Physics 214 UCSD/225a UCSB

Lecture 6

- Symmetries & QCD
  - Finishing off Isospin et al.
  - SU(3)
- Draw heavily from H&M chapter 2 today.

Isospin for anti-quarks

- We want charge to be conserved.
  - This requires putting the most positively charged state always at the top in our isospin doublets.
- We want to distinguish quark and antiquark formally.
  - Add a “bar” on top of the letter.

$$\begin{pmatrix} u \\ d \end{pmatrix} \xrightarrow{?} \begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}$$

- Want to make anti-quark doublet with same transformation properties as quarks
  - This will allow us to have the same clebsh-gordon coefficients for both, thus making it possible to combine quarks and antiquarks without thinking too much.

Isospin for anti-quarks

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \sigma_y$$

- Want to make anti-quark doublet with same transformation properties as quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

We need an extra minus sign in the definition, in addition to “bar and flip”.

$$\begin{pmatrix} u \\ d \end{pmatrix} \to e^{\frac{1}{2} i\theta_y \sigma_y} \begin{pmatrix} u \\ d \end{pmatrix} = \left[\cos\frac{\theta_y}{2} + i\sin\frac{\theta_y}{2}\sigma_y \right] \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} & \sin\frac{\theta_y}{2} \\ -\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

If you simply bar and flip then you get the wrong sign
In front of the “sin” terms.

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow e^{\frac{1}{2} i\theta_y \sigma_y} \begin{pmatrix} u \\ d \end{pmatrix} = \left[\cos\frac{\theta_y}{2} + i\sin\frac{\theta_y}{2}\sigma_y \right] \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} & \sin\frac{\theta_y}{2} \\ -\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \rightarrow e^{\frac{1}{2} i\theta_y \sigma_y} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \left[\cos\frac{\theta_y}{2} + i\sin\frac{\theta_y}{2}\sigma_y \right] \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} & \sin\frac{\theta_y}{2} \\ -\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{pmatrix} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

Note:
The point here is that you want to be able to derive the rotated doublet either via rotation to the quark doublet followed by Charge conjugation and flip, or by starting with the anti-q doublet and using the same rotation as for q doublet.

$$C\begin{pmatrix} u' \\ d' \end{pmatrix} = \begin{pmatrix} -\bar{d}' \\ \bar{u}' \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta_y}{2}\bar{u} - \cos\frac{\theta_y}{2}\bar{d} \\ \cos\frac{\theta_y}{2}\bar{u} + \sin\frac{\theta_y}{2}\bar{d} \end{pmatrix}$$

$$\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \rightarrow e^{\frac{1}{2}i\theta_y\sigma_y} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} & \sin\frac{\theta_y}{2} \\ -\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{pmatrix} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta_y}{2}\bar{u} - \cos\frac{\theta_y}{2}\bar{d} \\ \cos\frac{\theta_y}{2}\bar{u} + \sin\frac{\theta_y}{2}\bar{d} \end{pmatrix}$$

The point here is that you want to be able to derive the rotated doublet either via rotation to the quark doublet followed by Charge conjugation and flip, or by starting with the anti-q doublet and using the same rotation as for q doublet.

Quantum Numbers for Mesons

- J^{PC}

  J = total angular momentum = L+S

  P = parity

  C = charge conjugation

- Only neutral particles can be eigenstates of C, of course.

Generalized Pauli Principle

- The fermion-antifermion wave function must be odd under interchange of all coordinates (space, spin, charge).
  - Space interchange -> $(-1)^{L}$
  - Spin interchange -> $(-1)^{S+1}$
  - Charge interchange -> depends on eigenvalue of C
- Bottom line:

  $(-1)^{L+S+1}$ C = -1 => C = $(-1)^{L+S}$; P = $(-1)^{L+1}$

π^0 : C = $(-1)^{0+0}$ = 1 ; P = $(-1)^{0+1}$ = -1 => pseudoscalar meson

ρ^0 : C = $(-1)^{0+1}$ = -1; P = $(-1)^{0+1}$ = -1 => vector meson

b : C = $(-1)^{1+0}$ = -1; P = $(-1)^{1+1}$ = +1 => axial vector meson

You will use this in the homework.

