Recap

Discussed concept of hydrostatic equilibrium

(1) Balance between inward gravitational and outward pressure forces leads to:

$$\boxed{\frac{dP}{dr} = -\frac{G M(r)\rho(r)}{r^2}} \quad (1)$$

(2) Stability of hydrostatic equilibrium.
Any deviation from eq. (1) quickly erased.

$$T_{free\text{-}fall} = \sqrt{\frac{3\pi}{32G(1-\alpha)\langle\rho\rangle}} \; ; \; \alpha = \frac{\left|-\frac{1}{\rho}\frac{dP}{dr}\right|}{GM/r^2}$$

$$= \frac{0.58\,hr}{\sqrt{1-\alpha}} \quad \text{for the Sun}$$

(3) Crude estimates of central conditions

Central Pressure $\quad P_c \sim \frac{GM_\odot}{R_\odot}\langle\rho\rangle \sim \frac{GM_\odot^2}{R_\odot^4}$

Accurate: $P_c \simeq 2\times10^{17}$ dynes/cm²

(4) Ideal gas law:

$$P = \left(\frac{k}{m_H}\right)\frac{\rho T}{\mu} = \frac{\mathcal{R}\rho T}{\mu} \; ; \; \mathcal{R} = \frac{k}{m_H}$$

μ is mean molecular weight { mass of average free particle in units of m_H }

Central temperature

$$T_c \sim \frac{GM_\odot}{\mathcal{R}\cdot R_\odot} \simeq 2\times10^{7}\,K$$

(5) Virial theorem

when we integrated eq. (1) we found important result for this equilibrium object.

(1) thermal kinetic energy: E_{KE}

$$\boxed{E_{KE} = -\frac{1}{2}\Omega_G} \quad (2)$$

where Ω_G is gravitational potential energy

(2) Total energy $E = E_{KE} + \Omega_G$

$$\boxed{E = +\frac{1}{2}\Omega_G} \quad (3)$$

(6) Kelvin-Helmholtz Contraction

Loss of energy due to radiative output:

$$\Delta E = \frac{1}{2}\Delta\Omega_G$$

$$\therefore \quad \boxed{\frac{\Delta E}{\Delta t} = \frac{1}{2}\frac{\Delta\Omega_G}{\Delta t}} \quad (4)$$

Isolated hot gravitational bound body ~~loses~~ suffers net loss in energy per unit time

$$\frac{\Delta E}{\Delta t} < 0.$$

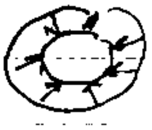
Eq. (4) implies $\Delta\Omega_G/\Delta t < 0$

Since $\Omega_G = -\int \frac{GM(r)\,dm}{r} \simeq \frac{\xi G M_\odot^2}{R_\odot}$

So, since $M_\odot = \text{const}$ the only way for Ω_G to become more negative is for $R_\odot$ to decrease; i.e., for star to contract.

(7) <u>Negative heat capacity</u>

As star contracts, gravitational potential energy decreases. Where does $|\Delta\Omega_G|$ released go?



ⓐ $\Delta E = \frac{1}{2}\Delta\Omega_G$: Half of released potential energy gets radiated away

ⓑ $\Delta E_{KE} = -\frac{1}{2}\Delta\Omega_G$: other half of released potential energy increases thermal kinetic energy. Since $\Delta\Omega_G < 0$, $\Delta E_{KE} > 0$. Star heats up.

ⓒ Negative heat capacity:

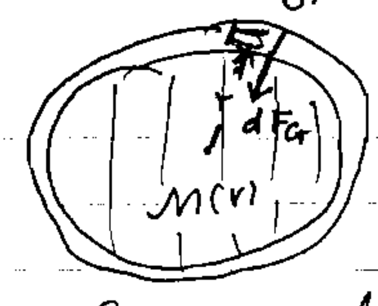
$$\Delta Q = C_V \Delta T \Rightarrow \Delta T = \frac{\Delta Q}{C_V}$$

So if $\Delta Q < 0$ is accompanied by $\Delta T > 0$, implication is that $C_V < 0$.

⑧ <u>Kelvin Contraction</u>

Slow Kelvin Contraction (<u>not</u> dynamic free fall) through sequence of hydrostatic equilibrium states keeps star shining for time scale, the Kelvin-Helmholtz contraction time, τ_{KH}.

First let's work out expression for Ω_G



(1) Gravitational force between mass element dM at edge of spherical mass distribution with mass $M(r)$ is given by

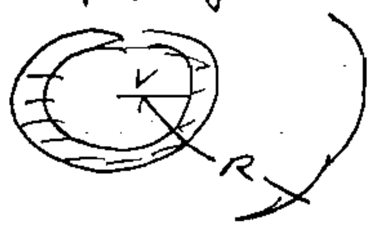
$$dF_G = G\frac{M(r)\,dM}{r^2}$$

where dF_G is directed toward center of the sphere. This is the same force that would exist if all mass of the sphere were located at $r=0$ from dM.

