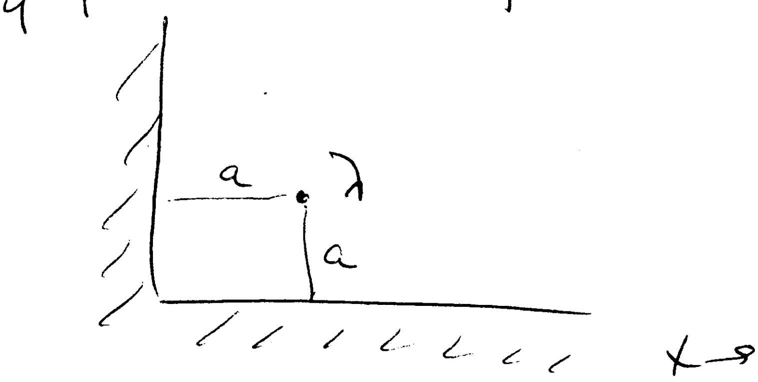
max. $\frac{d}{dz}\left(\frac{z}{(b^2+z^2)^{3/2}}\right) = 0 = \frac{1}{[b^2+z^2]^{3/2}} - \frac{3z^2}{[b^2+z^2]^{5/2}}$

$b^2 + z^2 - 3z^2 = 0 \qquad \underline{z = b/\sqrt{2}} \qquad F_{max} = \frac{\lambda q b^2}{2\sqrt{2}\epsilon_0\left(\frac{3}{2}\right)^{3/2}b^3}$

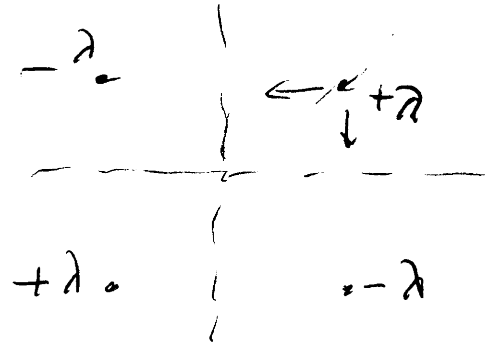
$= \underline{\frac{\lambda q}{3\sqrt{3}\,\epsilon_0 b}}$



4.)



images



For one line charge: $2\pi s l E = \frac{\lambda L}{\epsilon_0}$

(a) $F/\ell = q/\ell \, E$ $\quad E = \frac{\lambda}{2\pi s \epsilon_0}$

$$\underline{F/L} = \frac{\lambda^2}{2\pi\epsilon_0}\left[-\frac{\hat{x}}{2a} - \frac{\hat{y}}{2a} + \frac{\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}}{2\sqrt{2}a}\right]$$

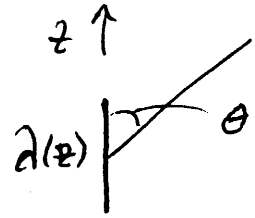
$$= \underline{\frac{-\lambda^2}{8\pi\epsilon_0 a}\left[\hat{x} + \hat{y}\right]} \qquad \left\{ \begin{array}{l} \hat{x} + \hat{y} = \sqrt{2}\,\hat{r} \\ a = r/\sqrt{2} \end{array} \right\}$$

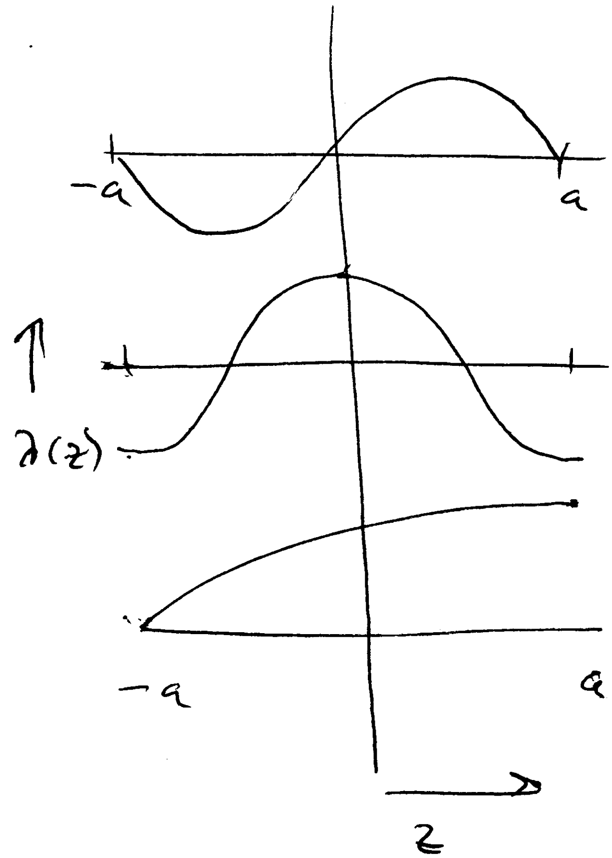
b)

$$W/L = -\int_{(5a,5a)}^{(a,a)} F/\ell \cdot d\vec{r} = +\frac{\lambda^2}{8\pi\epsilon_0} \int_{\sqrt{2}(5a)}^{\sqrt{2}a} \frac{\sqrt{2}\,\hat{r}\cdot\hat{r}\,dr}{r/\sqrt{2}}$$

$$= \frac{-\lambda^2}{4\pi\epsilon_0} \ln(5)$$

5)





