Final Exam 2009:

1 a) $\frac{35 \text{ events}}{74 pb \cdot 100/pb} = 0.5\% = 5\ 10^{-3}$

b) NO. For a given 3W final state, there is only one way to get same sign dilepton.

$W^+ W^+ W^-$
$W^+ \to$ lepton
$W^+ \to$ lepton

c) Verify WW:

uU	19.5 pb
dD	12.7 pb
sS	3.1 pb
cC	1.7 pb
	37.0 pb

37.0 pb $* 2 = 74$ pb

qQ vs Qq

d)

$W^+W^+W^-$

uD	$3.8\ 10^{-2}$ pb
uS	$0.14\ 10^{-2}$ pb
cD	$0.02\ 10^{-2}$ pb
cS	$0.23\ 10^{-2}$ pb
	$\simeq 4.2\ 10^{-2}$ pb

$* 2 = 8.4\ 10^{-2}$ pb

$W^+W^-W^-$

dU	$2.0\ 10^{-2}$ pb
dC	$0.06\ 10^{-2}$ pb
sU	$0.03\ 10^{-2}$ pb
sC	$0.2\ 10^{-2}$ pb
	$\simeq 2.2\ 10^{-2}$ pb

$* 2 = 4.4\ 10^{-2}$ pb

$t\bar{T}W^+$ $\simeq 2 * 2.1\ 10^{-1}$ pb $= 4.2\ 10^{-1}$ pb

$t\bar{T}W^-$ $\simeq 2 * 1.0\ 10^{-1}$ pb $= 2.0\ 10^{-1}$ pb

(e) The visible crossection in $\ell^+\ell^+$ combined with $\ell^-\ell^-$ is then

$t\bar{t}W = 6.2\ 10^{-1}\text{pb} * 5\ 10^{-3} = 3.1\ 10^{-3}\text{pb} = 3.1$ fb

$WWW = 12.8\ 10^{-2}\text{pb} * 5\ 10^{-3} = 6.4\ 10^{-4}\text{pb} = 0.64$ fb

To see 10 events, we thus need:

$t\bar{t}W \sim 3.3/\text{fb}$ of data @ 14TeV

$WWW \sim 15.6/\text{fb}$

As an aside, the difference in $W^+W^+W^-$ and $W^-W^-W^+$ is due to the valence quarks of the proton

$\left.\begin{matrix} u \\ u \end{matrix}\right\} \to \bar{d} \to W^+$

$d \to \bar{u} \to W^-$

The $W^+W^+W^-$ has a larger crossection because the u-quark is more likely to find in the proton than the d quark.

(f) The WW PAS note requires a jet veto in order to reduce $t\bar{t}$ background.

Our final states are:

$t\bar{t}W \to \ell^+\ell^+ b\bar{b} jj$ most of the time.

$WWW \to \ell^+\ell^+ jj$

This will make it very difficult to suppress $t\bar{t}$ bkg!

A jet veto would eliminate our signal.

Requiring a b-jet "far enough" away in ϕ, η from the ℓ^+ might help discriminate against top and certainly distinguishes WWW from $t\bar{t}W$.

$\boxed{2}$ a)

	ρ^+	ρ^0	ρ^-
ρ :	$-u\bar{d}$	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	$\bar{u}d$
ω :		$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	
ϕ :		$s\bar{s}$	

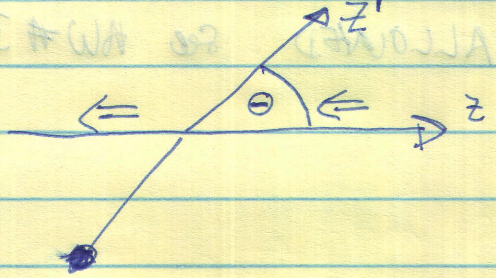
b) $\phi_{color} = \frac{1}{\sqrt{3}}(R\bar{R} + B\bar{B} + G\bar{G})$ singlet

c) $J=1$ with $L=0$, $S=1$ $\Rightarrow$ $\chi_{10} = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$

$\boxed{3}$ Two possibilities, by helicity conservation

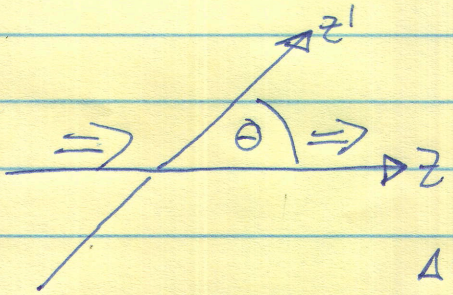
$$e_L^+ e_R^- \to \Upsilon(4S) \to B\bar{B}$$

$$e_R^+ e_L^+ \to \Upsilon(4S) \to B\bar{B}$$



$J_z = -1 \Rightarrow J_{z'} = \emptyset$

Amplitude $\propto d_{0-1} = \frac{-\sin\theta}{\sqrt{2}}$



$J_z = +1 \Rightarrow J_{z'} = \emptyset$

Amplitude $\propto d_{01} = \frac{+\sin\theta}{\sqrt{2}}$

Either way $\sigma \propto \sin^2\theta$

$$\sigma \propto |d_{0-1}|^2 + |d_{01}|^2 \Rightarrow \sigma \propto \sin^2\theta$$

