HOMEWORK ASSIGNMENT #1
October 01, 2009

1. Solve Problem 6.1.7 of Arfken and Weber (AW).

[Hint: Combine the two series, utilizing the fact that $\cos\theta + i\sin\theta = \exp(i\theta)$] .

2. (a) Solve Problem 6.1.8 of AW.

(b) Examine the limiting behavior of formulae (a) and (b) of this problem as p tends to 1. For the resulting formulae to be valid, what restrictions should be imposed on x?

(c) Can you obtain the same limiting forms for these sums from formulae (a) and (b) of the previous problem by letting N go to infinity?

3. The imaginary part of an analytic function $f(z)$ is $i\, e^{-y} \sin x$.
What is $f(z)$?
[Hint: Exploit Cauchy-Riemann conditions].

4. Using "Schwarz reflection principle" --- Eqn. 6.59 of AW --- as applied to the Gamma function, show that

(i) $|\Gamma(\tfrac{1}{2} + i y)| = [\pi / \cosh(\pi y)]^{1/2}$, and

(ii) $|\Gamma(i y)| = [\pi / y \sinh(\pi y)]^{1/2}$.

In passing, examine the limiting behavior of these function as y tends to zero or to infinity.

5. Derive Laurent expansion(s) of the function $1 / [z(1-z)(2+z)]$ around the point $z = 0$, such that
(a) the first expansion is valid for $0 < |z| < 1$,
(b) the second is valid for $1 < |z| < 2$, and
(c) the third is valid for $|z| > 2$.

Interpret the values of the coefficient a_{-1} obtained in each of these cases in terms of the residues of the function $f(z)$ at its singular points.

6. (a) Locate the poles of the function $\cot z$ and evaluate the residues of this function at those poles.

(b) Expand $\cot z$ as a Laurent series around the point $z = 0$, evaluating at least <u>three</u> non-vanishing terms of the series. What is the range of validity of this series?