7.4 Sirius is a visual binary with a period of 49.94 yr. Its measured trigonometric parallax is $0.37921'' \pm 0.00158''$ and, assuming that the plane of the orbit is in the plane of the sky, the true angular extent of the semimajor axis of the reduced mass is $7.61''$. The ratio of the distances of Sirius A and Sirius B from the center of mass is $a_A/a_B = 0.466$.

(a) Find the mass of each member of the system.

(b) The absolute bolometric magnitude of Sirius A is 1.36, and Sirius B has an absolute bolometric magnitude of 8.79. Determine their luminosities. Express your answers in terms of the luminosity of the Sun.

(c) The effective temperature of Sirius B is approximately $24{,}790 \text{ K} \pm 100 \text{ K}$. Estimate its radius, and compare your answer to the radii of the Sun and Earth.

7.5 ζ Phe is a 1.67-day spectroscopic binary with nearly circular orbits. The maximum measured Doppler shifts of the brighter and fainter components of the system are 121.4 km s^{-1} and 247 km s^{-1}, respectively.

(a) Determine the quantity $m \sin^3 i$ for each star.

(b) Using a statistically chosen value for $\sin^3 i$ that takes into consideration the Doppler-shift selection effect, estimate the individual masses of the components of ζ Phe.

7.6 From the light and velocity curves of an eclipsing, spectroscopic binary star system, it is determined that the orbital period is 6.31 yr, and the maximum radial velocities of Stars A and B are 5.4 km s^{-1} and 22.4 km s^{-1}, respectively. Furthermore, the time period between first contact and minimum light $(t_b - t_a)$ is 0.58 d, the length of the primary minimum $(t_c - t_b)$ is 0.64 d, and the apparent bolometric magnitudes of maximum, primary minimum, and secondary minimum are 5.40 magnitudes, 9.20 magnitudes, and 5.44 magnitudes, respectively. From this information, and assuming circular orbits, find the

(a) Ratio of stellar masses.

(b) Sum of the masses (assume $i \simeq 90°$).

(c) Individual masses.

(d) Individual radii (assume that the orbits are circular).

(e) Ratio of the effective temperatures of the two stars.

7.7 The V-band light curve of YY Sgr is shown in Fig. 7.2. Neglecting bolometric corrections, estimate the ratio of the temperatures of the two stars in the system.

7.8 Refer to the synthetic light curve and model of RR Centauri shown in Fig. 7.11.

(a) Indicate the approximate points on the light curve (as a function of phase) that correspond to the orientations depicted.

(b) Explain qualitatively the shape of the light curve.

7.9 Data from binary star systems were used to illustrate the mass–luminosity relation in Fig. 7.7. A strong correlation also exists between mass and the effective temperatures of stars. Use the data provided in Popper, *Annu. Rev. Astron. Astrophys.*, *18*, 115, 1980 to create a graph of $\log_{10} T_e$ as a function of $\log_{10}(M/M_\odot)$. Use the data from Popper's Table 2, Table 4, Table 7 (excluding the α Aur system), and Table 8 (include only those stars with spectral types in the Sp column that end with the Roman numeral V). The stars that are excluded in Tables 7 and 8 are evolved stars with structures significantly different from the main sequence stars.[9] The article by Popper may be available in your library or it can be downloaded from the NASA Astrophysics Data System (NASA ADS) at http://adswww.harvard.edu

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PROBLEMS

8.1 Show that at room temperature, the thermal energy $kT \approx 1/40$ eV. At what temperature is kT equal to 1 eV? to 13.6 eV?

8.2 Verify that Boltzmann's constant can be expressed in terms of electron volts rather than joules as $k = 8.6173423 \times 10^{-5}$ eV K^{-1} (see Appendix A).

8.3 Use Fig. 8.6, the graph of the Maxwell–Boltzmann distribution for hydrogen gas at 10,000 K, to estimate the fraction of hydrogen atoms with a speed within 1 km s^{-1} of the most probable speed, v_{mp}.

8.4 Show that the most probable speed of the Maxwell–Boltzmann distribution of molecular speeds (Eq. 8.1) is given by Eq. (8.2).

8.5 For a gas of neutral hydrogen atoms, at what temperature is the number of atoms in the first excited state only 1% of the number of atoms in the ground state? At what temperature is the number of atoms in the first excited state 10% of the number of atoms in the ground state?

8.6 Consider a gas of neutral hydrogen atoms, as in Example 8.1.3.

(a) At what temperature will equal numbers of atoms have electrons in the ground state and in the second excited state ($n = 3$)?

(b) At a temperature of 85,400 K, when equal numbers (N) of atoms are in the ground state and in the first excited state, how many atoms are in the second excited state ($n = 3$)? Express your answer in terms of N.

(c) As the temperature $T \to \infty$, how will the electrons in the hydrogen atoms be distributed, according to the Boltzmann equation? That is, what will be the relative numbers of electrons in the $n = 1, 2, 3, \ldots$ orbitals? Will this in fact be the distribution that actually occurs? Why or why not?