What's the J^{PC} of the initial state?
What J^{PC} can I construct from the final state particles?
J^{PC} is conserved in QCD, and thus not all final state combinations may be allowed, in general.

Example: π Wave Function

$$\left|T=1;T_3=1\right\rangle = -u\overline{d}$$

$$\left|S=0\right\rangle = \uparrow\downarrow - \downarrow\uparrow$$ -> Antisymmetric under interchange

$$\left|color\right\rangle = R\overline{R} + B\overline{B} + G\overline{G}$$

$$\left|\pi^+\right\rangle = \sqrt{\frac{1}{6}} \sum_{a=R,G,B} \left|u_a \uparrow \overline{d}_{\overline{a}} \downarrow\right\rangle - \left|u_a \downarrow \overline{d}_{\overline{a}} \uparrow\right\rangle$$

Note: C=+1 for pi0 because pi0 -> 2 photons.
Photons are C=-1 because the are produced
by moving charges which have C=-1, of course.

Accordingly, pi0 -> 3photons is heavily suppressed.

SU(3)

- Start with general characteristics
  - Generators and fundamental representation
  - T,U,V spin; SU(2) embedded in SU(3)
  - Graphical way to construct multiplets
  - Applications:
    - Flavor SU(3)
    - Color SU(3)

SU(3) Generators

3^2-1=8 generators λ_i :

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix};\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix};\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$$

$$\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix};\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix};\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

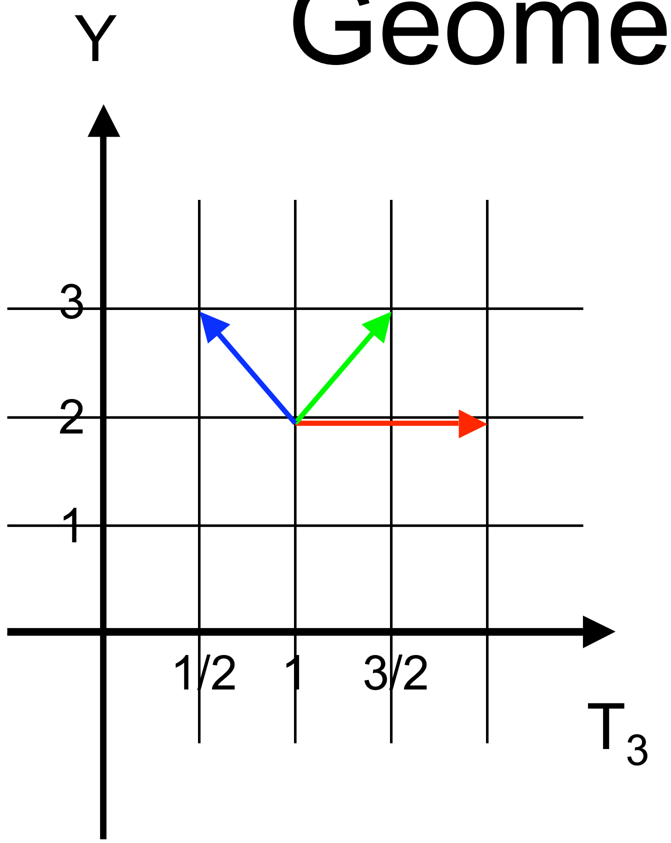
$$\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix};\sqrt{\frac{1}{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

Interesting structure in that there are three spin-1/2 subspaces.

Rank = 2
⇒D(a,b) to classify multiplets.
⇒T_3 and Y quantum numbers within multiplet.

$$T_3 = \lambda_3 /2$$
$$Y = \lambda_8 /\sqrt{3}$$

Geometric Construction



Lines of increasing:

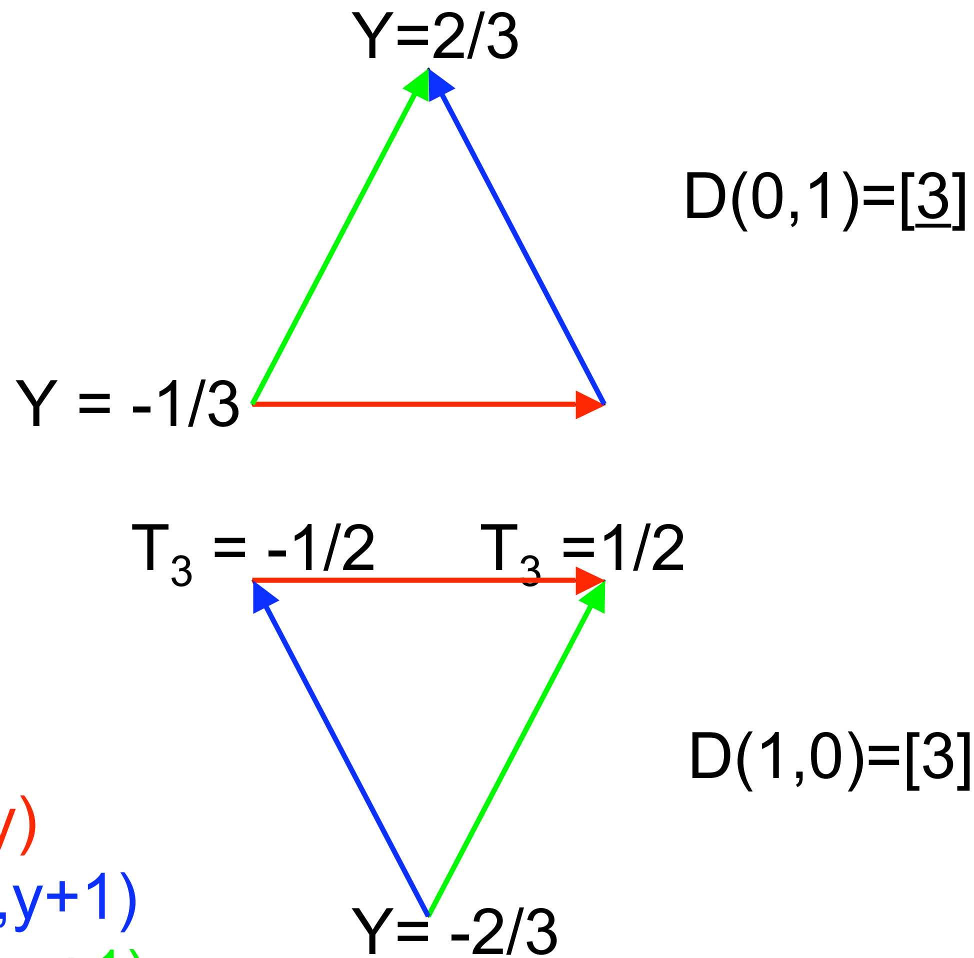
T_3,Y

Isospin: T_+ (m,y) = (m+1,y)

U-spin: U_+ (m,y) = (m-1/2,y+1)

V-spin: V_+ (m,y) = (m+1/2,y+1)

Fundamental Representations:

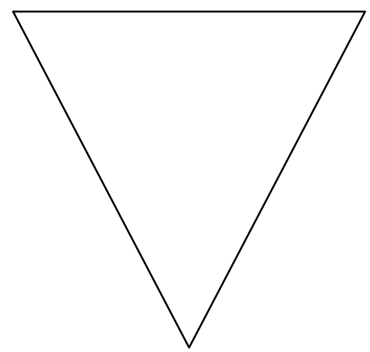


Convention: D(width at top, width at bottom)

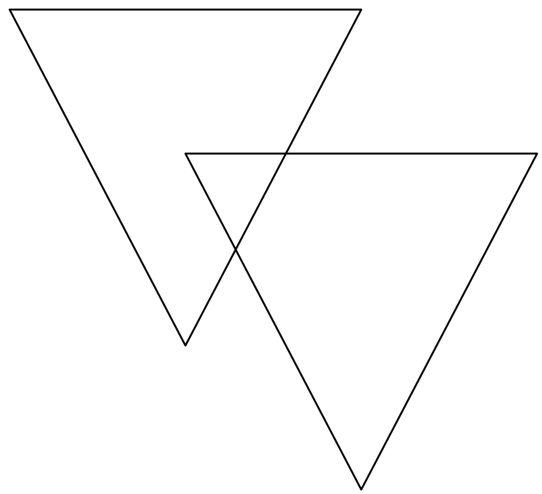
Three additional rules:

- For a given multiplet, the outer most ring can only occupy one state at each point.
- Going inwards, you get multiple occupation per site that belong to the same multiplet.
  - Going one ring in, add one more state per site on that ring.
- If you get more points than would fit on that ring at that site, then collect left-overs, because they will form a separate multiplet.

