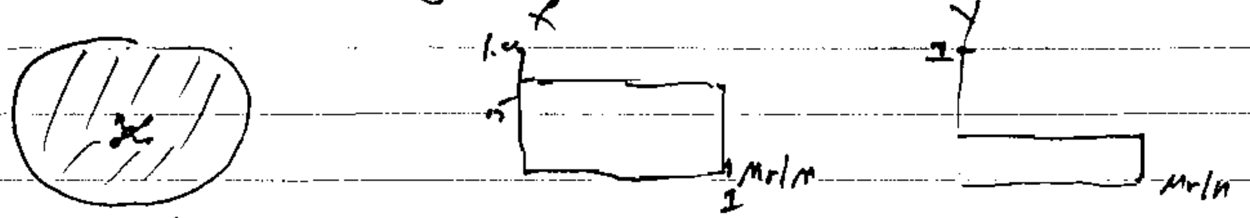
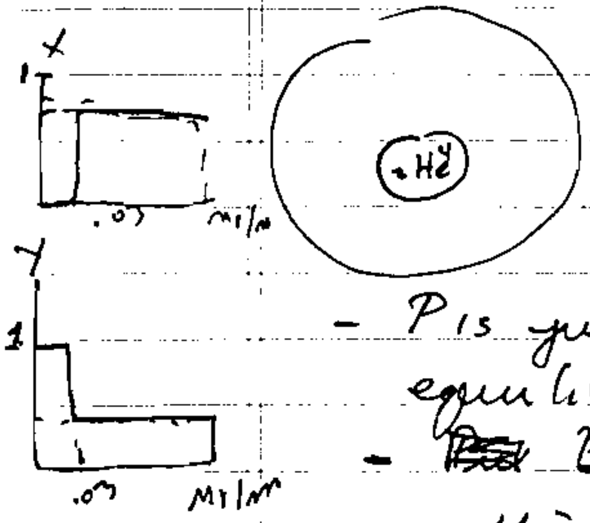
Evolution off the MS

(1) Zero-age MS: At start of its MS life, star has normal composition $X=0.7$, $Y=0.28$, $Z=0.02$ And ,H' burning core



t = t initial

(2) Development of isothermal $_2He^4$ core:

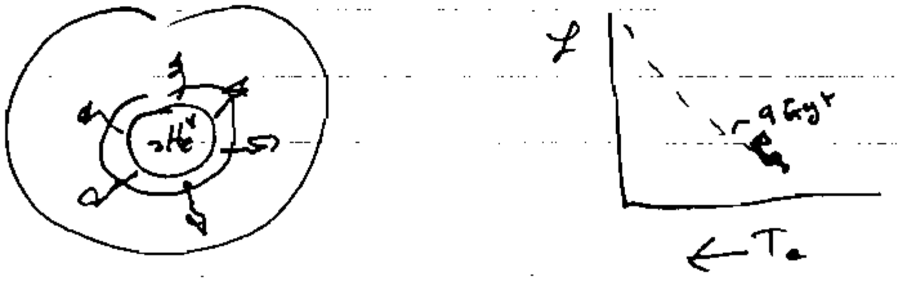


(a) But as time goes on composition of core is
$X=0$, $Y=0.98$, $Z=.02$

(b) Ideal gas law: $P=\frac{R\rho}{\mu}T$

- P is just given by hydrostatic equilibrium, so stays the same as before
- But μ has increased from $\mu \simeq 0.6 \rightarrow \mu = 4/3$
- To keep P the same, ρ & T must increase through core contraction.
- Energy generation: where does energy generation come from? Increased ρ & T result in ,H' burning shell.