In that case

$$d\Omega_G = -\frac{GM(r)\,dM}{r}$$

Now, assume dM is distributed uniformly in shell of thickness $dM = 4\pi r^2 \rho(r)\,dr$.



$$\therefore\ d\Omega_G = -\frac{GM(r)\cdot 4\pi r^2 \rho(r)\,dr}{r}$$

Integrate over all mass shells from $r=0$ to total radius, R, of the star:

$$\Omega_G = -G\int_0^R M(r)\,\rho(r)\,4\pi r\,dr$$

or

$$\boxed{\Omega_G = -G\int_0^R \frac{M(r)\,dM}{r}} \quad (5)$$

For a uniform shell, $\rho = \text{const.}$, one gets simple result:

$$\Omega_G = -\frac{3}{5}\frac{GM^2}{R}.$$

In general $$\boxed{\Omega_G = -\xi \cdot \frac{GM^2}{R}} \quad (6)$$

Back to Kelvin time-scale

Let $\frac{dE}{dt} = -L = \frac{1}{2}\frac{d\Omega_G}{dt}$

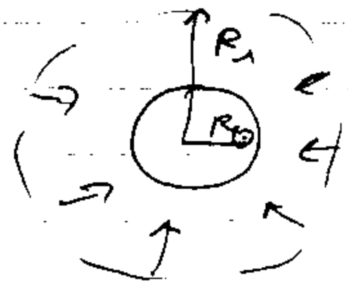
From eq. (6) we have $\frac{d\Omega_G}{dt} = \xi \frac{GM^2}{R^2}\frac{dR}{dt}$

Assume $L \simeq$ const. during contraction.

Therefore: $-L = \frac{1}{2}\left[\xi\frac{GM^2}{R^2}\frac{dR}{dt}\right]$

or $$dt = -\frac{\xi}{2}\frac{GM^2}{L} \times \frac{dR}{R^2}$$

$$\int_0^{\tau_{KH}} dt = -\frac{\xi}{2}\frac{GM^2}{L}\int_{R_i}^{R_0}\frac{dR}{R^2}$$



where R_i is the initial radius of the star as it starts Kelvin contraction.

$$\tau_{KH} = -\frac{\xi}{2}\frac{GM^2}{L}\left[-\frac{1}{R}\right]_{R_i}^{R_0} = -\frac{\xi}{2}\cdot\frac{GM^2}{L}\left[-\frac{1}{R_0}+\frac{1}{R_i}\right.$$

Assuming $R_{initial} \equiv R_i \gg R_\odot$, we get -

$$\tau_{KH} = \frac{3}{2}\frac{GM^2}{R_\odot L} \simeq \frac{1}{2}\frac{|\Omega_G(R_\odot)|}{L}$$

Kelvin-Helmholtz timescale ≃ time it takes for star to radiate away half its potential energy!

Numerics: $\tau_{KH} \simeq \frac{1}{2}\frac{6.7\times10^{-8}(2\times10^{33})^2}{7\times10^{10}\times4\times10^{33}}$

$$\tau_{KH} \simeq 4.8\times10^{14}\,s \Rightarrow \boxed{\tau_{K} \simeq 2\times10^7\,yr}$$

This is the maximum time that the sun could be in its current state. Future? shrinkage heats star up to higher T, which would increase L and shorten τ_{KH}.

Problem: radioactive dating of meteorites tells us that solar system (hence sun) formed 4.6×10^9 years ago. Something is wrong!
Sun in its current state for $\sim 4.6\times10^9$ yr!

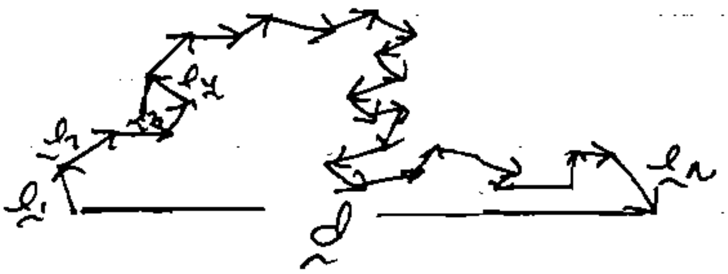
Implication: Gravitational potential energy is not the energy source that keeps the sun shining. Some other source of potential energy decrease is required!

But before we discuss this I want to discuss one more timescale, radiative diffusion time!

Diffusion Time

There is no direct outward flow of radiation streaming out of stars. Rather photons travel in st. direction for distance $d \simeq l_\lambda$ where l_λ is mean-free-path due to scattering and/or absorption. Photon is either scattered, i.e., changes direction due to Thomson scattering, or is absorbed and another photon is re-emitted in its place. In both cases photon changes direction!