$\boxed{4}$ A, B, C $J^P = 0^-$

D $J^P = 1^-$

a) $A \to BC$ $P = -1 \Rightarrow P' = (-1)(-1)(-1)^L$
$= (-1)^L$
$L = 0$ & $P' = +1$
$\Rightarrow$ NOT ALLOWED

b) $D \to BC$ $P = -1 \Rightarrow P' = (-1)(-1)(-1)^L$
$L = 1$ & $P' = -1 \Rightarrow$ ALLOWED

c) $D \to AA$; example $\rho^0 \to \pi^0 \pi^0$
NOT ALLOWED see HW # 3

$\boxed{5}$ $\Omega^- = sss$ state

$m_{\Omega^-} = 1672.5$ MeV

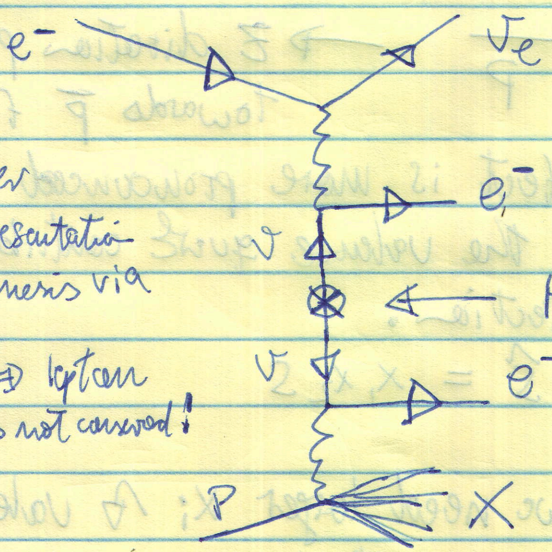
lifetime $= 0.8 \; 10^{-10}$ s $\Rightarrow$ weak decay

dominant decay modes all violate strangeness, i.e. are weak decays of $s \xrightarrow{W} u$

The reason for this is that the Ω is the lowest mass baryon with 3 s-quarks, and decays to ΞK are kinematically not allowed because of low Ω mass. Baryon conservation requires the final state to include a baryon. $\Rightarrow$ weak decay to Ξ $(S=-2)$ or ΛK

$S=-1$ $S=-1$

$\boxed{6}$ $e^- p \to \nu_e \, l^- l^- X^{++}$



a) For this process to take place, we need the ν_e to be a majorana neutrino.

← Majorana neutrino

b) Works just as well for $\mu^-\mu^-$ in the final state, with ν_μ being a majorana neutrino, of course!!!

Note: Remember Trevor's presentation about Baryogenesis via leptogenesis. Majorana $\nu \Rightarrow$ lepton number is not conserved!

a) $\overset{\Rightarrow}{\longrightarrow} \; \overset{\Rightarrow}{\longleftarrow}$

$\quad \bar{u} \qquad d$

Therefore the W^- will be fully polarized along the direction of motion of $\bar{u}$.

b) In pp collision there is an equal probability for pulling the $\bar{u}$ from either proton. If z is the beam line, we then get $J_z = +1$ and $J_z = -1$ with equal probability.

$\Rightarrow$ No forward-backward charge asymmetry for the W production cross section.

In $\bar{p}p$ you can pull the $\bar{u}$ from the $\bar{p}$. There is thus a tendency to find $J_z = +1$ more likely than $J_z = -1$ if we define z as

$\underset{\bar{p}}{\longrightarrow} \; \underset{p}{\longleftarrow} \qquad \longrightarrow z$ direction pointing towards $\bar{p}$ flight direction.

This polarization effect is more pronounced the more important the valence quark contribution to the W cross section.

$$m_W \sim \hat{s} = x_1 x_2 s$$

$\Rightarrow$ At lower s we need larger x_i $\Rightarrow$ valence quarks are more important.

c) $\Rightarrow$ Polarization effect more important at SppS than at Tevatron than at an LHC with $p\bar{p}$!!!