Extra energy released

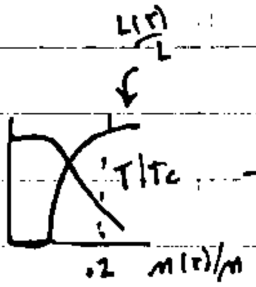


Star moves up & to the left on MS

(4) Isothermal core:

No energy generation in the core $\Rightarrow \epsilon = 0$

$$\frac{d\mathcal{L}}{dr} = 4\pi r^2 \rho \epsilon = 0 \quad \Rightarrow \mathcal{L}_r = \text{const}$$



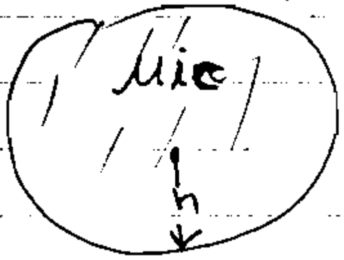
In fact $\mathcal{L}_r \simeq 0$ throughout inert $_2He^4$ core. But recall 4^{th} eq. of stellar structure

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L(r)}{4\pi r^2} \simeq 0 \Rightarrow T = T_i = \text{const}.$$

(5) Chandrasekhar-Schönberg Limit

For an isothermal core to support weight of envelope material above it in hydrostatic equilibrium, a pressure gradient must be generated by increase in density of the core. This can only happen by slow core contraction

But $_2He^4$ core contraction can only stabilize rest of star down to a critical radius. This is result of virial theorem of an object with external surface pressure! In this case



M_{env}

$$4\pi r_1^3 P_s(r_1) = 2E_{KE} + \Omega_G$$

(a) Uniform Sphere

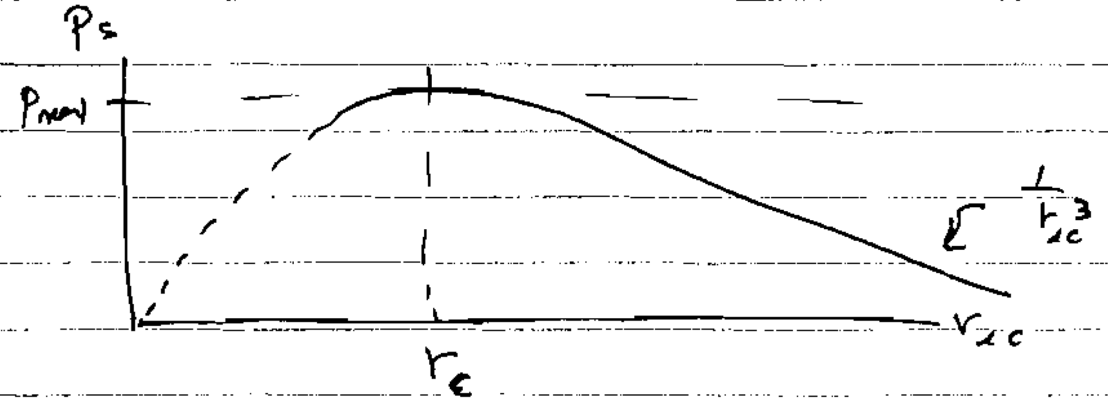
$$\Omega_G = -\frac{3}{5}\frac{G M_{ic}^2}{r_1}$$

$M_{ic} = \mu_{ic} m_H N$

$$E_{KE} = \frac{3}{2} N kT = \frac{3}{2}\frac{M_{ic}}{\mu_{ic}}\left(\frac{k}{m_H}\right) T_{ic}$$

As a result:

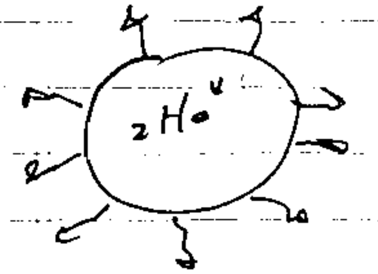
$$P_s = \frac{1}{4\pi}\left[\frac{3 R T_{1c} }{\mu_{1c}\, r_{1c}^3} - \frac{3}{5}\frac{M_{1c}^2}{r_{1c}^4}\right]$$



$r > r_c$: Core contraction results in rise in pressure on surface, sufficient to balance weight above it.