Result: Photons undergo haphazard random walk



Photons undergo random walk in N steps. Each step is a displacement given by $\underset{\sim}{l_i}$ where $|\underset{\sim}{l_i}| = l = mfp$

$$\underset{\sim}{d} = \underset{\sim}{l_1} + \underset{\sim}{l_2} + \underset{\sim}{l_3} + \cdots \underset{\sim}{l_N}$$

Scalar product $\underset{\sim}{d}\cdot\underset{\sim}{d} = \underset{\sim}{l_1}\cdot(\underset{\sim}{l_1} + \underset{\sim}{l_2} + \underset{\sim}{l_3} + \cdots \underset{\sim}{l_N}) + \underset{\sim}{l_2}\cdot(\underset{\sim}{l_1} + \underset{\sim}{l_2} + \underset{\sim}{l_3} \cdots \underset{\sim}{l_N})$
$+ \cdots \; \underset{\sim}{l_N}\cdot(\underset{\sim}{l_1} + \underset{\sim}{l_2} + \underset{\sim}{l_3} \cdots + \underset{\sim}{l_N})$

$$d^2 = \underset{\sim}{d}\cdot\underset{\sim}{d} = l_1^2 + l_2^2 + l_3^2 \cdots l_N^2 + l^2(\cos\theta_{12} + \cos\theta_{13} \cdots \cos\theta_{1N}$$
$$+ \cos\theta_{21} + \cos\theta_{23} + \cdots \cos\theta_{2N} + \cdots \cos\theta_{N1} + \cos\theta_{N2} + \cdots \cos\theta_{N,N-1})$$

where θ_{ij} is angle between $\underset{\sim}{l_i}$ and $\underset{\sim}{l_j}$

$$d^2 \simeq N\ell^2 + \ell^2 \sum_i \sum_{\substack{j \\ j \neq i}} \cos\theta_{ij}$$

on the average = no. of positive and negative steps:

In the limit $N \to \infty$, the $\cos\theta$ sum vanishes and

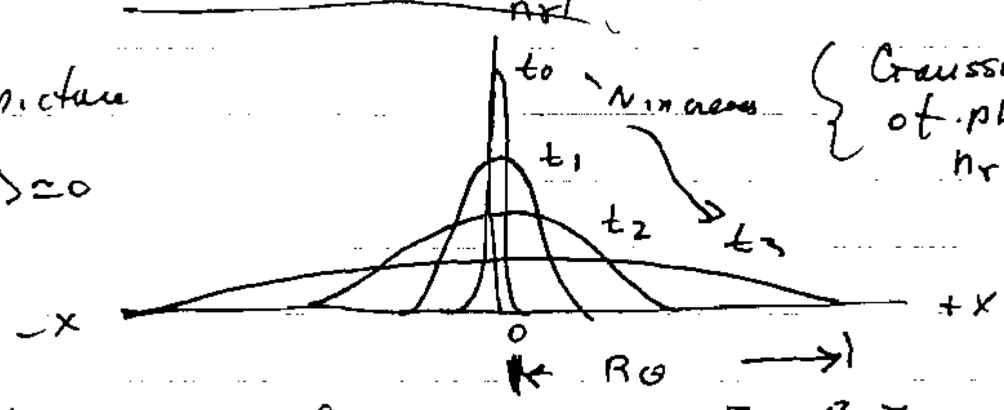
$$d^2 \simeq N \cdot \ell^2 \quad \text{or} \quad d = \sqrt{N}\,\ell$$

Diffusion time: Time taken for a photon to originate at $r=0$ and escape from $r = R_\odot$

$$\boxed{t_{diff} = N \times \frac{\ell}{c}} \quad (7)$$

Realistic picture

mean $\langle d \rangle \simeq 0$



Gaussian distribution of photons:
$n_\gamma dx$ = # photons in $(x, x+dx)$

dispersion of $n_\gamma \propto \exp\left[-\frac{x^2}{2\sigma_\gamma^2}\right]$ $\therefore \sigma_\gamma \simeq d = \sqrt{N}\,\ell$

where $N = c\,t/c$ as t increases so will σ_γ

Finally at t_3 $d \simeq R_\odot$.
So no. of scatterings $N_{av} = (d/\ell)^2 = (R_\odot/\ell)^2$

$$\therefore t_{diff} = \left(\frac{R_\odot}{\ell}\right)^2 \times \frac{\ell}{c}$$ ~~$R_\odot$~~

As a result $t_{diff} \simeq \frac{R_\odot^2}{\ell \cdot c}$

Expressed differently: $t_{diff} \simeq \left(\frac{R_\odot}{\ell}\right) \times \left(\frac{R_\odot}{c}\right)$

$(R_\odot/\ell) \simeq \tau$ optical depth

$(R_\odot/c)$ = light travel time directly through sun

$\therefore \boxed{t_{diff} \simeq \tau \cdot \frac{R_\odot}{c}}$

It takes much longer than direct light-travel time for photon to diffuse out of the sun!