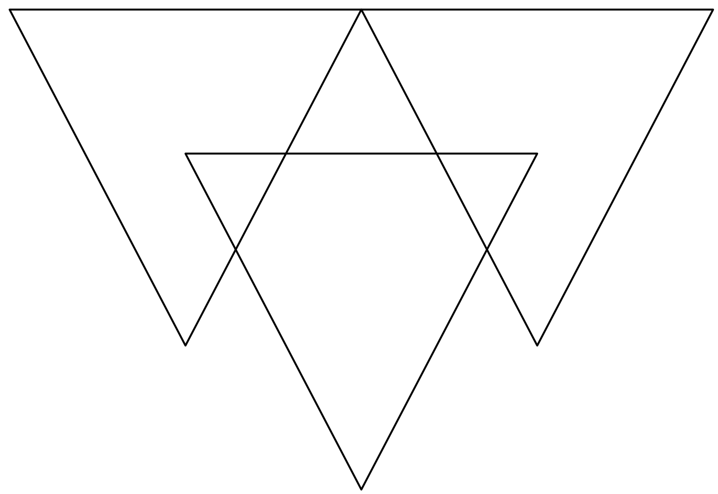
Constructing 3 x 3



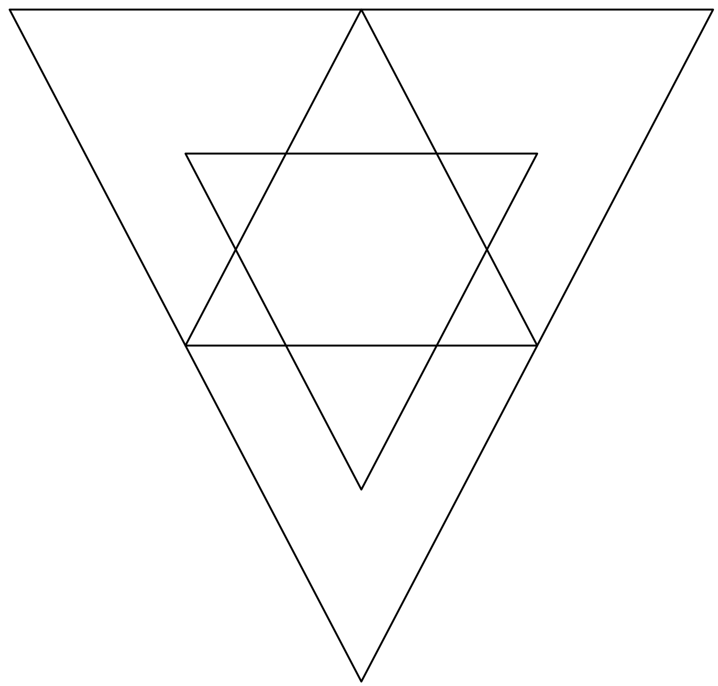
Constructing 3 x 3



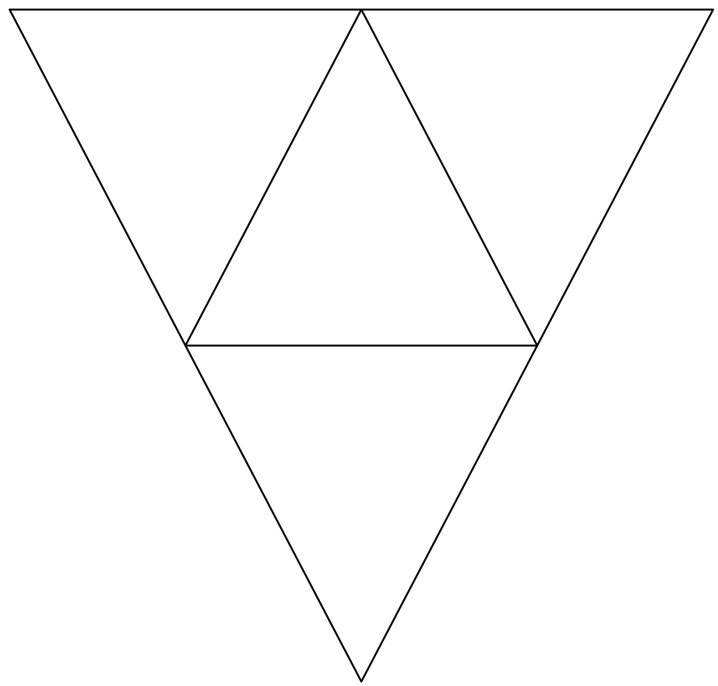
