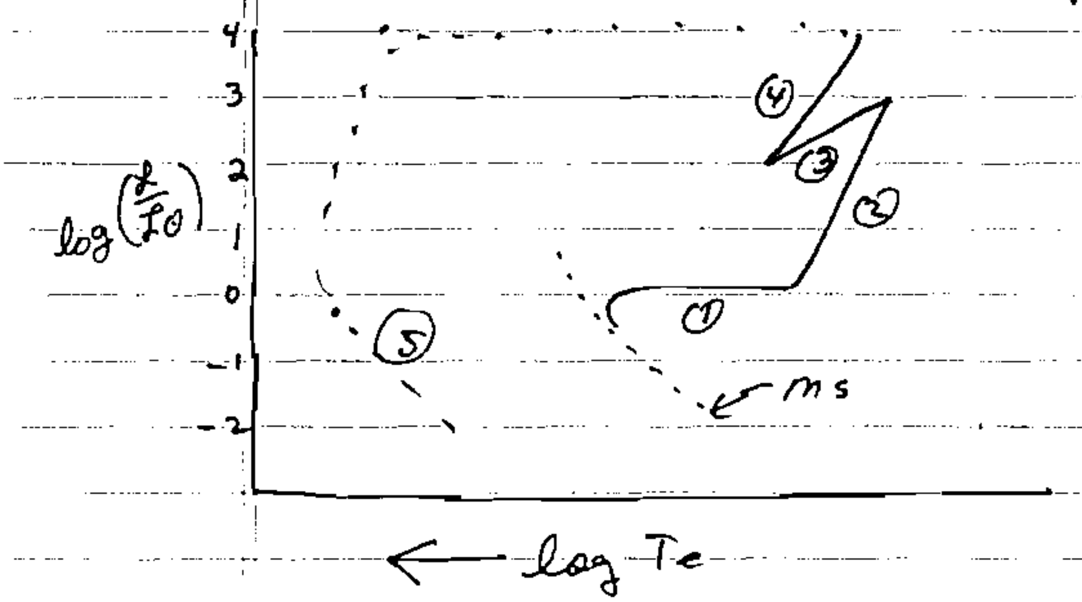
Recap.. Summary of evolution of ~ 1 $M_\odot$ star on the HR diagram



Major evolutionary states

① Sub-Giant branch: $_2He^4$ core mass exceeds ~8%. Core contracts surrounded by H burning shell. Increase ΔL_{nuc} released does not result in expansion of core. Rather ΔL pushes out envelope at const L_{rad}.

② Red-Giant Branch: Stellar envelope becomes convective, so excess ΔL reaches surface by convective motion at adds to [illegible] original L_r. Star ascends at Hayashi T.

③ Horizontal Branch: $_2He^4$ core reaches $T \sim 10^8 K$ and triple α process results in $_6C^{12}$ production in core. Core now expands, T goes down & ΔL reduced star contracts.

④ Asymptotic Branch. Double shell source inert $_6C^{12}$ core contracts, ΔL convectively transported to the surface and star ascends RG branch for the second time.

⑤ <u>Endpoints of stellar Evolution</u>: For modest mass stars like the sun, $_6C^{12}$ burning will not occur, since T in core is not sufficiently high to overcome high Coulomb barrier.

what happens when we turn off nuclear energy source?.

- $\frac{dE}{dt} = \frac{1}{2}\frac{d\Omega_G}{dt} = -L_{rad}$

  Star shrinks

- But $\frac{dE_{KE}}{dt} = -\frac{1}{2}\frac{d\Omega_G}{dt}$, so star heats up.

But ultimate fate depends on Mass

Ⓐ <u>stars with $M < 9M_\odot$</u> : Such stars become white dwarfs through track like that in the Figure.

Properties of WD's : (1) $\langle M_{WD}\rangle \simeq 0.6 M_\odot$

(2) $M_{WD} \leq 1.4 M_\odot$

(3) Supported by e^- degeneracy pressure

(4) $R_{WD} \simeq 10^{-2} R_\odot$

Densities : Since $\rho \sim M/R_3$

$$\rho_{WD} \simeq \rho_\odot \left(\frac{R_\odot}{R_{WD}}\right)^3 = \rho_\odot \left(\frac{10^{11}}{10^{9}}\right)^3 \simeq 10^6 \rho_\odot$$

$$\rho_{WD} > 10^6 \text{ g cm}^{-3}$$

Dense Objects: what happens at higher ρ?

(1) Pauli Exclusion principle: no $2e^-$ in same state

(2) Uncertainty Principle

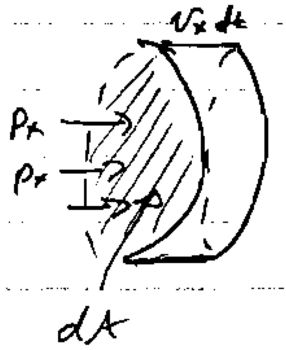
$$\Delta x \Delta p_x \geq \hbar$$

As density increases, mean separation between e^- decreases. As a result Δp_x must increase. But Δp_x results in pressure even if thermal $T \rightarrow 0$!!

So 2 nearby e^- must have momenta that differ by $\Delta p_x \geq \frac{\hbar}{\Delta x}$

- As Δx decreases Δp_x increases
- Speed for non-relativistic e^-: $v_x = \frac{p_x}{m_e}$ also ↑

Pressure:



number of particles hitting wall in time interval dt:

$$dN = n\, dV = n\, dA\, v_x dt$$

Total momentum imparted

$$dP_x = p_x dN = p_x n\, dA\, v_x dt$$

Force $\quad dF_x = \frac{dP_x}{dt} = p_x n\, dA\, v_x$

Pressure: $\boxed{P = \frac{d^2 P_x}{dt\, dA} = p_x n v_x}$ { momentum flux }

$$\frac{1}{n} = \frac{\text{volume occupied}}{\text{particle}}$$

$\simeq \frac{4\pi}{3}\Delta x^3$ or neglecting $\frac{4\pi}{3} \Rightarrow \Delta x \sim \frac{1}{n^{1/3}}$

Therefore: $\Delta p_x \sim \hbar n_e^{1/3}$; $v_x = \frac{\Delta p_x}{m_e} = \frac{\hbar n_e^{1/3}}{m_e}$ { pure QM concepts }

$$\therefore \boxed{P_{degen} \simeq \underbrace{(\hbar n_e^{1/3})}_{p_x}(n_e)\underbrace{\left(\frac{\hbar n_e^{1/3}}{m_e}\right)}_{v_x} \sim \frac{\hbar^2 n_e^{5/3}}{m_e}}$$

Accurate treatment: $P_{degen} \doteq \frac{3^{5/3}}{(8\pi)^{2/3} \times 15} \left[\frac{\hbar^2 n_e^{5/3}}{m_e} \right]$

$$\boxed{P_{degen} = (0.0485) \frac{\hbar^2 n_e^{5/3}}{m_e}}$$

Dependence on mass density ρ:

[redacted]

Suppose WD consists of ions with nuclear chg Z $\left(\begin{array}{ll} {}_2He^4 & Z = 2, A = 4 \\ {}_6C^{12} & Z = 6, A = 12 \text{ etc.} \end{array} \right)$

$\therefore n_e = Z n_{ion}$ (each ion supplies Z electrons)