What is ℓ: $\ell \simeq \frac{1}{n\,\sigma_{cross}}$

Thomson Scattering: Recall solar interior is very hot. As a result all of H and everything else is ionized.

Electron scattering cross-section: $\sigma_T = \frac{8\pi}{3} r_e^2$
where r_e is classical electron radius
$r_e = \frac{e^2}{m_e c^2}$ (cgs)

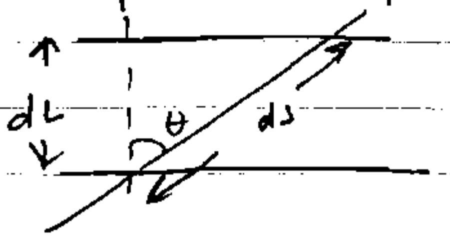
$\therefore \sigma_T = 6.65 \times 10^{-25}\ \mathrm{cm}^2$ $(6.65 \times 10^{-29}\ \mathrm{m}^2)$

recall $\langle\rho\rangle \sim 1.4\ \mathrm{g\,cm}^{-3} \Rightarrow n = \frac{\langle\rho\rangle}{\mu m_H} \sim \frac{1.4}{1.67\times10^{-24}} \sim 10^{24}\ \mathrm{cm}$

Therefore photon mean-free path in sun

$\ell_T \sim \frac{1}{10^{24}(.6\times10^{-24})} \sim 1\ \mathrm{cm}$

(2) Light stays in pill box for time $dt = ds/c$



But $dL = ds \cdot \cos\theta$

Therefore $dt = dL / c \cdot \cos\theta$

(3) Consequently $dE_\nu = \dfrac{I_\nu \, d\nu \, dA \, \cancel{\cos\theta} \, d\Omega \cdot dL}{c \cdot \cancel{\cos\theta}}$

(4) Energy Density:

$$U_\nu \, d\nu = (dE_\nu / dV)_{4\pi}$$

- where $d\Omega$ integrated over 4π sterad
- $dV = dA \cdot dL$

$$dE_\nu = \frac{I_\nu \, d\nu \, dV \, d\Omega}{c}$$

$$\therefore U_\nu \, d\nu = (dE_\nu / dV)_{4\pi} = \left(\int_{4\pi} \frac{I_\nu \, d\Omega}{c} \right) d\nu$$

(5) Mean Intensity

Recall: $J_\nu = (1/4\pi) \int I_\nu \, d\Omega$

Therefore $4\pi J_\nu = c \, U_\nu$

or $\boxed{U_\nu = \left(\frac{4\pi}{c}\right) J_\nu}$

(6) Black-Body Radiation

$$J_\nu = B_\nu(T) \Rightarrow U_\nu = \frac{4\pi}{c} \times B_\nu(T)$$

(7) Integrate U_ν over all frequencies

As a result:

$$t_{diff} \sim \left(\frac{\overbrace{7\times10^{10}}^{\ell}}{1}\right)\left(\overbrace{\frac{7\times10^{10}}{3\times10^{10}}}^{R_\odot/c}\right) \sim 2\times10^{11}\ \mathrm{sec} \approx 10^{4}\ \mathrm{years}$$

But we neglected other emission and absorption processes (bound-free, free-free, . . .)

Best guess $\boxed{t_{diff} \simeq 2\times10^{4}\,\mathrm{yr}}$

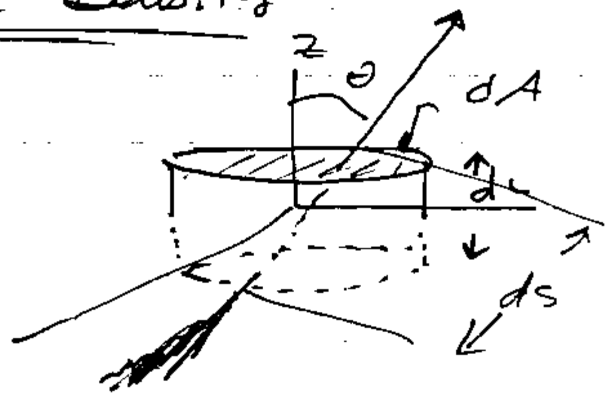
~~Consp~~ Consequences

(1) Comparison with other timescales:

$$t_{diff} << t_{KH} \simeq 2\times 7\ \mathrm{yr}.$$

What is total energy in photons?

Radiant Energy Density



Photons propagating in +z direction at angle θ wrt z axis encounter pill box with height dL, area dA.