Constructing 3 x 3



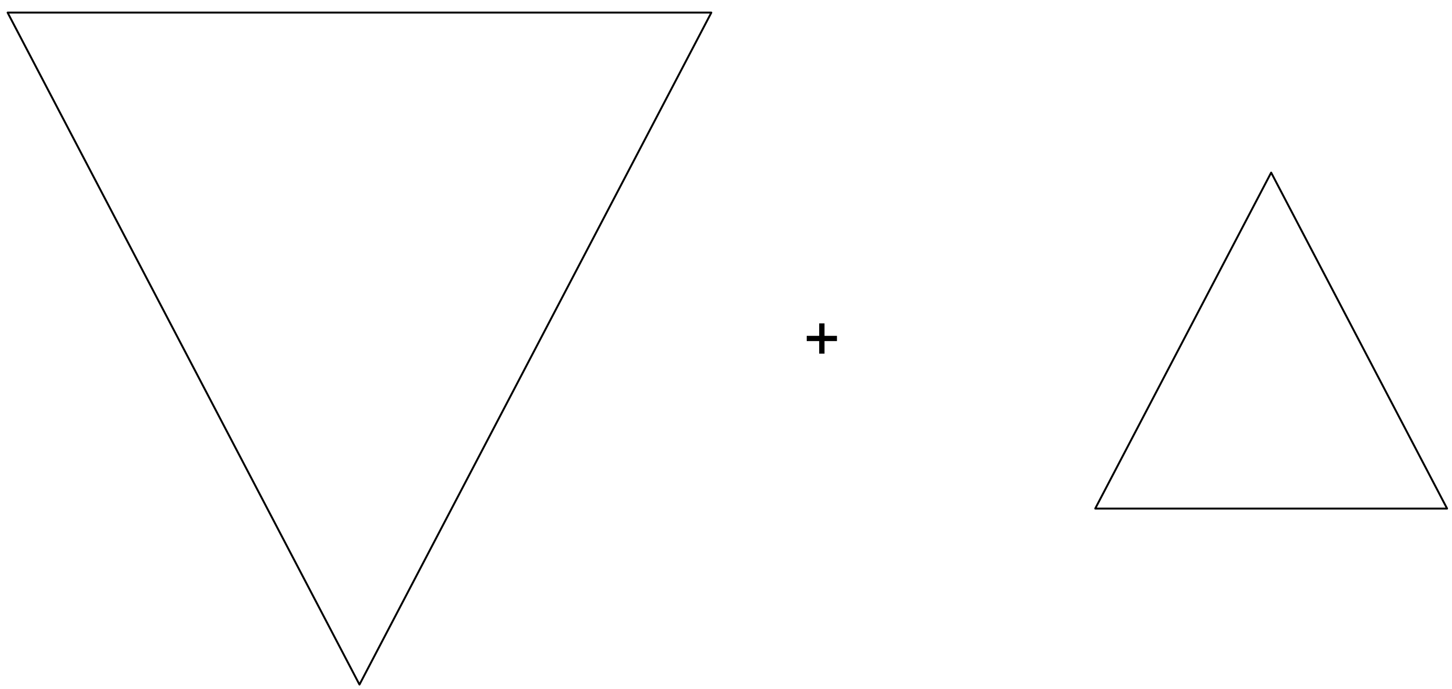
Constructing 3 x 3



Constructing 3 x 3

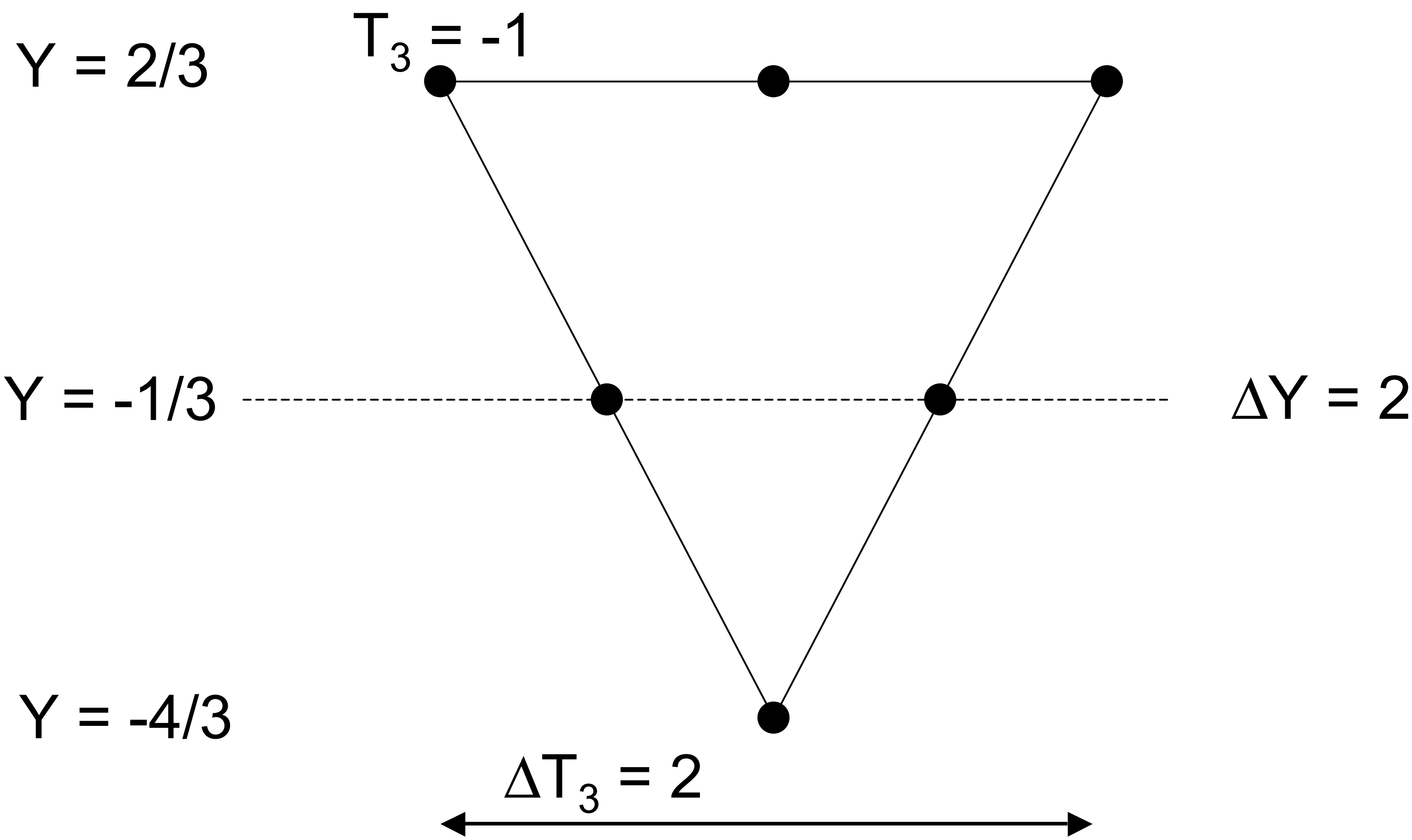


$3 \times 3 = 6 + \underline{3}$



This is a sextuplet, or in group notation
D(2,0) = [6]

D(width at top, width at bottom)