Mass density: $\rho = (\overset{\text{atom wt.}}{A} m_p n_{ion} + m_e n_e) = (A m_p + Z m_e) n_{ion}$

or $\rho \simeq A m_p n_{ion}$

$\therefore n_e \doteq Z \left(\frac{\rho}{A m_p} \right) = \frac{\rho}{\left(\frac{A}{Z} m_p \right)} = \frac{\rho}{\mu_e m_p} =$ [redacted]

where $\mu_e \equiv \frac{A}{Z}$ $\left(\frac{\text{no. H mass}}{\text{free electron}} \right) \Rightarrow n_e = \left(\frac{Z}{A m_p} \right) \rho$

$$\boxed{\therefore P_{degen} = (0.0485) \left(\frac{\hbar^2}{m_e} \right) \left(\frac{Z}{m_p A} \right)^{5/3} \rho^{5/3}}$$

Kinetic theory Picture of degeneracy

Consider phase- space density distribution

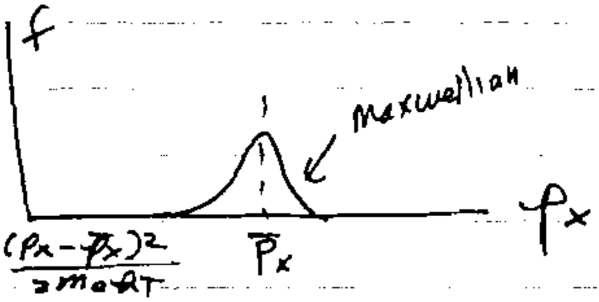
Let $dN = f(p, x)\, d^3\underset{\sim}{x}\, d^3\underset{\sim}{p}$

be no. of particles with position $(\underset{\sim}{x}, \underset{\sim}{x}+d\underset{\sim}{x})$ momentum $(\underset{\sim}{p}, \underset{\sim}{p}+d\underset{\sim}{p})$.

Cartesian Coordinates

$$dN = f(p_x, p_y, p_x; x, y, z)\, dx\,dy\,dz\,dp_x\,dp_y\,dp_z$$

Look along p_x axis:

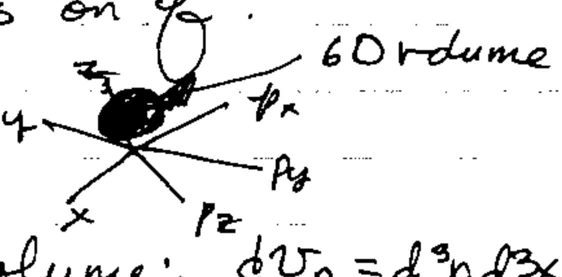


Usual densities we have Maxwellian: $e^{-\frac{p_x^2/2m_e}{kT}} = e^{-\frac{(p_x-\bar{p}_x)^2}{2m_e kT}}$

$(\bar{p}_x = 0)$

<u>But</u> f changes <u>shape</u> as we add Fermions because uncertainty & Pauli exclusion principles place upper limits on f !

6D phase-space :



6D differential volume: $dV_p = d^3p\, d^3x$

$$dV_p = (dp_x dx)(dp_y dy)(dp_z dz)$$

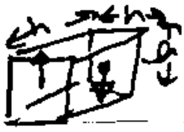
Uncertainty principle says: $dp_x dx \geq h,\ dp_y dy \geq h, \ldots$

$\therefore\quad dV_p \geq \hbar^3$: smallest physical resolvable volume

$$\therefore f = \underbrace{\frac{dN}{dV_p} \leq \frac{dN}{\hbar^3}}_{}\quad \text{(upper limit to f)}.$$

$dN = f \cdot dV_p$

But Exclusion principle says: $dN \leq 2$

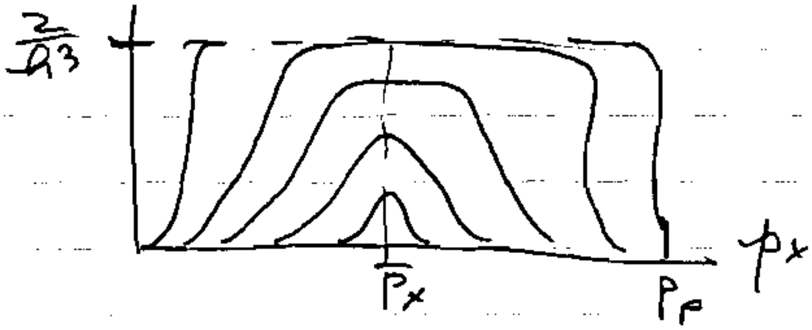


we cannot fit more than $2e^-$ in 6D cube: with opposite spins

$$\boxed{\therefore f \leq f_{lim} \leq \frac{2}{h^3}}$$

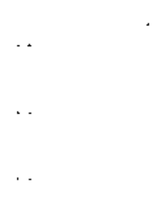
Maximum Fermion density

(Boson stars could have $dN \gg 2$).



As we add ~~[illegible]~~ more Fermions, we cannot place them at or near $\bar{p}_x$, since all those momentum states are occupied. Rather f spreads out ~~to~~ maximum momentum called Fermi momentum, P_F. This corresponds to given density!

$$n_e \equiv \frac{dN}{d^3x} = \int f\, d^3p \leq \frac{2}{h^3} \iiint dp_x dp_y dp_z$$



Go to spherical coordinates in momentum space:

$$\therefore n_e \leq \frac{2}{h^3} \int_0^{P_F} 4\pi p^2 dp = \frac{2}{h^3} \times \frac{4\pi P_F^3}{3}$$

$$\boxed{\therefore (n_e)_{max} = \frac{8\pi P_F^3}{3h^3}}$$

So e^- are degenerate. But, what about nuclei? Are they also degenerate?

Nuclei in White Dwarfs

In thermodynamic equilibrium, nuclei and electrons interact via Coulomb force and "relax" such that they have the same mean kinetic energy.

Both follow Maxwellian distribution with same T

$\langle KE \rangle = \langle p^2 \rangle / 2m$ or $\langle p^2 \rangle^{1/2} = \sqrt{2m} \times \sqrt{\langle KE \rangle}$

Momentum Ratio: $\frac{p_A}{p_e} = \sqrt{\frac{m_A}{m_e}} = \sqrt{A \cdot \frac{m_p}{m_e}} = \sqrt{A} \cdot 43$

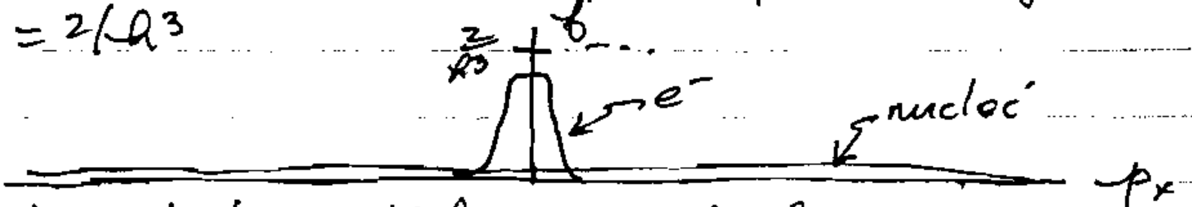
Since $A \geq 4$ in white dwarfs $\frac{p_A}{p_e} \geq 86$

Thus nuclei have much higher momenta than electrons in white dwarfs

Phase Space Volume : $\Delta V_p \propto p^3$

As a result: $\frac{(\Delta V_p)_A}{(\Delta V_p)_e} = \left(\frac{p_A}{p_e}\right)^3 \geq (86)^3 \sim 10^6$

Consequently: We can pack in $\sim 10^6$ more nuclei than electrons before nuclei phase-space density reaches $f_{max} = 2/h^3$



Nuclei behave like an ideal gas.