$\boxed{8}$ $\pi\pi$ states must be symmetric under $1 \leftrightarrow 2$

because $J(B) = \emptyset$ so $L(\pi\pi) = \emptyset$

Denote $|\pi^+\pi^-\rangle = |+-\rangle$

$|\pi^+\pi^0\rangle = |+0\rangle$

$|\pi^0\pi^0\rangle = |00\rangle$

$$|+-\rangle = \frac{1}{\sqrt{2}}|+-\rangle + \frac{1}{\sqrt{2}}|-+\rangle$$

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{6}}|20\rangle + \frac{1}{\sqrt{2}}|11\rangle + \frac{1}{\sqrt{3}}|00\rangle\right]$$

$$+ \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{6}}|20\rangle - \frac{1}{\sqrt{2}}|11\rangle + \frac{1}{\sqrt{3}}|00\rangle\right]$$

$$= \frac{1}{\sqrt{3}}|20\rangle + \frac{2}{\sqrt{3}}|00\rangle$$

$$|00\rangle = \sqrt{\frac{2}{3}}|20\rangle - \frac{1}{\sqrt{3}}|00\rangle$$

This refers to Isospin $|00\rangle$

$$|+0\rangle = \frac{1}{\sqrt{2}}|+0\rangle + \frac{1}{\sqrt{2}}|0+\rangle = |21\rangle$$

Therefore $B^+ \to \pi^+\pi^0$ measures the reduced matrix element $S_{3/2}$

$$\langle 1/2 | S_{3/2} | 2 \rangle = A^{+0}$$

Now, you have a choice. You can either assign isospin to the transition operator $S_{3/2}$ or pull that $\Delta I = 3/2$ into the B^+ wave function. The latter is sometimes referred to as adding a $\Delta I = 3/2$ "spurion" to the $|1/2\ 1/2\rangle$ state

Let's do it that way:

$$A^{+0} = \langle +0 | S_{3/2} | B^+ s\rangle$$

$$|B^+ s\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle = \frac{\sqrt{3}}{2}|2\,1\rangle - \frac{1}{2}|1\,1\rangle$$

$$|+0\rangle = |2\,1\rangle$$

$$\Rightarrow \quad A^{+0} = \frac{\sqrt{3}}{2}\langle 2\,1 | S_{3/2} | 2\,1\rangle = \boxed{\frac{\sqrt{3}}{2} S_{3/2} = A^{+0}}$$

Having introduced the "spurion" we now have reduced $S_{3/2}$ to be an Isospin conserving operator!

$B^0 \to \pi^+\pi^-$ has a $\Delta I = 3/2$ and a $\Delta I = 1/2$ amplitude. We thus introduce two "spurions"

$$|s\rangle = |\tfrac{3}{2}\,\tfrac{1}{2}\rangle$$
$$|s'\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$A^{+-} = \langle +- | S_{3/2} | B^0 s\rangle + \langle +- | S_{1/2} | B^0 s'\rangle$$

$$|B^0 s\rangle = \frac{1}{\sqrt{2}}|2\,0\rangle + \frac{1}{\sqrt{2}}|1\,0\rangle$$
$$|B^0 s'\rangle = \frac{1}{\sqrt{2}}|1\,0\rangle + \frac{1}{\sqrt{2}}|0\,0\rangle$$
$$|+-\rangle = \frac{1}{\sqrt{3}}|2\,0\rangle + \frac{2}{\sqrt{3}}|0\,0\rangle$$

$\Rightarrow$ $|1\,0\rangle$ states in $|B^0 s^{(\prime)}\rangle$ give you nothing!

$$\Rightarrow \quad \boxed{A^{+-} = \frac{1}{\sqrt{6}} S_{3/2} + \frac{1}{\sqrt{3}} S_{1/2}}$$

We then play the same game for $|\pi^0\pi^0\rangle$

$$A^{00} = \langle \pi^0\pi^0 | S_{3/2} | B^0 S\rangle + \langle \pi^0\pi^0 | S_{1/2} | B^0 S'\rangle$$

The algebra I'll skip, and you get

$$\boxed{\begin{aligned} A^{00} &= \frac{1}{\sqrt{3}} S_{3/2} - \frac{1}{\sqrt{6}} S_{1/2} \\ A^{+0} &= \frac{\sqrt{3}}{2} S_{3/2} \\ A^{+-} &= \frac{1}{\sqrt{6}} S_{3/2} + \frac{1}{\sqrt{3}} S_{1/2} \end{aligned}}$$

From this you can immediately see that 8c) is correct.

$$\boxed{\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0}}$$

We thus have an amplitude triangle relationship!!!

b) The tree diagrams have combinations of $I=1$ in the final state while the $u\bar{u}$ or $d\bar{d}$ from the gluon is an isospin $\emptyset$.
The penguin digram thus contributes only to $\Delta I = 1/2$

There is no penguin diagram in $B^+ \to \pi^+\pi^0$ because there is no $\Delta I = 1/2$ contribution as we have shown above.

Points

Problem 1	a) b) c) d) e) f)	26p total 26/100
2	10/100	
3	8/100	
4	10/100	
5	10/100	
6	8/100	
7	8/100	
8	20/100	