Homework # 2

Problem 1

(a) intrinsic : $\frac{\sigma(P_T)}{P_T} = \frac{\sigma_{R\phi} P_T}{0.3 BL^2} \sqrt{\frac{720}{N+4}}$

multiple scattg : $\frac{\delta(P_T)}{P_T} = \frac{0.05}{BL} \sqrt{\frac{1.43 \cancel{X}}{X\emptyset}}$

For CMS we have:

$\sigma_{R\phi} \sim 40\,\mu m$ we take this to be the same for all layers, even though it isn't.

$B = 4T$

$L = 110$ cm

The tracker consists in the barrel of:

$$\left.\begin{array}{l} \text{Pixel} = 3 \text{ layers} \\ \text{TIB} = 4 \text{ layers} \\ \text{TOB} = 6 \text{ layers} \end{array}\right\} N = 13 \;@\; \eta = 0$$

At $\eta = 1.6$, tracks go through 3 pixel layers, 1 TIB, $\emptyset$ TOB but instead 11 disks

$\Rightarrow N = 15 \;@\; \eta = 1.6$

The L that enters the intrinsic resolution formula is the same for $\eta = 0$ and $\eta = 1.6$ because it comes from the radius measurement.

Problem 1 a) continued:

$$\frac{\sigma(P_T)}{P_T} = \frac{40\,\mu m}{0.3\ 4T\ 1.2\,m^2}\sqrt{\frac{720}{N+4}}\ P_T$$

$$\sqrt{\frac{720}{17}} = 6.5$$

this is the same for the precision we are after.

$$\sqrt{\frac{720}{19}} = 6.2$$

$$\frac{4\ 10^{-5}}{0.3 \cdot 4 \cdot 1.2} = 2.8\ 10^{-5}$$

$$\boxed{\frac{\sigma(P_T)}{P_T} \sim 2\ 10^{-4} \cdot P_T}$$

roughly the same for $\eta = 0$ and $\eta = 1.6$

For multiple scattering:

$$\frac{X}{X_0}(\eta=0) = 0.4 \qquad \frac{X}{X_0}(\eta=1.6) \simeq 1.4$$

To be fair, we note that each layer only sees the material inside of itself. So let's use 0.4 and 1.0 for the radiation length.

$$\frac{\sigma(P_T)}{P_T} = \frac{0.05}{4 \cdot 1.1} \cdot \sqrt{1.43 \cdot 0.4} = 0.01 \cdot 0.76 = 9\ 10^{-3}\ @\eta=0$$

$$12\ 10^{-3}\ @\eta=1.6$$

~~It is worthwhile noting that the difference in radiation length does not deteriorate the resolution dramatically~~

Problem 1 a) continued:

$$\left.\frac{\delta p_T}{p_T}\right|_{MS} \cong 10^{-2}$$

$$\left.\frac{\delta p_T}{p_T}\right|_{\text{intrinsic}} \approx 2\,10^{-4} \cdot p_T$$

The two are the same at roughly $p_T \approx 50\,\text{GeV}$

This is surprisingly consistent with Figure 1.5.
From Fig. 1.5 I read off a cross over at ~ 50-60 GeV
This is roughly the momentum of leptons from Z decay. $\frac{m_Z}{2} \simeq 45\,\text{GeV}$

1(b) $N(\lambda) = N_0\, 1 - e^{-\lambda/\lambda_0}$ = # of π that undergo hadronic interactions

$$\frac{\lambda}{\lambda_0} = \begin{cases} 0.13 \\ 0.4 \end{cases} \Rightarrow \frac{N_\lambda}{N_0} \simeq \begin{matrix} 0.12\ @\ \eta=0 \\ 0.33\ @\ \eta=1.6 \end{matrix}$$

The inefficiency is roughly 0.1 @ $\eta=0$
0.2 @ $\eta=1.6$

Though it's almost 0.3 for 1 GeV particles at $\eta=1.6$
Some fraction of the π^+ that undergo hadronic interactions are reconstructed as tracks. Most likely because they interact in layers at larger radius and are thus reconstructed in the inner layers.

Problem 1c:

$P_T = 0.3\,B\,R$

$R = \frac{1}{2} \cdot 1.1\,m \Rightarrow P_T = 0.656\,eV$

$B = 4T$

Problem 1d:

$$\frac{E}{E_0} = e^{-X/X_0} \qquad \frac{X}{X_0} = 0.4 \ @\ \eta = 0$$
$$= 1.4 \ @\ \eta = 1.6$$

you thus radiate off
about 33% of the e^- energy @ $\eta = 0$
and about 75% @ $\eta = 1.6$

In comparison, the resolution at 50GeV for photons is 0.6%.

It is an interesting reconstruction problem to figure out a way to measure the electron energy given the precision of the ECAL (very good!) but the amount of radiation (terrible!).

Problem 1e: $\frac{N}{N_0} = e^{-\frac{7}{9}\frac{X}{X_0}}$ relevant $\frac{X}{X_0} \simeq \frac{1}{3} \cdot 0.075$ @ $\eta = 0$

$\frac{1}{3} \cdot 0.35$ @ $\eta = 1.6$

$\Rightarrow$ $\sim 2\%$ @ $\eta = 0$
$\sim 9\%$ @ $\eta = 1.6$ of photons convert in first layer.