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Under what conditions does degeneracy occur?

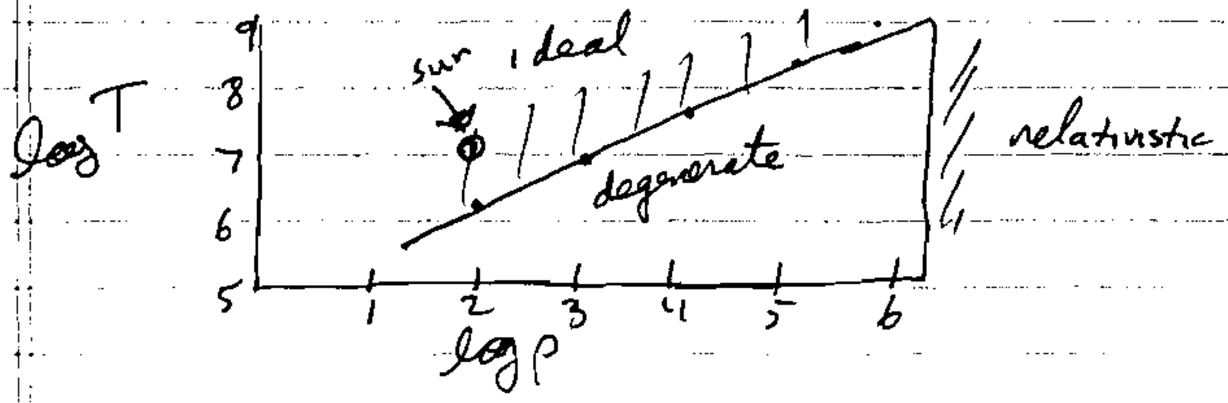
$P_{degen} = K_1 \rho^{5/3}$ (p.9-7) ; $P_{ideal} = \frac{R}{\mu} \rho T$ {

$K_1 \propto (Z/A)^{5/3}$

Gas degenerate when $P_{degen} \geq P_{ideal}$

$$\left(\frac{Z}{A}\right)^{5/3} \rho^{5/3} > \rho T \Rightarrow T \leq \rho^{2/3}$$

$$\text{or } T \leq 1.46\times10^5 \left(\frac{Z}{A}\right)^{2/3} \rho^{2/3} \quad \left(\frac{A}{Z}=2\right)$$



Hydrostatic Equilibrium : $\frac{dP}{dr} = -\frac{GM\rho}{r^2}$

when $P \propto \rho^{5/3}$: $P_c = 0.770\, GM^2/R^4$ $\left\{ \begin{array}{l} \text{central pressure required to hold} \\ \text{up star. not Fermi statistics} \end{array} \right\}$

$\rho_c = 1.43\, M/R^3$

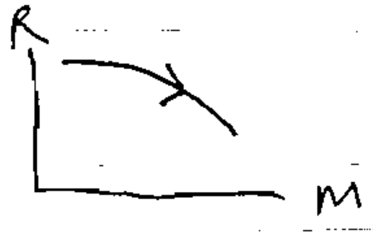
Equate hydrostatic requirements with pressures of degenerate e^- gas

$$\text{at center: } K_1 \underset{}{\overset{\text{degen}}{(\rho_c)^{5/3}}} = \overset{\text{hydrostatic}}{0.77\, GM^2/R^4}$$

$$K_1 \left(\frac{1.43 M}{R^3}\right)^{5/3} = 0.77\, GM^2/R^4$$

$$\Rightarrow \frac{M^{5/3}}{R^5} \propto \frac{M^2}{R^4}$$

$$\rightarrow \boxed{R \propto M^{-1/3}}$$



So, as we increase mass ~~M~~ M of WD, it shrinks. Very different from MS star where $R \propto M^a$; $a \simeq 1$.

for WDs

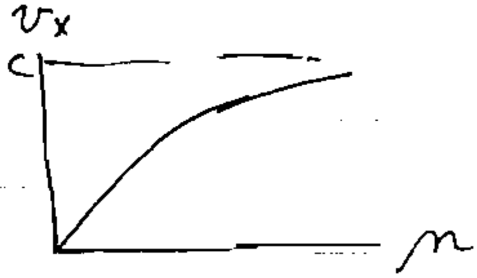
But, $\rho_c \propto \frac{M}{R^3} \propto \frac{M}{M^{-1}} = M^2$

So as M increases, ρ increases and Fermi momentum must increase to balance increase in the gravitational force.

Recall $p_F \propto n_e^{1/3} \propto \rho^{1/3}$

Relativistic limit

$$v_x = \frac{\hbar n_e^{1/3}}{m_e} \propto M^{2/3}$$



So if we increase M enough, $v_x \to c$: gas becomes relativistic:

In this case

$$P_{degen} = v_x n_e p_x = c\, n_e p_x \quad (V_x \text{ saturates})$$
$$= c n_e(\hbar n_e^{1/3}) = c\hbar n_e^{4/3}$$

Accurate: $P^{rel}_{degen} = 0.123\, \hbar c\, n_e^{4/3}$

$$P^{rel}_{degen} = 0.123\, \hbar c \left(\frac{Z}{m_p A}\right)^{4/3} \rho^{4/3}$$

Recall $P^{non\text{-}rel}_{degen} \propto \rho^{5/3}$

$$P^{non\text{-}rel}_{degen} = (0.0485)\left(\frac{\hbar^2}{m_e}\right)\left(\frac{Z}{m_p A}\right)^{5/3} \rho^{5/3}$$

Recap White Dwarfs

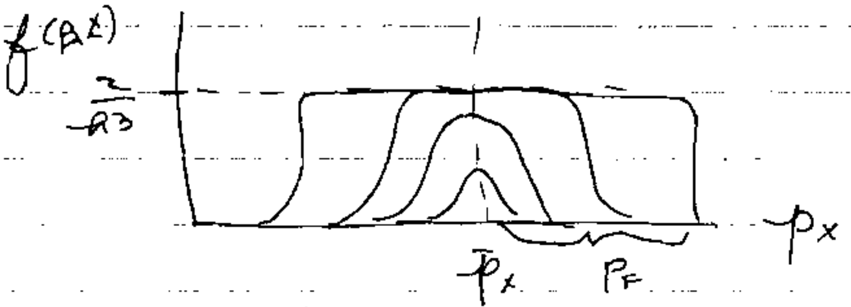
(1) Quantum Mechanical Effects resulting from high densities

(A) Large momentum of electrons: $\Delta p_x \geq \hbar n^{1/3} \simeq p_F$

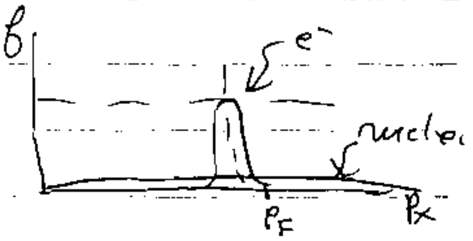
- Electron degeneracy pressure

$$P_{degen} = (.0485)\left(\frac{\hbar^2}{m_e}\right)\left(\frac{Z}{m_p A}\right)^{5/3}\rho^{5/3}$$

- Maximum limit to phase-space densities



- Nuclei are not degenerate due to larger momentum



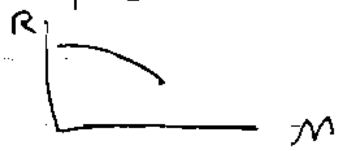
(B) Hydrostatic Equilibrium

Non-Relativistic $P_{degen} = P_{hydrostatic}$

$K_1 \rho_c^{5/3} = 0.77\, G M^2/R^4$

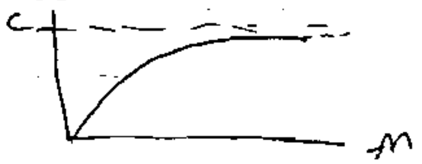
and $\rho_c \propto M/R^3 \Rightarrow R \propto M^{-1/3}$