(a) <u>Symmetry in z odd</u>

<u>no $n=0$</u> $\int \lambda\, dz = 0$

<u>$n = 1 \neq 0$</u> <u>lowest order moment</u>

<u>$n=2 = 0$</u> odd symmetry

(b) even symmetry but $\int \lambda\, dz = 0$

no $n=0$ $\int \lambda\, dz = 0$

no $n=1$ even symmetry

$n=2 \neq 0$ <u>lowest order moment</u>

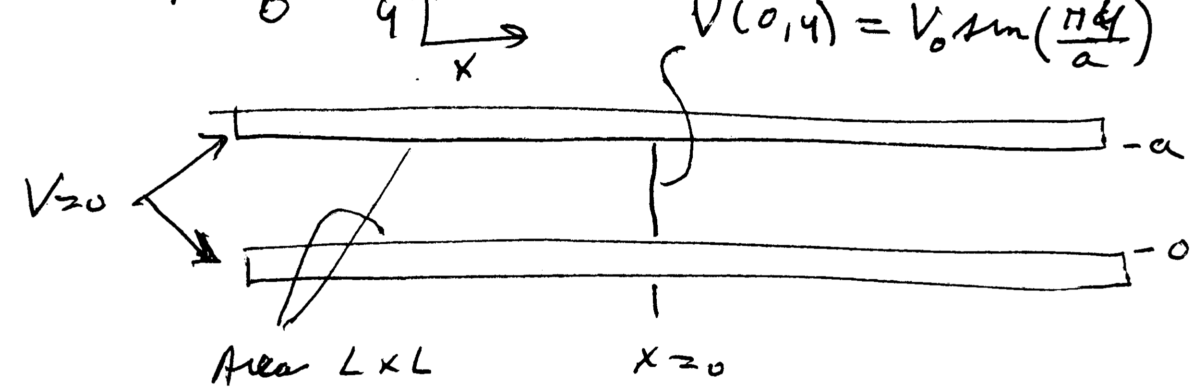
(c) $\int \lambda\, dz \neq 0$ <u>$n=0$ is $\neq 0$</u> <u>lowest order moment</u>

$n=1 \neq 0$

$n=2 \neq 0$

F-6

6.



(a) $\nabla^2 V = 0$

One dimension harmonic function
Exponential in the other.

Choose exponential in x, harmonic in y

$\sin\left(\frac{\pi y}{a}\right)$ works so

$$V = V_0\, e^{-\left(\frac{\pi x}{a}\right)} \sin\left(\frac{\pi y}{a}\right) \qquad x \geq 0$$

(b) Conductors so

$$\vec{E}\cdot\hat{n} = \frac{\sigma}{\epsilon_0}$$

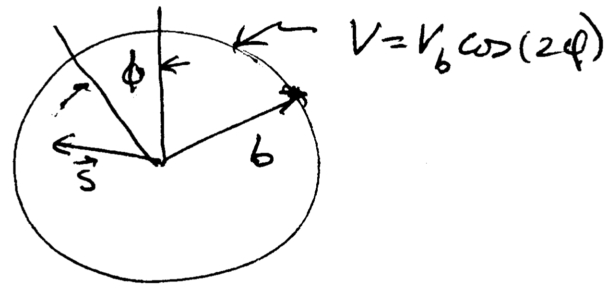
$$\sigma = -\left.\frac{\partial V}{\partial y}\hat{n}\right|_{surf} = -\left.\frac{\pi V_0}{a} e^{-\left(\frac{\pi x}{a}\right)} \cos\left(\frac{\pi y}{a}\right)\right|_{surf} \hat{n}\cdot\hat{z}$$

$y = a$ $\quad \sigma = -\frac{\pi V_0}{a} e^{-\left(\frac{\pi x}{a}\right)}$

$y = 0$ $\quad \sigma = -\frac{\pi V_0}{a} e^{-\left(\frac{\pi x}{a}\right)}$

7.

