HOMEWORK ASSIGNMENT #2

Oct. 08, 2009

1. Using the calculus of residues, show that

$$\int_0^{2\pi} \frac{\sin^2\theta \, d\theta}{a + b\cos\theta} = \frac{2\pi}{b^2}\left(a - \sqrt{a^2 - b^2}\right)$$

$$(a > b > 0).$$

Examine the limiting value of this result as $b \to 0$.

2. Evaluate the contour integral

$$\oint_C \frac{\cosh(az)}{\cosh(\pi z)}\, dz \qquad (|a| < \pi)$$

along the contour sketched in Fig. 7.5 of AW, to show that, for $|a| < \pi$,

$$\int_{-\infty}^{\infty} \frac{\cosh(ax)}{\cosh(\pi x)}\, dx = 2\sum_{n=0}^{\infty} (-1)^n \cos\left(n + \tfrac{1}{2}\right) a = \sec\left(\tfrac{1}{2} a\right).$$

3. Solve problem 2, using a rectangular contour defined by its corners $(-R, R, R+i, -R+i)$, with $R \longrightarrow \infty$.

4. Evaluate the contour integral

$$\oint_C e^{iz^2}\, dz$$

along a contour that consists of the radial line $\theta = 0$ going from $x = 0$ to $x = R$; the circular arc $r = R$ going from $\theta = 0$ to $\theta = \pi/4$ and the radial line $\theta = \pi/4$ going from $r = R$ to $r = 0$, with $R \to \infty$. Deduce that

$$\int_0^\infty \cos(x^2)\, dx = \int_0^\infty \sin(x^2)\, dx = \sqrt{\frac{\pi}{8}}.$$

5. Evaluate the <u>principal value</u> of the contour integral

$$\oint_C \frac{z + i e^{iz}}{z^3}\, dz$$

along the contour shown in Fig. 7.7 of Arfken, to show that

$$\int_{-\infty}^{\infty} \frac{x - \sin x}{x^3}\, dx = \frac{\pi}{2}.$$

6. Using a suitable contour, evaluate the integral

$$\oint_C \frac{z^{1/2}}{(z+1)^2}\, dz ,$$

to show that

$$\int_0^{\infty} \frac{x^{1/2}}{(x+1)^2}\, dx = \frac{\pi}{2}.$$