$\Rightarrow$



Relativistic Limit

$v_e = \frac{\hbar n_e^{1/3}}{m_e}$; but $n_e \propto \rho \propto M^2 \Rightarrow v_e \propto M^{2/3}$



$P_{degen} = 0.123\hbar c \left(\frac{Z}{m_p A}\right)^{4/3}\rho^{4/3}$

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Hydrostatic equilibrium: Models for gas with $P \propto \rho^{4/3}$

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

$\Rightarrow$ Central Pressure: $P_c^{\text{hy}} = 11.0\, GM^2/R^4$

Central Density: $\rho_c^{\text{hy}} = 12.9\, M/R^3$

Letting $P^{\text{rel}}_{\text{degen}} = P_c^{\text{hy}}$

$$0.123\, \hbar c \left(\frac{Z}{m_p A}\right)^{4/3} \rho_c^{4/3} = 11.0\, GM^2/R^4$$

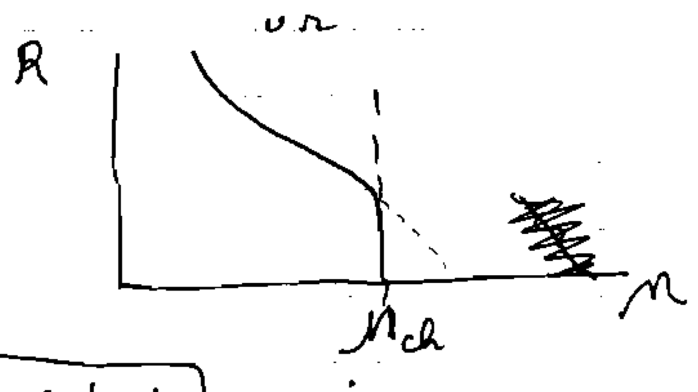
$$\Rightarrow \hbar c \left(\frac{Z}{m_p A}\right)^{4/3} \frac{M^{4/3}}{\cancel{R^4}} \propto M^2/\cancel{R^4}$$

Come up with a unique mass: Chandrasekhar mass

$$\boxed{M_{ch} = 0.2 \left(\frac{Z}{A}\right)^2 \left(\frac{c\hbar}{Gm_p^2}\right)^{3/2} m_p}$$

For $\frac{Z}{A} = 1/2 \Rightarrow M_{ch} = 1.4 M_\odot$

Recall: non-relativistic result $R \propto M^{-1/3}$



Non-relativistic

As we increase M, v_x increases, so momentum flux increases, and degenerate pressure sufficient to balance increased gravitational pull. Result is that radius shrinks.

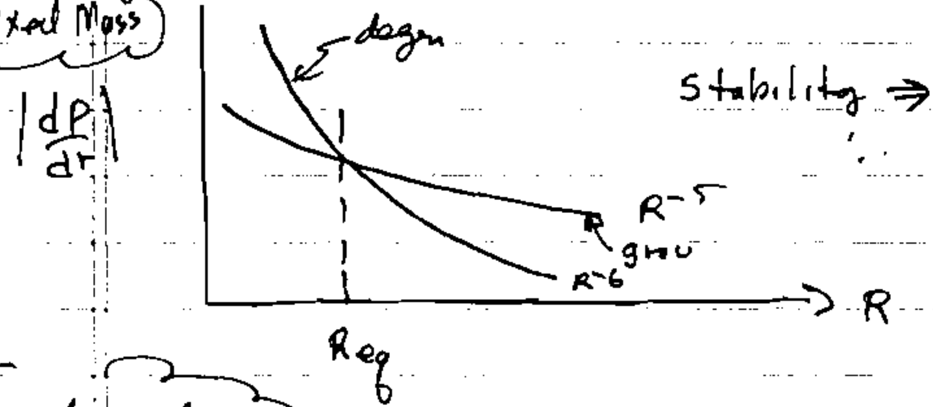
Non-relativistic : Gravity & degenerate pressure have different dependence on R which allows star to adjust to smaller R as M increases.

Look at dP/dr

grav $\left(\frac{dP}{dr}\right)_{grav} = \frac{GM\rho}{R^2} \propto \frac{M^2}{R^5}$

degenerate pressure: $\left(\frac{dP}{dr}\right)_{d.p} \propto \frac{P_c}{R} \propto \frac{\rho_c^{5/3}}{R} \propto \frac{(M/R^3)^{5/3}}{R} = \frac{M^{5/3}}{R^6}$

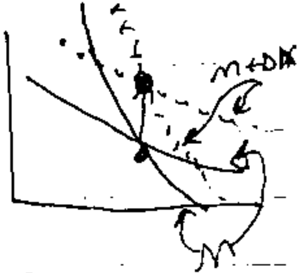
Fixed Mass



Stability ⇒ if $\left|\frac{dP}{dr}\right|_{degn} > \left|\frac{dP}{dr}\right|_{grav}$ (
star expands
$\left|\frac{dP}{dr}\right|_{d.p.}$ falls off faster than $\left|\frac{dP}{dr}\right|_{grav}$: & equil. reached.

Mass dependence

$$\frac{|dP/dr|_{grav}}{\left|\frac{dP}{dr}\right|_{d.p}} \propto \frac{M^2}{R^5} \times \frac{1}{\frac{M^{5/3}}{R^6}} = M^{1/3}R$$



So if M increases then just decrease R to maintain hydrostatic equilibrium:

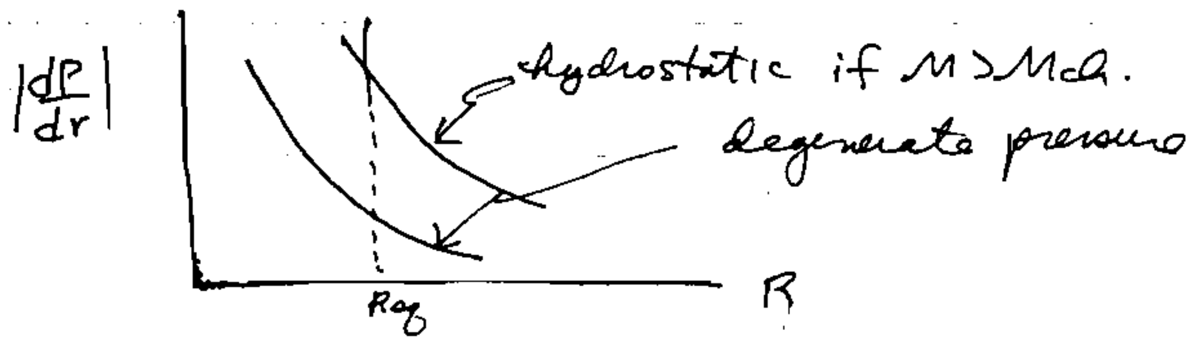
Microphysics

increase v_x increases momentum flux required to balance increase in gravity as M increases. But this increase in flux saturates as $v_x \to c$

Relativistic Pressure gradients:

Hydrostatic: $\left|\frac{dP}{dr}\right|_{hy} \propto \frac{M^2}{R^5}$

Relativistic degenerate: $\left|\frac{dP}{dr}\right|_{r\text{-}deg} \propto \frac{P_c}{R} \propto \frac{\rho^{4/3}}{R} = \frac{M^{4/3}}{R^5}$



So in WD in which $M \geq M_{ch}$, and in which $\left|\frac{dP}{dr}\right|_{hydro} > \left|\frac{dP}{dr}\right|_{r\text{-}deg}$, both Pressure gradients have same dependence on R.