(a)



Only need one term $n = 2$

$$\underline{V = V_b \left(\frac{s}{b}\right)^2 \cos(2\phi)}$$

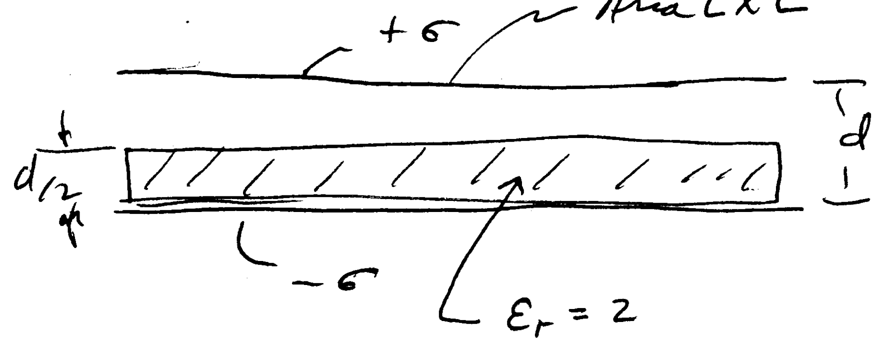
(b) $s \leq b$

$$\vec{E} = -\vec{\nabla} V = -2 V_b \left(\frac{s}{b^2}\right) \cos(2\phi)\, \hat{s} + 2 V_b \left(\frac{s}{b^2}\right) \sin(2\phi)\, \hat{\phi}$$

(c) $s \geq b$

$$V(s, \phi) = V_b \left(\frac{b}{s}\right)^2 \cos(2\phi)$$

8.)



(a)

$$D = \sigma_f = \sigma$$

in dielectric

$$\bar{E} = \frac{D}{\epsilon_0 \epsilon_r} = \frac{\sigma}{\epsilon_0 \epsilon_r} = \underline{\frac{\sigma}{2\epsilon_0}}$$

in vacuum

$$\bar{E} = \underline{\frac{\sigma}{\epsilon_0}}$$

$$V = -\int \bar{E} \cdot dl$$

(b) $$V = \frac{(d)\sigma}{2\epsilon_0}\left(1 + \frac{1}{2}\right) = \underline{\frac{3\sigma d}{2\epsilon_0}}$$

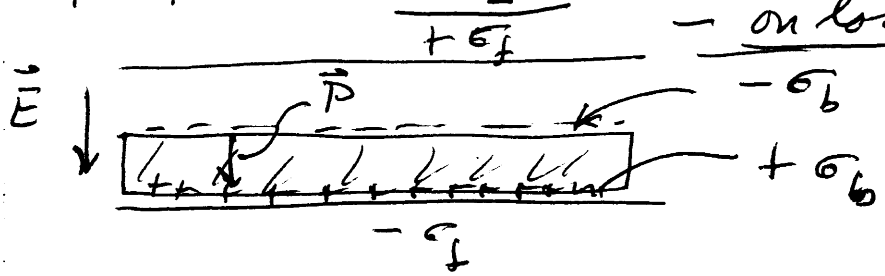
(c) $\sigma_b = P \cdot \hat{n}$

$$P = D - \epsilon_0 \bar{E}_{in} = \sigma - \frac{\sigma}{2} = \underline{\frac{\sigma}{2}}$$

$$|\sigma_b| = |P \cdot \hat{n}| = \underline{\frac{\sigma}{2}}$$

+ on upper surface of dielectric

− on lower



9.) $\vec{P}(r) = kr\hat{r}$

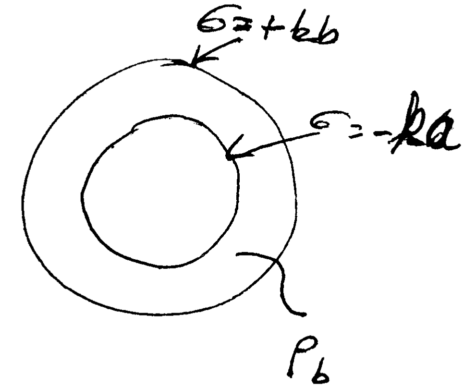
(a) surface charge $\sigma_b = \vec{P}\cdot\hat{n}$

at b, $\underline{\sigma_b = +kb}$

at a, $\underline{\sigma_b = -ka}$

$$\underline{\rho_b} = -\nabla\cdot P = \frac{-1}{r^2}\frac{\partial}{\partial r}\left(r^2 P\right)$$

$$= \underline{-3k} \qquad a \le r \le b$$



(b) No free charge, so outside,

$E = 0$ and also for $r \le a$

$a \le r \le b$

$$Q_{encl} = 4\pi k a^3 + \int_a^r 4\pi r^2 (-3k)\, dr$$

$$= 4\pi k a^3 + 4\pi k\left[a^3 - r^3\right] = -4\pi k r^3$$

$$\underline{\vec{E}} = \frac{Q_{enc}}{4\pi r^2 \epsilon_0} = \underline{-\frac{kr}{\epsilon_0}\hat{r}}$$