(1) $dE_\nu = I_\nu \, d\nu \, dA \cos\theta \, d\Omega \, dt$

(~~photon~~ Energy in radiation crossing pill box into $d\Omega$)

$$U_{rad} = \int U_\nu \, d\nu$$

$$U_{rad} = \frac{4\pi}{c} \int B_\nu \, d\nu$$

$$= \frac{4\pi}{c}\left(\frac{\sigma}{\pi} T^4\right) = \left(\frac{4\sigma}{c}\right) \cdot T^4$$

(8) Total radiant energy $U_{rad} = a \cdot T^4$ where $a = 4\sigma/c = 7.56\times10^{-15}$ (cgs)
7.56×10^{-16} (mks)

$$E_{rad} = U_{rad} \cdot 4\pi R_\odot^3/3$$

$$\cancel{(9)\ P_{rad} = 7.56\times10^{-15}\times(2\times10^{7})^4}$$

Estimate E_{rad} by assuming $\langle T \rangle \simeq 5\times10^6$ K

$$\therefore E_{rad} \simeq 7.6\times10^{-15}\times(.5\times10^{7})^4 \cdot 4\times(7\times10^{10})^3$$

$$\boxed{E_{rad} \sim 6\times10^{45} \text{ ergs}}$$

Recall $E_{KE} \sim 3\times10^{48}$ ergs

Thus $\boxed{E_{rad} \sim 10^{-3} E_{KE}}$

$$L \sim \frac{E_{rad}}{t_{diff}} \sim 10^{33} \text{ ergs/s}$$ which is about right almost equal to $L_\odot = 4\times10^{33}$ ergs/s measured value.

How much energy radiated in 1 diffusion time?

$(\Delta E)_1 = L \cdot t_{diff} = E_{rad} \sim 10^{-3} E_{KE}$

Thus, to radiate away all thermal K.E. present in sun:

$(\Delta E) = L \cdot 10^3 \cdot t_{diff} = E_{KE}$ requires 10^3 diffusion times.

Nuclear Faucet

We saw last lecture that if gravitational potential energy is the only source of energy emitted by the sun, it would last in its present state no longer than one Kelvin-Helmholtz contract time scale, τ_{KH}. Since $\tau_{KH} \simeq 2\times10^7$ yr, this is unacceptable physical explanation, because age of the sun $t_\odot \simeq 4.6\times10^9$ y.

Solution: We need an energy source to balance radiative energy output.

Virial theorem: Recall $\frac{dE}{dt} = -\mathcal{L}_{rad} = +\frac{1}{2}\frac{d\Omega_g}{dt}$

Since $\mathcal{L}_{rad} > 0$, $\frac{d\Omega_g}{dt} < 0$ and star contracts.

Energy Source:

But core of a star is so hot ($T \sim 2\times10^7$ K for the sun) that, as we shall see, nuclear fusion reactions are possible. Fusion reaction result in the release of nuclear binding energy; i.e., decrease in nuclear potential energy.

Basic Reaction: $4\,({}_1H^1) \longrightarrow {}_2He^4 + Q \quad ({}_ZX^{\overbrace{Z+n}^{A}})$

Release of Q per unit time, integrated over all ${}_1H^1$ nuclei $= \mathcal{L}_{nuc}$

New Virial Theorem:

$$\frac{dE}{dt} = \underset{\text{sink}}{\underset{\Uparrow}{-\mathcal{L}_{rad}}} + \underset{\text{source}}{\underset{\Uparrow}{\mathcal{L}_{nuc}}} = \frac{1}{2}\frac{d\Omega_g}{dt}$$

If $\mathcal{L}_{nuc} = \mathcal{L}_{rad}$, then $\frac{dE}{dt} = 0$. Result is <u>no</u> <u>net</u> change in total energy. Result is that $\frac{d\Omega_g}{dt} = 0$ and star will not contract.

But, this mechanism works, provided nuclear energy supply lasts longer than $t_{age} \simeq 4.6 \times 10^9$ y.

<u>Stated differently</u>: Release of gravitational binding energy energy $B = |\Omega_g|$ cannot account for energy radiated by the sun in its 4.6×10^9 y lifetime!

$$|\Delta E_{TOT}| \simeq \mathcal{L}_\odot \times t_\odot = 4 \times 10^{33} \times 4.6 \times 10^9 \times 3 \times 10^7$$
$$\simeq 55 \times 10^{49} \simeq 0.6 \times 10^{51} \text{ ergs} \gg |\Omega_G| \simeq \text{few} \times 10^{48} \text{ ergs}$$

<u>Comparison of Energy Sources</u>

Gravity

Suppose $\Omega_G = -\frac{3}{2}\frac{GM^2}{R}$

Further suppose star contracts a significant fraction of its radius, say $\Delta R = -R/2$

$$E = \frac{1}{2}\Omega_G = -\frac{3}{4}\frac{GM^2}{R}$$

$$dE = \frac{3}{4}\frac{GM^2}{R^2}dR \Rightarrow \Delta E = \frac{3}{4}GM^2\int_R^{R/2}\frac{dR}{R^2}$$

$$\Delta E = -\frac{3}{4}\frac{GM^2}{R}$$

Energy released per unit mass

$$\left|\frac{\Delta E}{M}\right| = \frac{3}{4}\frac{GM}{R} \sim \frac{GM}{R} \text{ (order of mag.)}$$

$$\left|\frac{\Delta E}{M}\right| \sim \frac{6.7\times10^{-8} \times 2\times10^{33}}{7\times10^{10}} \sim 2\times10^{15}\,\mathrm{erg/g}$$

This is rough estimate of amount of energy that gravitational contraction extracts from stars like the sun.