Consequently, star ~~[illegible]~~ can not readjust such that two pressure gradients are equal; i.e.

Ratio $\frac{|dP/dr|_{hydro}}{\left|\frac{dP}{dr}\right|_{rdeg}} \propto \frac{M^2}{R^5} \times \frac{1}{\frac{M^{4/3}}{R^5}} = M^{2/3}$ ← (no R depend

So if M increases, no adjustment of R returns star to hydrostatic equilibrium

HR Diagram

$\mathcal{L} = 4\pi R^2 \cdot \sigma T_e^4$

fixed M $\mathcal{L} \propto T_e^4$

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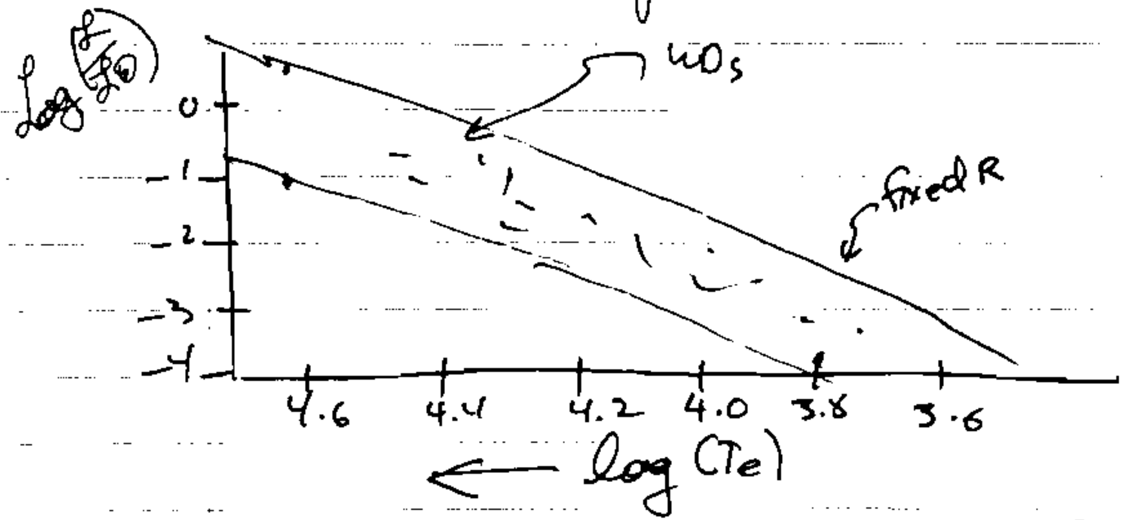
But $\mathcal{R} \propto M^{-1/3}$

$\therefore$ For fixed T_e ; $\mathcal{L} \propto M^{-2/3}$

What does HR diagram look like? We need to explain this!.



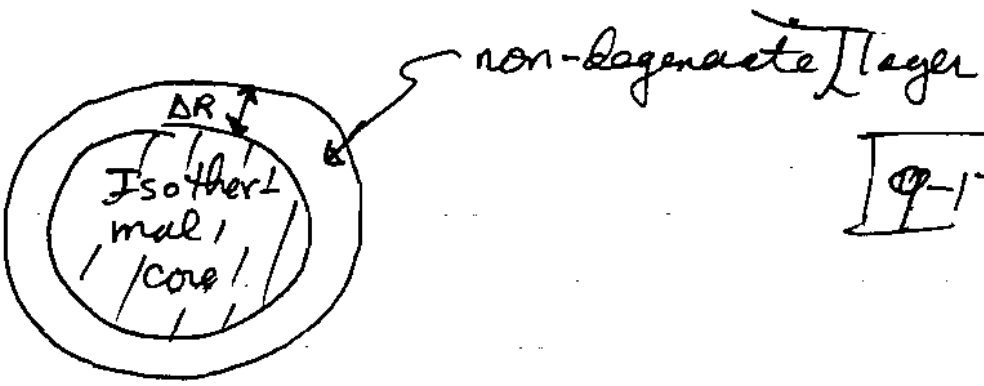
Most WDs lie along narrow strip for hot underluminous stars.

White Dwarfs Interiors

What is T_c? ; What is source of luminosity?

① Isothermal Core

Because of e^- degeneracy, all e^-'s cannot change their momenta. Since all scattering processes change momenta, collisional m.f.p is enormous. As a result heat transport to surface by conduction is very efficient. In same way that heat conduction erases T differences between different parts of a metal bar, White dwarf interiors are isothermal!



Energy Transport: Interior - Conduction e's
Surface - Radiation

Surface: Recall:

$$\mathcal{L}(r) = -4\pi r^2 \cdot \mathcal{F}(r) = -4\pi r^2 \left(\frac{c}{3\kappa\rho}\right) 4aT^3 \frac{dT}{dr}$$

Radiative Transport }
$$\frac{dT}{dr} = -\left(\frac{3\kappa\rho}{ca}\right) \times \frac{\mathcal{L}(r)}{4\pi r^2} \times \frac{1}{4T^3}$$

Hydrostatic Equilibrium }
$$\frac{dP}{dr} = -\frac{G\,M(r)\,\rho}{r^2}$$

$$\frac{dP}{dT} = \frac{dP/dr}{dT/dr} = \frac{GM\rho/\cancel{r^2}}{\frac{3\kappa\rho}{ca} \times \frac{\mathcal{L}}{4\pi \cancel{r^2}} \times \frac{1}{4T^3}} = \left(\frac{4ca}{3}\right)\left(\frac{4\pi GM}{\kappa\mathcal{L}}\right)T^3$$

Free-free opacity }
$$\kappa = \kappa_0 \rho T^{-3.5}$$

$$\Rightarrow \frac{dP}{dT} = \left(\frac{4ca}{3}\right)\left(\frac{4\pi GM}{\kappa_0 \mathcal{L}}\right)\frac{\times T^{6.5}}{\rho}$$

Surface Layer: $\Delta R << R$ ∴ therefore $M(r) = M$

$$P = \frac{\mathcal{R}}{\mu}\rho T \Rightarrow \frac{1}{\rho} = \frac{\mathcal{R}}{\mu}T/P$$

$$\frac{dP}{dT} = \left(\frac{4ca}{3}\right)\left(\frac{4\pi GM}{\kappa_0\mathcal{L}}\right)\left(\frac{\mathcal{R}}{\mu}\frac{T^{7.5}}{P}\right)$$

Integrate from $T=0, P=0 \longrightarrow P, T$

$$dP \cdot P = (\text{const})\, T^{7.5}\, dT$$

$$\left[\frac{P^2}{2}\right]_0^P = \text{const}\left[\frac{T^{8.5}}{8.5}\right]_0^T$$

$$P^2 = \left(\frac{2}{8.5} \times \text{const}\right) T^{8.5}$$

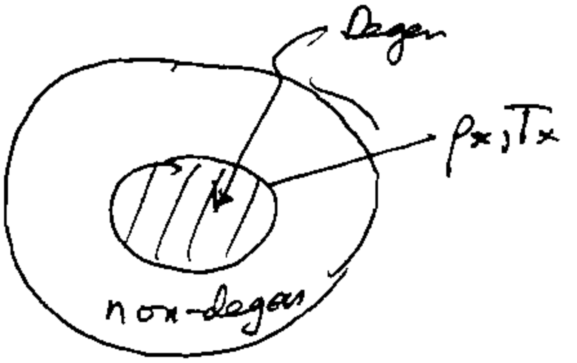
$$P = \left[\frac{2}{8.5}\ \text{const}\right]^{\frac{1}{2}} T^{4.25}$$