$r \leq r_c$: pressure exerted by isothermal core hits a maximum. No stable solutions at $r \leq r_c$ allowed. Gravity overwhelms thermal energy, ~~and~~ Maximum mass $\frac{M_c}{M} = 0.37\left(\frac{\mu_{env}}{\mu_{1c}}\right)^2 \simeq 0.08$

(6) Core Contraction: The core now undergoes more rapid Kelvin-Helmholtz contraction ($P_s = 0$) on a KH time-scale! ($\sim 10^7$ yrs.)



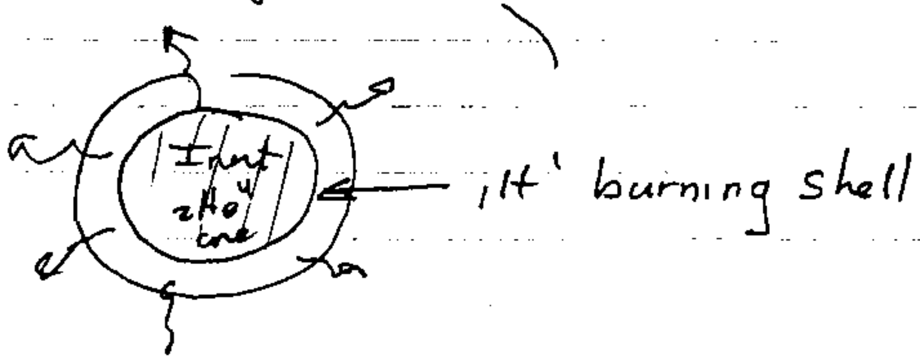
- Luminosity still being emitted by core, but $L_{nuc} = 0$
- $\frac{dE}{dt} = -L_{rad}$

again: $E = E_{KE} + \Omega_G = \frac{1}{2}\Omega_G$

$$\frac{1}{2}\frac{d\Omega_G}{dt} = -\mathcal{L}_{rad}$$

and core continues to contract.

- As core contracts, T rises in $_1H^1$ layers above it. A new high T a ~~thin~~ dense $_1H^1$ shell surrounding core ignites.



- Contraction of $_2He^4$ core (and surrounding $_1H^1$ shell) aided by deposition of $_2He^4$ "ash" on the core

Result: Gravitational field felt by $_1H^1$ burning shell gets stronger. Also–

- ρ and T in shell increase to balance stronger gravitational force

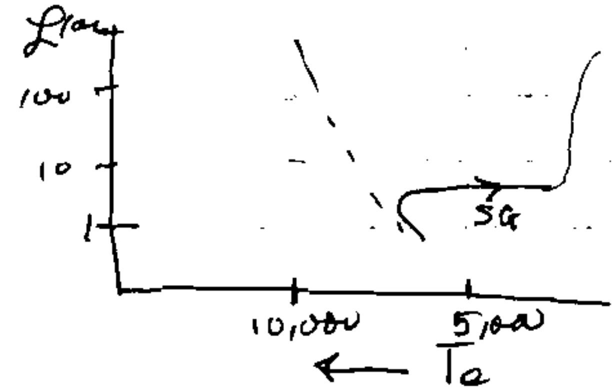
• At this point $(\mathcal{L}_{nuc})_{shell} > \mathcal{L}_{rad}$.

• Normally in H-burning core, this is alleviated by expansion of core accompanied by decrease in T. But in shell / core geometry this cannot occur!

• So what happens to $\Delta\mathcal{L} = \mathcal{L}_{nuc} - \mathcal{L}_{rad}$?
  - It cannot be radiated away since $\mathcal{L}_{rad}$ fixed by global structure of the star
  - Instead, $\Delta\mathcal{L}$ heats up intermediate layers at $R > R_{shell}$. As a result the layers expand! $\mathcal{L}_{rad} \approx$ const. $\Delta\mathcal{L}$ converted into PdV work!
  - Result: R increase while $\mathcal{L}_{rad}$ is constant

• Subgiant
  Recall $\mathcal{L} = 4\pi R^2 \sigma T_e^4$. As a result T_e decreases. Star moves horizontally to the right as it enters subgiant phase!