Fraction of rest-mass Energy

$$\left|\frac{\Delta E}{Mc^2}\right| \sim \frac{GM}{Rc^2} \sim \frac{2\times10^{15}}{(3\times10^{10})^2} \sim 2\times10^{-6}$$

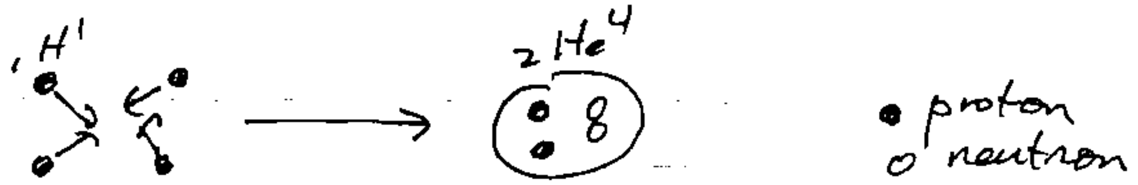
Fusion Reactions

By contrast to long-range gravitational interactions, fusion reactions between nuclei are short range. These occur when nuclei are within a few $\times10^{-13}$ cm of each other. At these scales, strong interaction turns on. Like gravity, strong interaction is attractive

protons attract protons
neutrons " neutrons
protons " neutrons

Just as in case of KH contraction, energy of individual nuclei decreases. And released energy is what powers stars

Essentially what happens



Energetics :

Start out with 4 free particles.

Special relativity tells us that

$$E = \gamma m_H c^2 \quad \text{for each proton, where}$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

On sun: $\langle \frac{1}{2} m_H v^2 \rangle = \frac{3}{2} kT \Rightarrow \left(\frac{v}{c}\right)^2 = \frac{3\left(\frac{k}{m_H}\right)T}{c^2}$

$$\left(\frac{v}{c}\right)^2 = \frac{3\times10^8 \times 2\times10^7}{(3\times10^{10})^2} = \frac{6\times10^{15}}{9\times10^{20}} \simeq .66\times10^{-5} \ll 1$$

Therefore most of the energy is rest-mass energy, i.e.

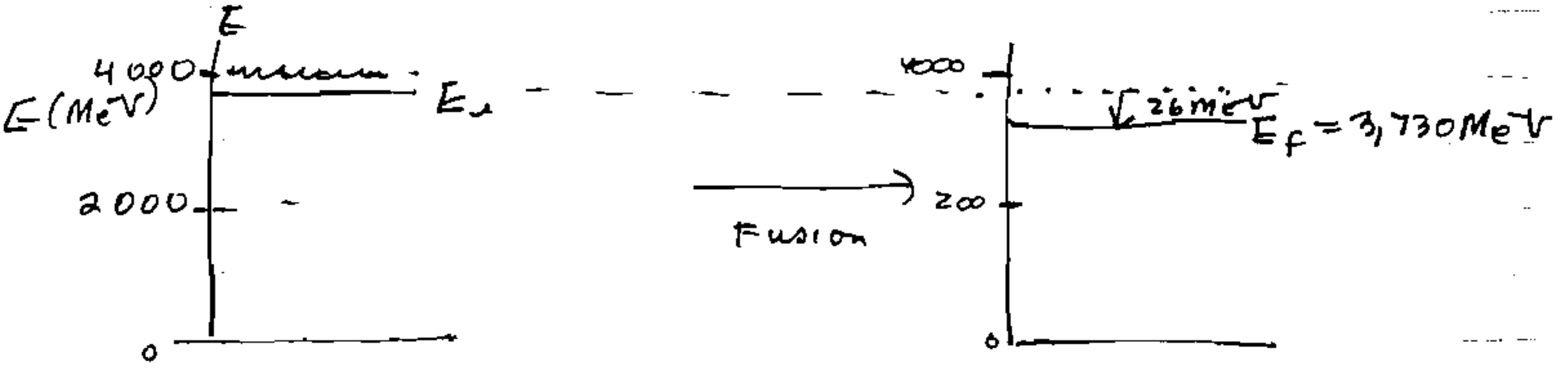
$$E \simeq mc^2 \quad (\gamma \simeq 1)$$

Proton rest-mass energy }

$$m_H c^2 = 1.67\times10^{-24} \times 9\times10^{20} = 15\times10^{-4}\,\text{ergs}$$

$$m_H c^2 = \frac{15\times10^{-4}\,\text{ergs}}{1.6\times10^{-12}\,\text{ergs/eV}} = 9.39\times10^{8}\,\text{eV} \simeq 939\,\text{MeV}$$

$$E_{TOT} = 4 \times m_H c^2 = 3{,}756\,\text{MeV}$$



initial

$4 \times {_1H^1}$

Final

$_2He^4$

Final mass, i.e., rest-mass energy of $_2He^4$ is lower than that of 4 protons by 26 MeV.