Since $P = \frac{\mathcal{R}}{\mu}\rho T$ in surface layer, we have

$$\frac{\mathcal{R}}{\mu}\rho T = \left[\frac{2}{8.5} \times \text{const}\right]^{1/2} T^{4.25}$$

$$= \left[\frac{2}{8.5} \times \left(\frac{4ca}{3}\right)\left(\frac{4\pi G M}{K_0 \mathcal{L}}\right)\left(\frac{\mathcal{R}}{\mu}\right)\right]^{1/2} T^{4.25}$$

$$\boxed{\rho = \left[\frac{2}{8.5} \times \left(\frac{4ca}{3}\right)\left(\frac{4\pi G M}{K_0 \mathcal{R}}\right)\left(\frac{\mu}{\mathcal{R}}\right)\right]^{1/2} T^{3.25}} \quad ①$$



Boundary Condition:

at $\rho = \rho_*, T = T_*$, $P^{\text{ideal}} = P^{\text{degen}}$

$$\therefore \frac{\mathcal{R}}{\mu}\rho_* T_* = 0.0485\left(\frac{h^2}{m_e}\right)\left(\frac{Z}{m_p A}\rho_*\right)^{5/3}$$

$$\Longrightarrow T_* \propto \rho_*^{2/3} \Rightarrow \rho_* \propto T_*^{3/2} \approx 2.4\times10^{-8}\, T_*^{3/2}$$

Put in eq. (1) $T_*^{3/2} \propto \left(\frac{M}{L}\right)^{1/2} T_*^{3.25} \Rightarrow T_*^3 \propto \left(\frac{M}{\mathcal{L}}\right) T_*^{6.5}$

$$\boxed{\mathcal{L} \propto M T_*^{3.5}}$$

~~[illegible]~~

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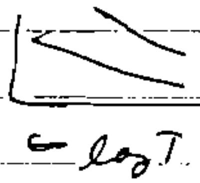
Numerics	$\mathcal{L}/\mathcal{L}_\odot$	T_* ; $M = M_\odot$
	10^{-2}	1.7×10^7
	10^{-3}	9×10^6
	10^{-4}	4×10^6



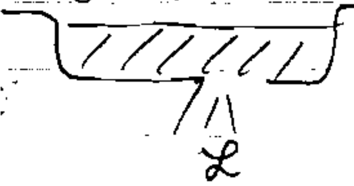
In isothermal WD interiors, temperatures $T \simeq 10^6 - 10^7$ not hot enough to burn, He, C, or O.
So no nuclear fusion in WDs

Source of thermal Energy

(1) Thermal energy of e^- not radiated away since e^- are degenerate
(2) Only ions, atoms can radiate via acceleration.

(3) Unlike stars in which $P = P^{ideal}$, star won't undergo Kelvin contraction, since R already at minimum {Pressure in WDs independent of T).

Cooling time:



$$~~a~~t_{coo} = \frac{U_{nuclei}}{\mathcal{L}} = \frac{\frac{3}{2}\frac{R}{\mu}T\cdot M}{\mathcal{L}} \quad \begin{pmatrix} \mu = 4 \\ \text{for He} \\ \text{no } e^- \end{pmatrix}$$

$$t_{cool} = \frac{\frac{3}{2}\left(\frac{10^8}{4}\right)(2\times10^{33})\left(\frac{\mu}{\mu_{He}}\right)^{-1}\times10^6\,(T/10^6)}{(\mathcal{L}/\mathcal{L}_\odot)\,4\times10^{33}}$$

$$t_{cool} = \frac{5.2\times10^{5}\,(T/10^{6}K)}{(\mathcal{L}/\mathcal{L}_\odot)}\ \text{yrs}$$

$\mathcal{L}/\mathcal{L}_\odot$	T(K)	t_{cool} (yrs.)
10^{-2}	1.7×10^{7}	8.5×10^{8}
10^{-3}	9×10^{6}	4.5×10^{9}

So, cooling times long enough that WDs have not yet faded from view. But short enough that typical $\mathcal{L}$ is low! Eventually they will just fade

Massive stars: What about evolution of stars with $M > 9M_\odot$?

(1) Because of greater core mass, temperatures achieved through contraction of inert core can be high enough to ultimately synthesize $_{26}Fe^{56}$

(2) After stars leave MS, initial contraction of $_2He^4$ cores surrounded by H^1 burning shell.

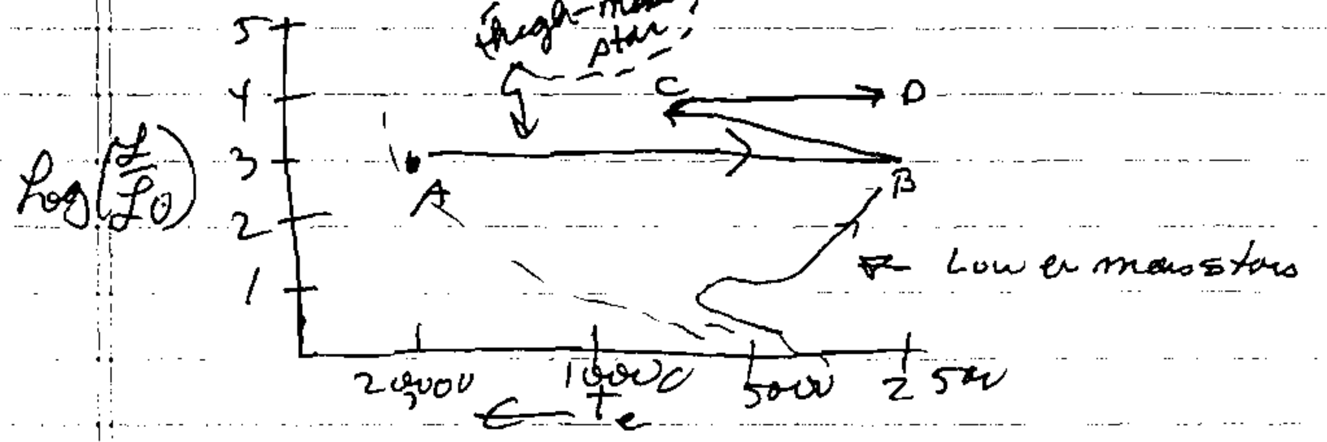
But here's difference with lower mass stars: Outer layers are so hot, that they do not cool sufficiently with expansion to the point where

where

(a) outer layers become partially ionized which as we saw before leads to $\gamma \to 1$

(b) H^- opacity dominates.