HR Diagram



Why does the star ascend the giant branch?

- Shell Source

  As shell source contracts, T increases and as a result so does L_{nuc}.

- Envelope

  As ΔL increases, envelope of star gets pushed out further. As a result, initially that envelope, in which gas was ionized, is now lower in T.

  Consequences

  (a) H^- opacity increases

  (b) Gas becomes partially ionized

- Convection

  In that case entire envelope of star becomes convective: Why?.

Recall:
$$\frac{d\ln T}{d\ln P} = \frac{3}{16\pi a c G} \cdot \frac{\kappa P}{T^4} L(r)$$

Star becomes convective when $\frac{d\ln T}{d\ln P} > \left(\frac{\gamma-1}{\gamma}\right)$ where $\gamma = c_p/c_v$.

- So, increase in H^- opacity increases κ, which increases $\frac{d\ln T}{d\ln P}$ → more unstable to convection

- Partial ionization drives $\gamma \to 1$ which also enhances convection.

why?

- Squeeze a sphere of fully ionized gas: Work goes into increasing E_{KE} and T ↑

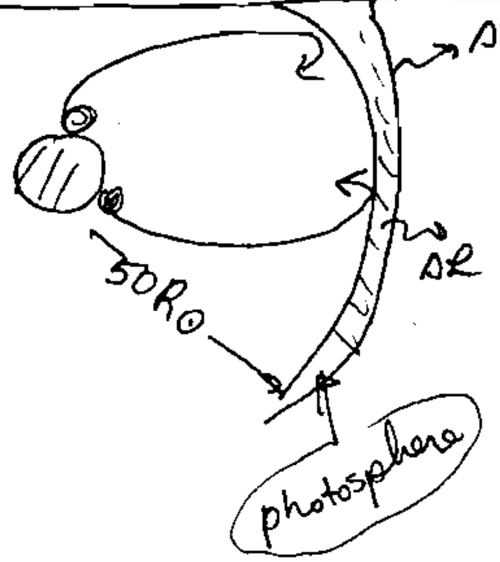
- Squeeze a sphere of partially ionized gas: Work goes into ionizing neutral H atoms, i.e., into internal degrees of freedom rather than increasing center-of-mass motions, hence T.
  - Lg. energy input with small T increase

$$dQ = c_v dT + P dV$$

Since adiabatic, $dQ = 0$ and $dT = \frac{-P dV}{c_v}$

Since $P dV$ may be large, dT is small only if $c_v \to \infty$. Since $\boxed{c_p = c_v + \frac{R}{\mu}}$
$\gamma = \frac{c_p}{c_v} \to \frac{\infty}{\infty} \to 1$

Result: Entire envelope of star becomes convective



- ΔL which previously went into work that pushed out envelope, now gets convected to radiative photosphere

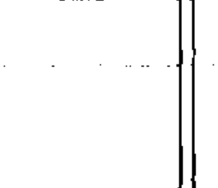
- ΔL gets radiated away from photosphere!

Hayashi Temperature:

So luminosity of star must climb dramatically. But on H R diagram star ascends red giant branch almost vertically; i.e., at a constant effective temperature: why?

(1) In photosphere:

$$\frac{dP}{dr} = -\frac{G M(r)}{r^2}\rho \simeq -g\rho \quad \left\{ \text{since } \frac{M(r)}{r^2} \simeq \text{const} \right\}$$



Integrate over scale-height H

$$\int_P^0 dP = -g\rho H \Rightarrow \boxed{P = g\rho H}$$

(2) mean-free path:

Earlier I showed that $H \sim l_\lambda$

Therefore $P \simeq \rho g l_\lambda$.