He nucleus is a bound particle containing 2 protons + 2 neutrons. No e^- since they have been ejected.

Rest-mass energy: $M_{He}c^2 = 2m_pc^2 + 2m_nc^2 + \Omega_{nuc}$ — nuclear potential energy

But $\Omega_{nuc} = -26\,MeV$ or binding energy $B_{He} = -\Omega_{nuc}$

For simplicity assume $m_n = m_p$;

Therefore: $M_{He}c^2 = 4m_pc^2 - B_{He}$

Conservation of energy:

$$E_i = 4m_pc^2 \quad \text{(initial)}$$
$$E_f = M_{He}c^2 + Q \quad \text{(final)}$$

Q = energy liberated

Energy Conservation: $E_i = E_f$

$$4m_pc^2 = M_{He}c^2 + Q$$
$$4m_pc^2 = 4m_pc^2 - B_{He} + Q$$
$$\Rightarrow \boxed{Q = B_{He}}$$

(Liberated energy = $_2He^4$ binding energy)

Mass difference:

$$\Delta Mc^2 = 4m_pc^2 - M_{He}c^2$$
$$\Delta Mc^2 = 4m_pc^2 - (4m_pc^2 - B_{He})$$

or $\Delta M_pc^2 = B_{He} = Q$

Conversion of mass into energy!

Comparison between Energy Production Efficiencies

1. Gravity $\left(\frac{|\Delta E|}{M}\right)_{grav} = 2\times10^{15}\ \text{erg/g}$

2. Nuclear

$$\left(\frac{|\Delta E|}{M}\right)_{nuc} = \left(\frac{Q}{M}\right) = \left(\frac{B_{He}}{4m_p}\right)$$

$$\left(\frac{|\Delta E|}{M}\right)_{nuc} = \left(\frac{26\times1.6\times10^{-6}}{4\times1.67\times10^{-24}}\right) = 6\times10^{18}\ \text{erg/g}$$

Therefore: $\left(\frac{|\Delta E|}{M}\right)_{nuc} \simeq 3500 \left(\frac{|\Delta E|}{M}\right)_{grav}$.

or $\frac{Q}{Mc^2} \simeq \frac{6.\cancel{8}\times10^{18}}{9\times10^{20}} \simeq 7\times10^{-3}$

$\boxed{\text{0.7\% of rest mass liberated as energy!}}$

Nuclear Time Scale

Total Energy Available: $E_{nuc} = \left(\frac{B_{He}}{4m_p}\right)\times(\underbrace{X\cdot M_\odot}_{M(H)})$

H mass

where $X \equiv \frac{\rho(H)}{\rho_{TOT}}$: H abundance by mass

$E_{nuc} = 6.8\times10^{18} \times (X\cdot2\times10^{33}) = 1.\cancel{2}\times10^{52}\,X$ ergs

$t_{nuc} = \frac{E_{nuc}}{L} = \frac{1.\cancel{2}\times10^{52}}{4\times10^{33}} \sim 3\times10^{18}\,X$ sec

$= 10^{11}\cdot X$ years

However, we shall see that this phase in the life of sun, i.e., fusion of $_1H^1 \to {}_2He^4$ ends when only 10% of sun's ~~$\mathcal{B}$~~ mass gets converted into He.

In that case $E_{nuc} \simeq 0.1 \times \left(\frac{B_{He}}{4m_p}\right)(X \cdot M_\odot)$

or $t_{main\text{-}sequence}(\odot) \simeq 10^{10}$ years

Closer Look at Nuclear Physics (Particle Physics)

(1) All ordinary matter made up of
Leptons (light)
Quarks (heavy)
Both are Fermions (Spin = 1/2 in units of $\hbar$)

(2) Fermions posses 4 types of charge

strength ↑
strong (color)
electric
weak
gravitational (mass)

(3) Boson Exchange:

Modern concept of force: we associate a force with each type of charge arising from exchange of Bosons between 2 particles:
- Bosons with odd integer spin (=1) {repulsion between like particles}
- Bosons with even integers spin (=2, 0) {attraction between like particles}

Goal → Unify 4 interactions → 1 super interaction

(4) Families

(A) Leptons: 6 kinds: come in 3 pairs

electron and its neutrino (e^-, ν_e)

muon and its neutrino (μ_e, ν_μ)

τ lepton " " " (τ, ν_τ)

(B) Quarks: 6 kinds: come in 3 pairs

up and down (u and d)

strange & charmed (s and c)

top & bottom (t and b)

(C) Unified Theories: Leptons, Quarks, paired in 3 generations

- e^-, ν_e and u, d quarks
- μ, ν_μ and s, c "
- τ, ν_τ and t, b "

(5) Protons and Neutrons

Primary stuff we will be dealing with since all ~~nu~~ nuclei are made up of protons and neutrons (p and n)

But, p and n are not elementary particles. Each consists of 3 quarks.