As a result, envelope of such stars does not become convective! As a result when $L_{nuc} > L_{rad}$ in shell, $\Delta L = L_{nuc} - L_{rad}$ does not reach the surface. Rather ΔL does work expanding envelope (stars evolve like sub-giants)

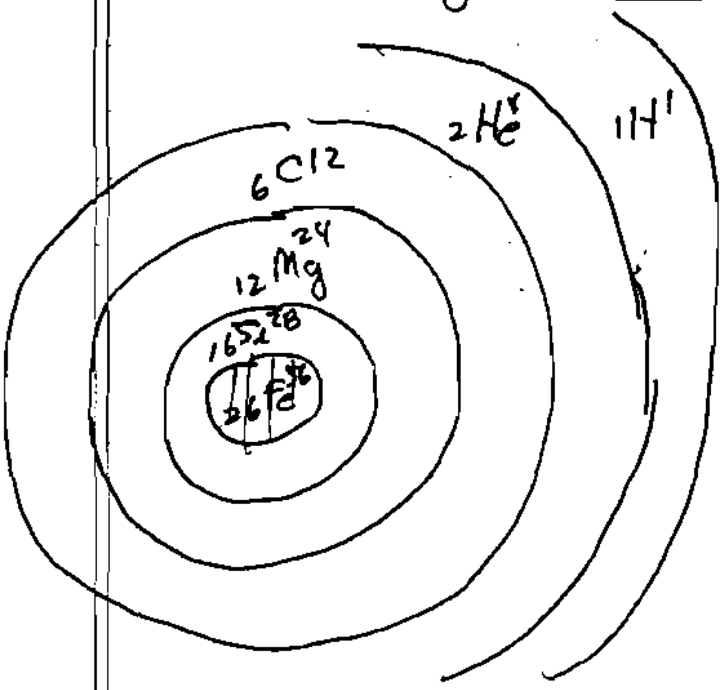


AB: H-burning shell surrounding inert $_2He^4$ core. Envelope expands and $T_e \propto R^{-1/2}$

BC: At B $_2He^4$ core ignites because $T \to 10^8$ K in contracted core. Core expands such that $\Delta L \to 0$. So no ΔL to support envelope. $L \approx$ const so T_e increases as R decreases

CD: He shell ignition etc. Contracting $_6C^{12}$ core surrounded by double shell source. Again ΔL pushes out envelope. But lack of convection implies L_{rad} does not increase!

Ultimate Layer Cake (Core)

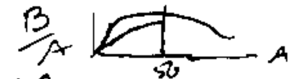


Degenerate and inert $_{26}Fe^{56}$ core surrounded by successive burning shells.

(1) Hot $_{26}Fe^{56}$ core is an incipient White Dwarf
$M_{core} \simeq 1.4 M_\odot$ since $\frac{Z}{A} \simeq \frac{26}{56} \simeq 1/2$

(2) But as additional mass from burning $_{16}Si^{28}$ shell drops on $_{26}Fe^{56}$ core, $M_{core} > M_{ch}$. As a result, the core contracts.

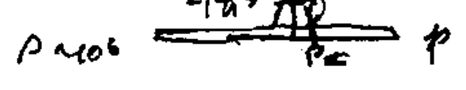
(3) But, no synthesis to heavier elements occur since all reactions to elements with $A > 56$ are <u>endothermic</u>



(4) So as $T \rightarrow 10^9 K$, <u>nucleosynthesis</u> gets reversed. Nuclei
by X-rays & γ-rays
are photodisintegrated & And we are left with bare nucleons: $_1H^1$ and $_0n^1$

$\rho \sim 10^{15} g\,cm^{-3}$

(5) But $\rho >> 10^6 g\,cm^{-3}$ for protons and neutrons
Recall Fermi-momentum $p_F \sim \hbar n^{1/3}$. So now
$p_F >> p$ (nuclei) which become degenerate



Nuclei turn into neutrons:

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(1) Inverse β decay

$$\boxed{\begin{array}{l} e^- + p \longrightarrow n + {}_0\nu^0 \\ {}_{-1}e^0 + {}_1H^1 \longrightarrow {}_0n^1 + {}_0\nu^0 \end{array}} \quad ③$$

Reaction ③ goes when $Q > 0$

$$m_ec^2 + m_pc^2 = m_nc^2 + Q$$

when $Q = m_ec^2 - (m_n - m_p)c^2 > 0$

$$m_ec^2 > (m_n - m_p)c^2 = 1.29\,MeV$$

how can $m_ec^2 > 1.29\,MeV$? rest mass $m_ec^2 = 0.51\,MeV$

Answer: is energy of e^- is not m_ec^2, but rather γm_ec^2. In ordinary stars we set $\gamma = 1$, But here if

$$\gamma m_ec^2 > 1.29\,MeV$$

$$\gamma > \frac{1.29}{0.511} = 2.6$$

So inverse β decay can occur if gas is relativistic, which it is (e^- are relativistic) : $v = \sqrt{\frac{56}{6c}}c$

Note, inverse reaction

$$\boxed{{}_0n^1 \longrightarrow {}_1H^1 + {}_{-1}e^0 + {}_0\bar{\nu}^0} \quad ④$$

cannot occur? Why? Because all electron momentum states are filled in the degenerate gas.

- Density is now high enough for n's to be degenerate.

n's $\quad P_{degen} = 0.0485\left(\frac{\hbar^2}{m_p^{8/3}}\right)\rho^{5/3} \qquad \left\{ \begin{array}{l} n = \frac{\rho}{m_p} \\ v_p = p_F/m_p \end{array} \right.$

e⁻'s $\quad P_{degen} = 0.0485\left(\frac{\hbar^2}{m_e}\right)\left(\frac{Z}{A}\times\frac{1}{m_p}\right)^{5/3}\rho^{5/3}$

Hydrostatic Equilibrium: (Non-relativistic nucleons)

Hydrostatic equilibrium $\left\{ \begin{array}{l} \text{central Pressure} : P_c^{hy} = 0.77\, GM^2/R^4, \quad \rho_c = 1.43M/R^3 \\ \text{hydrostatic} \quad \text{~~Pdegen~~ [illegible]} \quad P_{degen}(R=0) = P_c^{hy} \end{array} \right.$

$$0.0485\left(\frac{\hbar^2}{m_p^{8/3}}\right)\rho_c^{5/3} = 0.77\, GM^2/R^4$$

$$0.0485\left(\frac{\hbar^2}{m_p^{8/3}}\right)\left(\frac{1.43M}{R^3}\right)^{5/3} = \frac{0.77\, GM^2}{R^4}$$

Solve for R: $\quad R_{NS} = \frac{0.0485\left(\frac{\hbar^2}{m_p^{8/3}}\right)(1.43)^{5/3} M^{5/3}}{0.77\; G \times M^2}$

$$\therefore \boxed{R_{NS} = \frac{0.114\,\hbar^2}{G\, m_p^{8/3}} \times \frac{1}{M^{1/3}}}$$

By comparison WD radius is given by

$$\boxed{R_{WD} = \frac{0.114\,\hbar^2}{G m_e m_p^{5/3}}\left(\frac{Z}{A}\right)^{5/3} \times \frac{1}{M^{1/3}}}$$

Ratio $\dfrac{R_{NS}}{R_{WD}} = \dfrac{m_e m_p^{5/3}}{m_p^{8/3}}\left(\dfrac{A}{Z}\right)^{5/3} = \dfrac{m_e}{m_p}\left(\dfrac{A}{Z}\right)^{5/3} = \dfrac{m_e}{m_p}(2)^{5/3}$