Since $P = \frac{R}{\mu}\rho T_e$, we have

$$\boxed{T_e = \frac{\mu}{R} g l_\lambda}$$

(3) H^- mean free path:

For H^- ion l_λ is very sensitive to T_e but insensitive to ρ. $l_\lambda = \frac{\text{const}\, T_e^{-b}}{\rho^a}$

$b \gg 1$; $a \ll 1$.

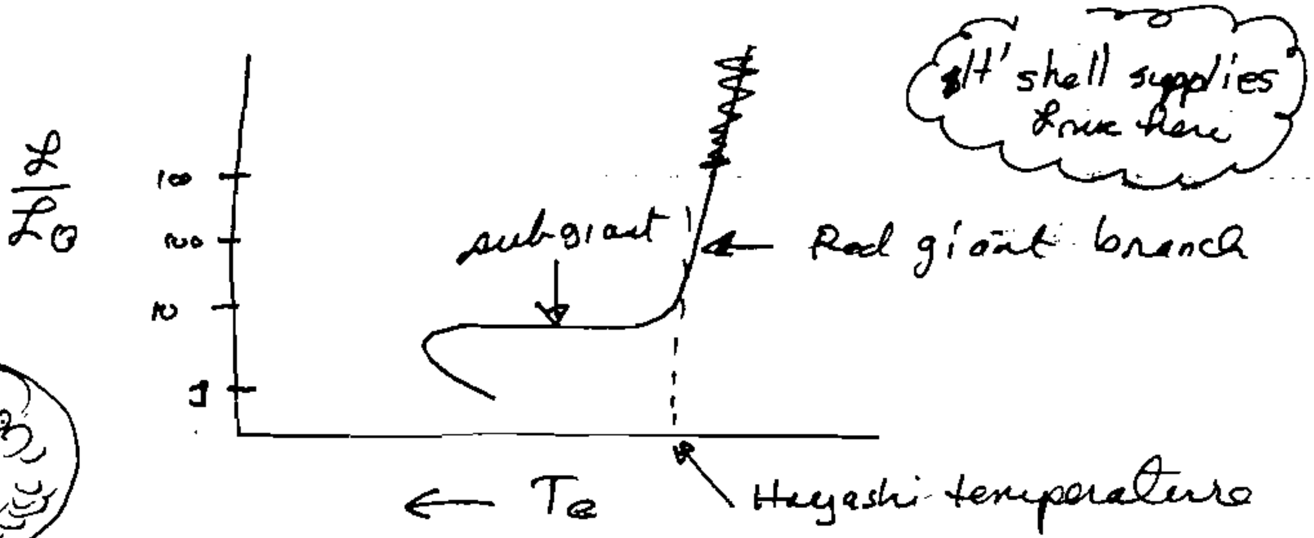
Therefore: $T_e \propto \frac{T_e^{-b}}{\rho^a} \Rightarrow T_e^{b+1} \propto \rho^a$

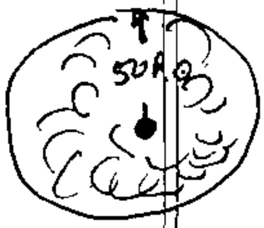
$$\boxed{T_e \propto (\rho)^{\frac{a}{b+1}}} \quad \text{where } \frac{a}{b+1} \ll 1$$

H^- ion is H atom with extra bound electron
$H^0 + e^- \rightleftarrows H^- + \gamma$
$\chi_{H^-} = 0.754\,eV$
any photon with $\lambda < 1.64\,\mu m$ can be absorbed

opacity source in cool stars

So T_e cannot fall below this "Hayashi T_e".





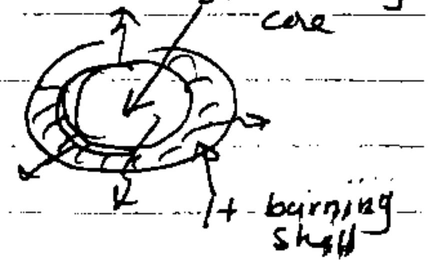
Back to Core

Core continues to contract.

