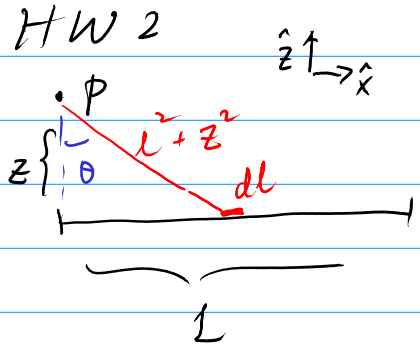
HW 2

2.3*



$$dE_x = -\frac{\lambda\, dl}{4\pi\varepsilon_0(l^2+z^2)} \cdot \frac{l}{\sqrt{l^2+z^2}}$$

$$= -\frac{\lambda}{8\pi\varepsilon_0} \frac{d(l^2)}{(l^2+z^2)^{\frac{3}{2}}}$$

$$\Rightarrow E_x = \int_{l=0}^{L} dE_x = \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{\sqrt{l^2+z^2}}\Bigg|_{l=0}^{L}$$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{L^2+z^2}}\right)$$

$$dE_z = \frac{\lambda\, dl}{4\pi\varepsilon_0(l^2+z^2)} \cdot \frac{z}{\sqrt{l^2+z^2}}$$

$$l = z\tan\theta \Rightarrow dl = z\sec^2\theta\, d\theta$$

$$\sqrt{l^2+z^2} = \frac{z}{\cos\theta} = z\sec\theta$$

$$\Rightarrow dE_z = \frac{\lambda}{4\pi\varepsilon_0} \frac{z\sec^2\theta\, d\theta \cdot z}{z^3\sec^3\theta} = \frac{\lambda}{4\pi\varepsilon_0 z}\cos\theta\, d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 z}\, d\sin\theta$$

$$\Rightarrow E_z = \frac{\lambda}{4\pi\varepsilon_0 z}\int_{\theta=0}^{\tan^{-1}(\frac{L}{z})} d\sin\theta = \frac{\lambda}{4\pi\varepsilon_0 z} \cdot \frac{L}{\sqrt{L^2+z^2}}$$

2.6 E field by a ring of $r \to r+dr$ at z:

$$dE = \frac{2\pi r\,dr \cdot \sigma}{4\pi\varepsilon_0 \sqrt{r^2+z^2}} \frac{z}{\sqrt{r^2+z^2}}$$

$$= \frac{\sigma z}{4\varepsilon_0} \frac{d(r^2)}{(r^2+z^2)^{3/2}}$$

$$\Rightarrow E = \int_{r=0}^{R} dE = \frac{\sigma z}{2\varepsilon_0} \frac{1}{\sqrt{r^2+z^2}} \Bigg|_{r=R}^{0}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+\left(\frac{R}{z}\right)^2}}\right)$$

$R \to \infty \Rightarrow E \to \frac{\sigma}{2\varepsilon_0}$, infinite plane

$z \gg R \Rightarrow \frac{1}{\sqrt{1+\left(\frac{R}{z}\right)^2}} \simeq 1 - \frac{1}{2}\left(\frac{R}{z}\right)^2$

$\Rightarrow E \to \frac{\sigma R^2}{4\varepsilon_0 z^2}$, point charge $\sigma \cdot \pi R^2$

2.7* $z < R$, $\Rightarrow E = 0$ (screening)

$z > R$, $\Rightarrow$ same as point charge $4\pi R^2 \sigma = q$

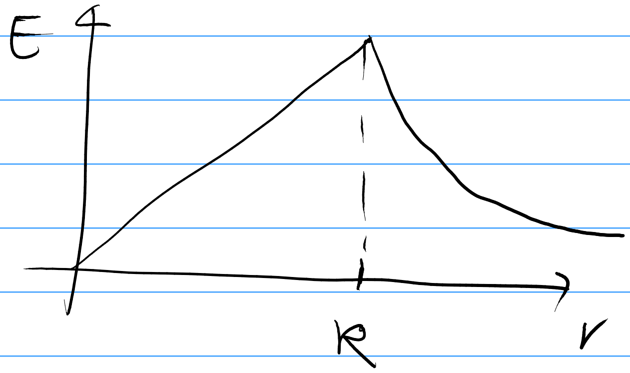
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{z^2} \hat{z}$$

2.8 For $r<R$, $E = \frac{1}{4\pi\varepsilon_0} \cdot \underbrace{\frac{4}{3}\pi r^3 \cdot \rho}_{\text{enclosed charge}} \cdot \frac{1}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{r^3}{R^3} q \cdot \frac{1}{r^2}, \quad q = \frac{4}{3}\pi R^3 \rho$$

$$= \frac{q}{4\pi\varepsilon_0 R^3} \cdot r$$

$r>R$, $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$



2.9 (a) $\nabla \cdot \vec{E} = k\left[\nabla(r^3) \cdot \hat{r} + r^3 \nabla \cdot \hat{r}\right]$

$$= k \cdot \left(3r^2 + r^3 \cdot \frac{2}{r}\right)$$

$$= 5kr^2$$

$\nabla \cdot \vec{E} = \rho/\varepsilon_0 \Rightarrow \rho = \varepsilon_0 \nabla \cdot \vec{E} = 5k\varepsilon_0 r^2$

(b) • $Q = \int^R 4\pi r^2 dr\, \rho(r) = 20\pi k \varepsilon_0 \int_0^R r^4 dr$

$$= 4\pi k \varepsilon_0 R^5$$

• $Q = \varepsilon_0 \oint \vec{E} \cdot dA \Big|_R = \varepsilon_0 \cdot 4\pi R^2 \cdot E_R = 4\pi k \varepsilon_0 R^5$

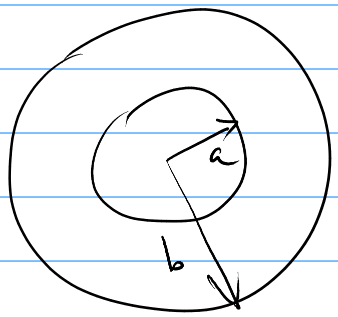
2.14

$$E(r) = \frac{1}{4\pi\varepsilon_0}\,\frac{\int_0^r 4\pi s^2 \rho(s)\,ds}{r^2}$$

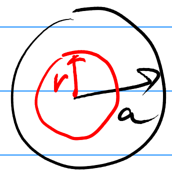
$$= \frac{1}{4\pi\varepsilon_0}\,\frac{4\pi k\int_0^r s^3\,ds}{r^2}$$

$$= \frac{kr^2}{4\varepsilon_0}$$

2.16



(i).

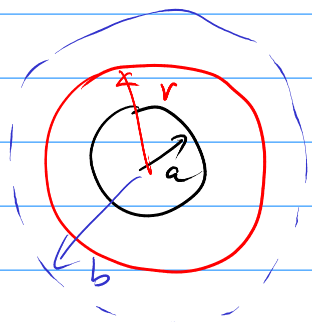


$$\oint E\cdot dA = E\cdot 2\pi r\cdot \delta l$$

$$= \int \frac{\rho}{\varepsilon_0}\,dV = \frac{\rho}{\varepsilon_0}\cdot \pi r^2\cdot \delta l$$

$$\Rightarrow E = \frac{\rho r}{2\varepsilon_0}$$

(ii).



$$\oint E\cdot dA = E\cdot 2\pi r\,\delta l$$

$$= \frac{\rho}{\varepsilon_0}\cdot \pi a^2 \delta l$$

$$\Rightarrow E = \frac{\rho a^2}{2\varepsilon_0 r}$$

(iii) $E = 0$ b/c of overall charge neutrality.