$= 3.17\, m_e/m_p \sim 10^{-3}$

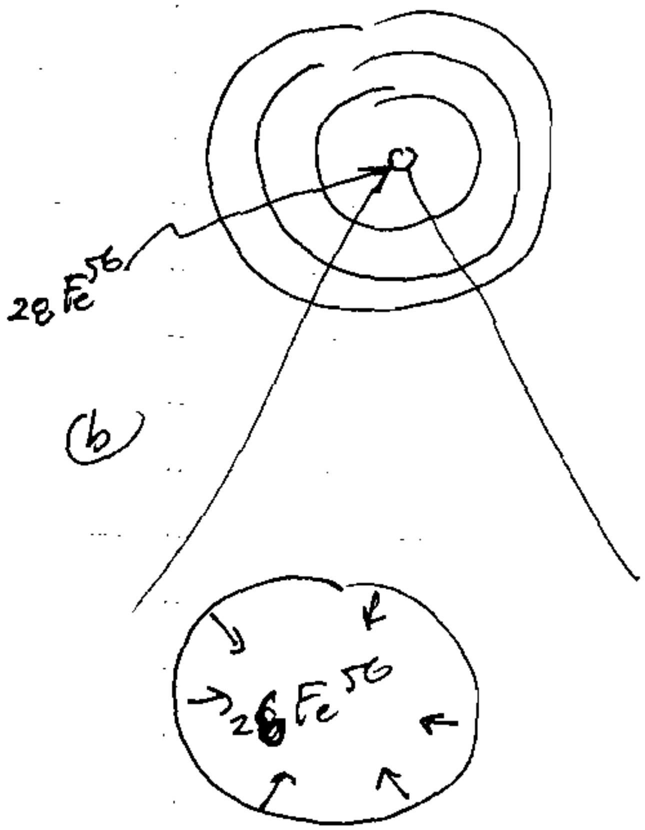
Recall $R_{WD} \simeq 10^{-2} R_\odot \sim 10^9$ cm

$\therefore R_{NS} \simeq 10^{-5} R_\odot \sim 7$ km!

$\left\{ \rho_{NS} \cong \left(\dfrac{R_{WD}}{R_{NS}}\right)^3 \cdot \rho_{WD} \simeq (10^3)^3 \rho_{WD} \simeq 10^{15}\,\text{g cm}^{-3} \right.$

Sequence of events leading to SN explosion

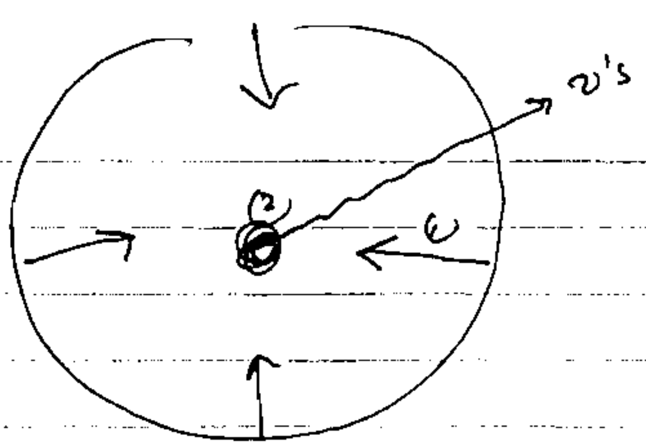
ⓐ Onion-layered shells with $_{26}Fe^{56}$ core



$_{26}Fe^{56}$ core held up by degenerate e^- pressure

$_{26}Fe^{56}$ core mass increases until it exceeds $M_{ch} = 1.4 M_\odot$. At that point $_{26}Fe^{56}$ core starts to collapse

(c)



(1) Outer core collapse at $V \doteq 0.23\,c$

(2) Inner core: heats up producing γ rays that photodisintegrate nuclei $\rightarrow {}_1H^1, {}_0n^1$

(3) T high enough so e^- become relativistic and inverse β decay converts ${}_1H^1$ into ${}_0n^1$.

$$ {}_{-1}e^0 + {}_1H^1 \rightarrow {}_0n^1 + {}_0\nu^0 $$

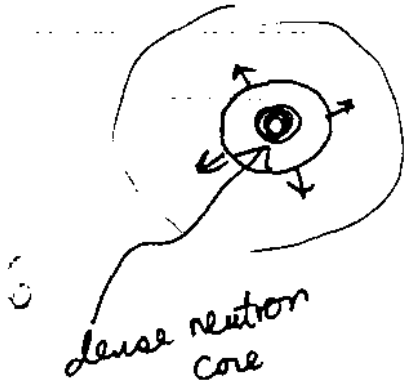
Energy transported out by neutrinos further accelerates collapse:

time scale: free-fall time $t_{ff} \sim \frac{1}{\sqrt{G\rho}}$

$$ \rho \simeq \frac{M}{\frac{4\pi}{3}R^3} = \frac{2\times10^{33}}{4\,(7\times10^5)^3} \sim 10^{15}\,g\,cm^{-3} $$

$t \simeq 0.1$ milliseconds!.

(4)



inward falling material bounces off dense neutron core forming outward propagating shock

strong interaction repulsive n-n interactions prevent further collapse

$T \sim 10^{11}$ K! in core: Thermal neutrinos >> inverse β decay neutrinos. These were detected in SN 1987A in ~~[illegible]~~ SMC

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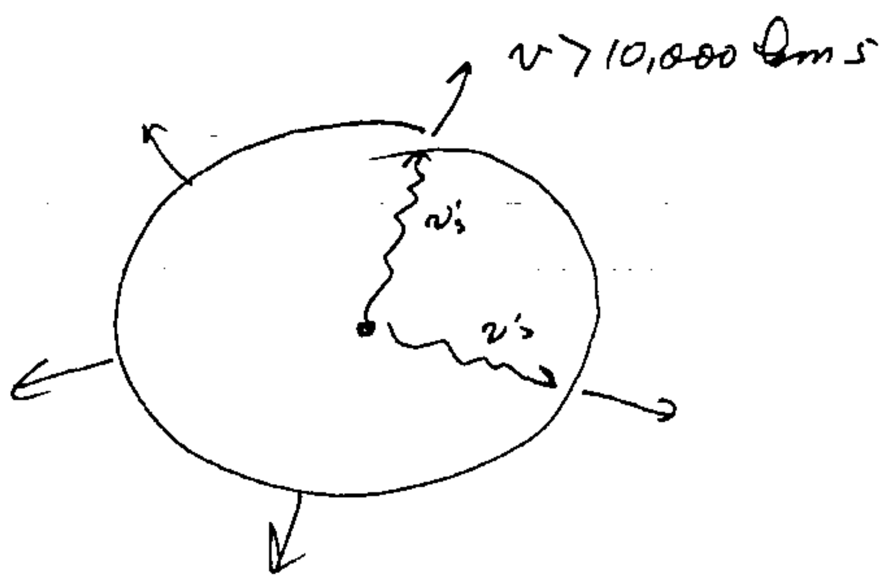
⑤



Neutrinos heat up shell giving it further outward speed. Result is:

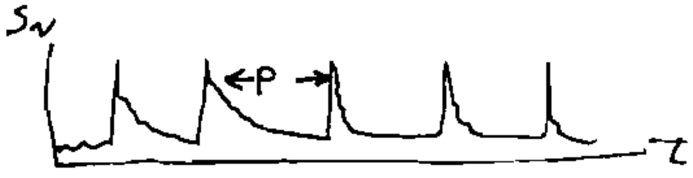
ⓐ Most of the progenitor star mass of $M > 9M_\odot$ ejected in SN explosion

ⓑ Emitted ~~light~~: $\mathcal{L} \simeq 10^{10}\mathcal{L}_\odot$!!

ⓒ Remnant $M_{NS} = 1.4 M_\odot$

<u>Evidence for NS?</u> Pulsars

1968: Hewish & Bell at Cambridge detected pulsating radio sources, i.e., pulsars



Extremely regular periods: (1) $P = 1.6\times10^{-3}$ s → 4.5 s

(2) $\Delta P/\Delta t > 0$ period increases slowly

Crab, Vela: Detected at center of SN remnants: about 60 objects (ouch)