- Low mass star:
  Density builds up to such high values that electrons become degenerate. Core becomes a white dwarf
- High mass star:
  Lower densities assure ideal gas law is valid and density keeps on increasing.

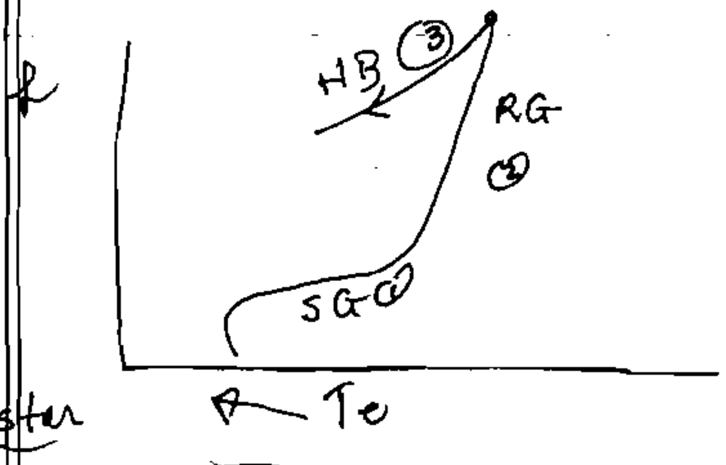
In both cases $T \rightarrow 10^8 K$ and 3-α reaction ignited!

Onset of $2He^4$ burning:
- Core heatsup.
- Now core expands



Results of core expansion
$T \downarrow$ even though 2 energy sources,
shell source luminosity decreases

Result: Excess ΔR, that was convected to the surface before, decreases. So lowered $\mathcal{L}$ of star is too little to keep star in its extended red giant phase.
Result:
Star shrinks in size
Star becomes dimmer: But Te actually increases

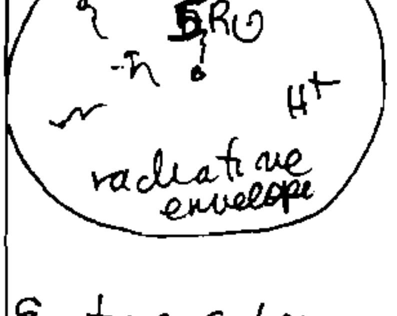


$\mathcal{L} = 4\pi R^2 T_e^4$

$$T_e \propto \frac{\mathcal{L}^{1/4}}{R^{1/2}}$$

more sensitive to R than $\mathcal{L}$

HB star



Entire star

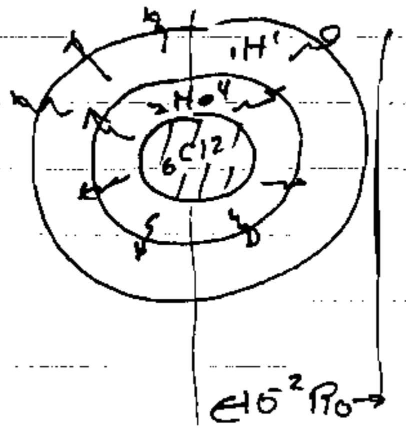
Core

Asymptotic Branch Ascension

Eventually $_2He^4$ in core is exhausted, and we end up with an inert $_6C^{12}$ core.

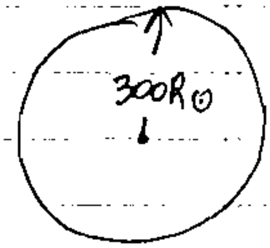
Triple α process: $3(_2He^4) \rightarrow {}_6C^{12} + Q$

(a) Structure: Double Shell Source



- Outer $_1H^1$ burning shell
- Inner $_2He^4$ " "
- Inert $_6C^{12}$ core

(b) $_6C^{12}$ core contracts: (History repeats itself)



Again more ΔL released that exceeds amount of L_r that radiative transfer can support. Star becomes convective again & excess ΔL transported by convection to photosphere and radiated away. Star ascends RG branch again "asymptotic giant branch"