To see how this works, let's take a look at particle properties

Particle	Spin	Electric charge	Baryon number	Lepton Number
u quark	1/2	+2/3	+1/3	0
d quark	1/2	-1/3	+1/3	0
p (uud)	1/2	+1	1	0
n (udd)	1/2	0	1	0

Specifics

Spin

Proton: 2 u quarks (Pauli exclusion says they must be ↑↓)
+
1 d quark (spin = 1/2)
Net spin = $1/2 - 1/2 + 1/2 = 1/2$

Neutron: 2 d quarks (↑↓)
1 u quark
Net spin $1/2 - 1/2 + 1/2 = 1/2$

Charge

Proton: $\overset{2u}{2(2/3)} \overset{d}{-1/3} = 4/3 - 1/3 = +1$

Neutron: $\overset{2d}{2(-1/3)} \overset{u}{+2/3} = -2/3 + 2/3 = 0$

Baryn no.

Proton or neutron: $3(1/3) = 1$

Designation of any Particle: $_{chg}X^{\text{bary no.}}$
Proton $_1H^1$
Neutron $_0n^1$

(8) Leptons (electrons and neutrinos)

Electron is a true elementary particle. No internal structure

Particle	Spin	Electric charge	Baryon number	Lepton number
Electron	1/2	−1	0	1
Designation		$_{-1}e^0$	(shorthand e^-)	
Neutrino	1/2	0	0	−1
Designation		$_0\nu^0$	(shorthand ν_e)	

(9) Antimatter : Relativistic Quantum Mechanics requires that each particle has corresponding anti-particle with opposite electric charge.

Example: (A) Anti proton :
2 anti up quarks ($2\cdot\bar{u}$)
1 anti down quark ($1\bar{d}$)

Charge: $2(-2/3) + 1/3 = -1$
Baryon no. $3(-1/3) = -1$
Spin $= 1/2$
Designation $_{-1}\bar{p}^{-1}$

(B) anti-electron; positron
$_{+1}\bar{e}^0$ (e^+ shorthand)
Lepton no. $= -1$

Particles interact through exchange Forces

All these Fermions interact through emission and absorption of virtual Bosons.

(1) Uncertainty principle:

Suppose ΔE is uncertainty in particle energy. Then Δt is uncertainty about time in which particle possesed that energy, where

$$\Delta E \cdot \Delta t \geq \hbar$$
$$\text{or} \quad (\Delta p_x \Delta x) \geq \hbar$$

(2) Force: Q. M. picture push and pull mediated by exchange of Bosons (integer or 0 spin).

(3) Example: Coulomb force

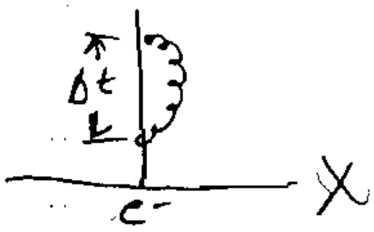
Coulomb force between e^- and e^-, e^- and p^+, etc. mediated by exchange of photons. But such photons are virtual, i.e., undetectable

(a) Not detected because they are emitted and absorbed on time-scale Δt, that is too short for detectability

(b) Suppose photon created with energy E, but absorbed within

$$\Delta t \leq \hbar/E$$

world line



(c) Dosorption. Photon violates energy conservation because energy $\Delta E = E$ extracted from stationary charge. Recall, only accelerated charges radiate detectable energy, not stationary charges.

But, if E is absorbed on timescale $\Delta t \leq \hbar/E$ after emission, then product, $\Delta E \Delta t < \hbar$, will not satisfy uncertainty principle and thus photon is virtual, it is impossible to detect.

(d) distance traveled by virtual photon:

$\Delta d = c\,\Delta t$, but if $\Delta t \leq \hbar/E$

$$\Delta d \leq \frac{c\hbar}{E}$$

Since $E = \hbar\nu$ we have

$$\Delta d \leq \frac{c}{\nu} = \lambda$$

Distance traveled by virtual photon is less than 1 wavelength.

Result:

Electric $\underset{\sim}{E}$ field transmitted by virtual photons is not wavey transverse electromagnetic. Rather it is longitudinal static Coulomb field.

(e) Long range

as E decreases, Δd (λ) increases, and virtual photon gets far from its source.