8.7 In Example 8.1.4, the statement was made that "nearly all of the H I atoms are in the ground state, so Eq. (8.7) for the partition function simplifies to $Z_I \simeq g_1 = 2(1)^2 = 2$." Verify that this statement is correct for a temperature of 10,000 K by evaluating the first three terms in Eq. (8.7) for the partition function.

8.8 Equation (8.7) for the partition function actually diverges as $n \to \infty$. Why can we ignore these large-n terms?

8.9 Consider a box of electrically neutral hydrogen gas that is maintained at a constant volume V. In this simple situation, the number of free electrons must equal the number of H II ions: $n_e V = N_{II}$. Also, the total number of hydrogen atoms (both neutral and ionized), N_t, is related to the density of the gas by $N_t = \rho V/(m_p + m_e) \simeq \rho V/m_p$, where m_p is the mass of the proton. (The tiny mass of the electron may be safely ignored in this expression for N_t.) Let the density of the gas be 10^{-6} kg m^{-3}, typical of the photosphere of an A0 star.

(a) Make these substitutions into Eq. (8.8) to derive a quadratic equation for the fraction of ionized atoms:

$$\left(\frac{N_{II}}{N_t}\right)^2 + \left(\frac{N_{II}}{N_t}\right)\left(\frac{m_p}{\rho}\right)\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT} - \left(\frac{m_p}{\rho}\right)\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT} = 0.$$

(b) Solve the quadratic equation in part (a) for the fraction of ionized hydrogen, N_{II}/N_t, for a range of temperatures between 5000 K and 25,000 K. Make a graph of your results, and compare it with Fig. 8.8.

8.10 In this problem, you will follow a procedure similar to that of Example 8.1.4 for the case of a stellar atmosphere composed of pure helium to find the temperature at the middle of the He I partial ionization zone, where half of the He I atoms have been ionized. (Such an atmosphere would be found on a white dwarf of spectral type DB; see Section 16.1.) The ionization energies of neutral helium and singly ionized helium are $\chi_I = 24.6$ eV and $\chi_{II} = 54.4$ eV, respectively. The partition functions are $Z_I = 1$, $Z_{II} = 2$, and $Z_{III} = 1$ (as expected for any completely ionized atom). Use $P_e = 20$ N m^{-2} for the electron pressure.

(a) Use Eq. (8.9) to find N_{II}/N_I and N_{III}/N_{II} for temperatures of 5000 K, 15,000 K, and 25,000 K. How do they compare?

(b) Show that $N_{II}/N_{\text{total}} = N_{II}/(N_I + N_{II} + N_{III})$ can be expressed in terms of the ratios N_{II}/N_I and N_{III}/N_{II}.

(c) Make a graph of N_{II}/N_{total} similar to Fig. 8.8 for a range of temperatures from 5000 K to 25,000 K. What is the temperature at the middle of the He I partial ionization zone? Because the temperatures of the middle of the hydrogen and He I partial ionization zones are so similar, they are sometimes considered to be a single partial ionization zone with a characteristic temperature of $1\text{–}1.5 \times 10^4$ K.

8.11 Follow the procedure of Problem 8.10 to find the temperature at the middle of the He II partial ionization zone, where half of the He II atoms have been ionized. This ionization zone is found at a greater depth in the star, and so the electron pressure is larger—use a value of $P_e = 1000$ N m^{-2}. Let your temperatures range from 10,000 K to 60,000 K. This particular ionization zone plays a crucial role in pulsating stars, as will be discussed in Section 14.2.

8.12 Use the Saha equation to determine the fraction of hydrogen atoms that are ionized, N_{II}/N_{total}, at the center of the Sun. Here the temperature is 15.7 million K and the number density of electrons is about $n_e = 6.1 \times 10^{31}$ m^{-3}. (Use $Z_I = 2$.) Does your result agree with the fact that practically *all* of the Sun's hydrogen is ionized at the Sun's center? What is the reason for any discrepancy?

8.13 Use the information in Example 8.1.5 to calculate the ratio of doubly to singly ionized calcium atoms (Ca III/Ca II) in the Sun's photosphere. The ionization energy of Ca II is $\chi_{II} = 11.9$ eV. Use $Z_{III} = 1$ for the partition function of Ca III. Is your result consistent with the statement in Example 8.1.5 that in the solar photosphere, "nearly all of the calcium atoms are available for forming the H and K lines of calcium"?

8.14 Consider a giant star and a main-sequence star of the *same spectral type*. Appendix G shows that the giant star, which has a lower atmospheric density, has a slightly lower temperature than the main-sequence star. Use the Saha equation to explain why this is so. Note that this means that there is not a perfect correspondence between temperature and spectral type!

8.15 Figure 8.14 shows that a white dwarf star typically has a radius that is only 1% of the Sun's. Determine the average density of a 1-$M_\odot$ white dwarf.

8.16 The blue-white star Fomalhaut ("the fish's mouth" in Arabic) is in the southern constellation of Pisces Austrinus. Fomalhaut has an apparent visual magnitude of $V = 1.19$. Use the H–R diagram in Fig. 8.16 to determine the distance to this star.

CHAPTER 9

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