HOMEWORK ASSIGNMENT # 6

November 12, 2009

1. Determine the values of the parameters α and β for which the following four functions of x are <u>NOT</u> linearly independent

$$y_1(x) = x \cosh x + \sinh x,$$

$$y_2(x) = x \sinh x + \cosh x,$$

$$y_3(x) = (x+\alpha)\, e^{x}, \quad \text{and}$$

$$y_4(x) = (x+\beta)\, e^{-x}.$$

2. Show that the differential equation

$$y'' + P(x)\, y' + Q(x)\, y = 0$$

can be transformed into

$$z'' + q(x)\, z = 0,$$

where

$$q(x) = Q(x) - \frac{1}{2} P'(x) - \frac{1}{4} P^2(x).$$

[Hint: Set $y(x) = f(x)\, z(x)$ and select $f(x)$ so that the term in z' vanishes.]

3. The simplest solutions of the Legendre equation

$$(1-x^2)\,y'' - 2x\,y' + \ell(\ell+1) = 0$$

are

(i) for $\ell = 0$, $Y_1(x) = 1$, and

(ii) for $\ell = 1$, $Y_1(x) = x$.

Determine the <u>other</u> solutions pertaining to these cases.

4. Find the general solution of the differential equation

$$x^2 y'' - x y' + y = x$$

that satisfies the conditions $y(1) = 1$ and $y(e) = e$.

5. Solve the differential equation

$$\frac{d}{dx}\left(\frac{1}{y}\frac{dy}{dx}\right) + \frac{2a}{\tanh(2ax)}\left(\frac{1}{y}\frac{dy}{dx}\right) = 2a^2 \qquad (a>0),$$

adjusting the "constants of integration" so that, for small x, $y(x)$ is linear in x.

6. As was shown in the class, the particular solution of the differential equation

$$y''(x) + p_1(x)\, y'(x) + p_0(x)\, y(x) = f(x)$$

is given by the expression

$$y_p(x) = -y_1(x) \int^x \frac{y_2(t)\, f(t)}{W(t)}\, dt + y_2(x) \int^x \frac{y_1(t)\, f(t)}{W(t)}\, dt.$$

Derive this expression by following this procedure:

(i) Write $y_p(x) = y_1(x)\, v(x)$ and substitute it into the given equation, to obtain a <u>first-order</u> diff. eqn. for $v'(x)$.

(ii) Solve this equation for $v'(x)$, integrate $v'(x)$ to get $v(x)$ and, hence, $y_p(x)$.

→ [To do this problem, you'll have to exploit some salient properties of the function W.]