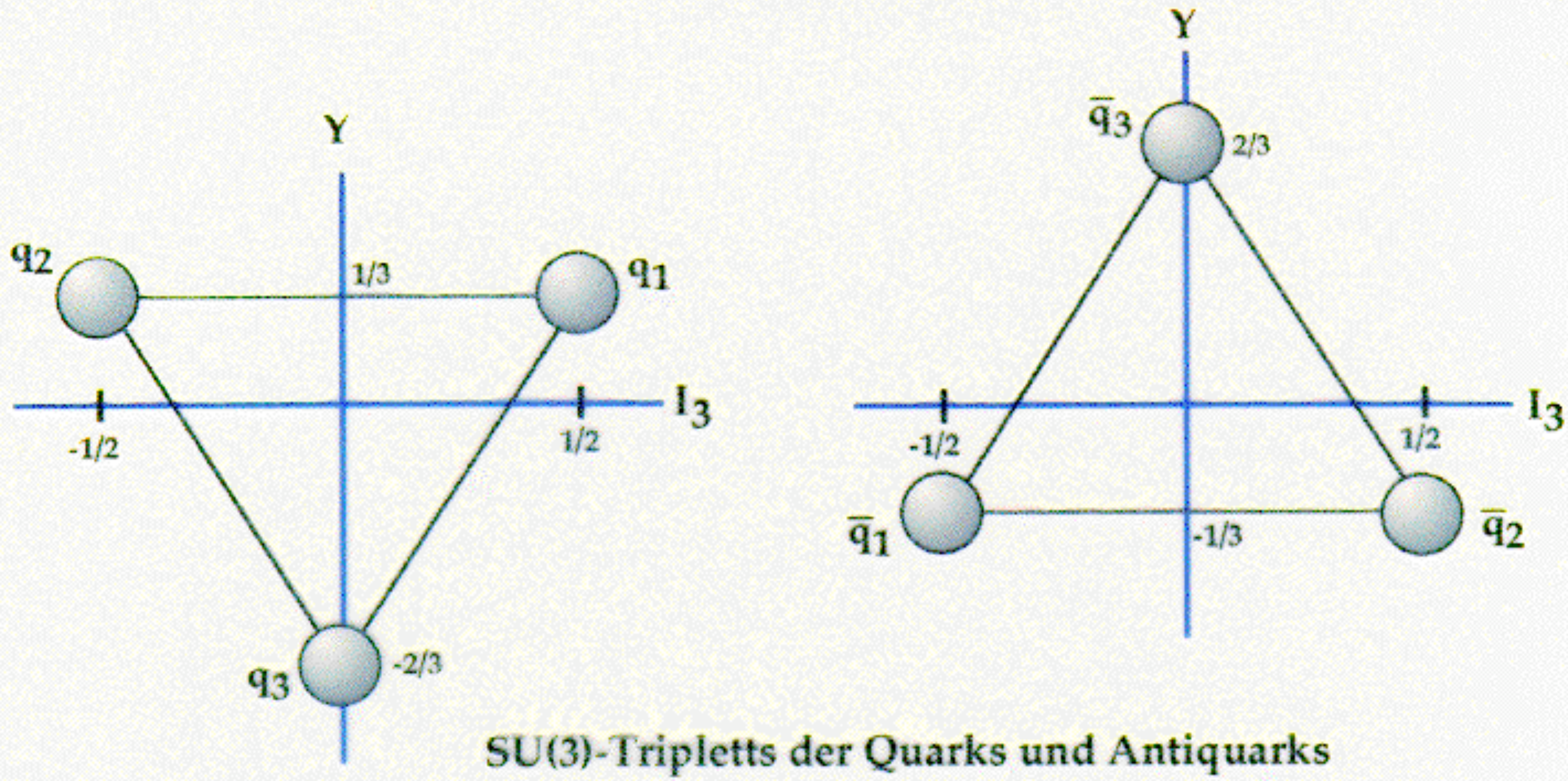


All multiplets of SU(3) can be constructed in this fashion.

Significance to Physics Flavor SU(3)

- Identify:
    - T = isospin
    - Y = B + S = baryon number + strangeness

    => Charge = Q = T_3 + Y/2
- Identify 3 with quarks u,d,s and $\underline{3}$ with antiquarks $\underline{u}$, $\underline{d}$, $\underline{s}$
- Not all SU(3) multiplets are physically meaningful !!!
    – A physical state needs to simultaneously satisfy SU(3) flavor and SU(3) color, and has to have the appropriate overall symmetry under interchange.
    – The two symmetries operate on completely separate hilbert spaces. The fact that both are SU(3) is an accident of nature.



SU(3)-Tripletts der Quarks und Antiquarks

$d = q_2 = (-1/2, 1/3) \quad Q = -1/2 + 1/6 = -1/3$

$u = q_1 = (+1/2, 1/3) \quad Q = +1/2 + 1/6 = +2/3$

$s = q_3 = (0, -2/3) \quad Q = 0 - 1/3 = -1/3$

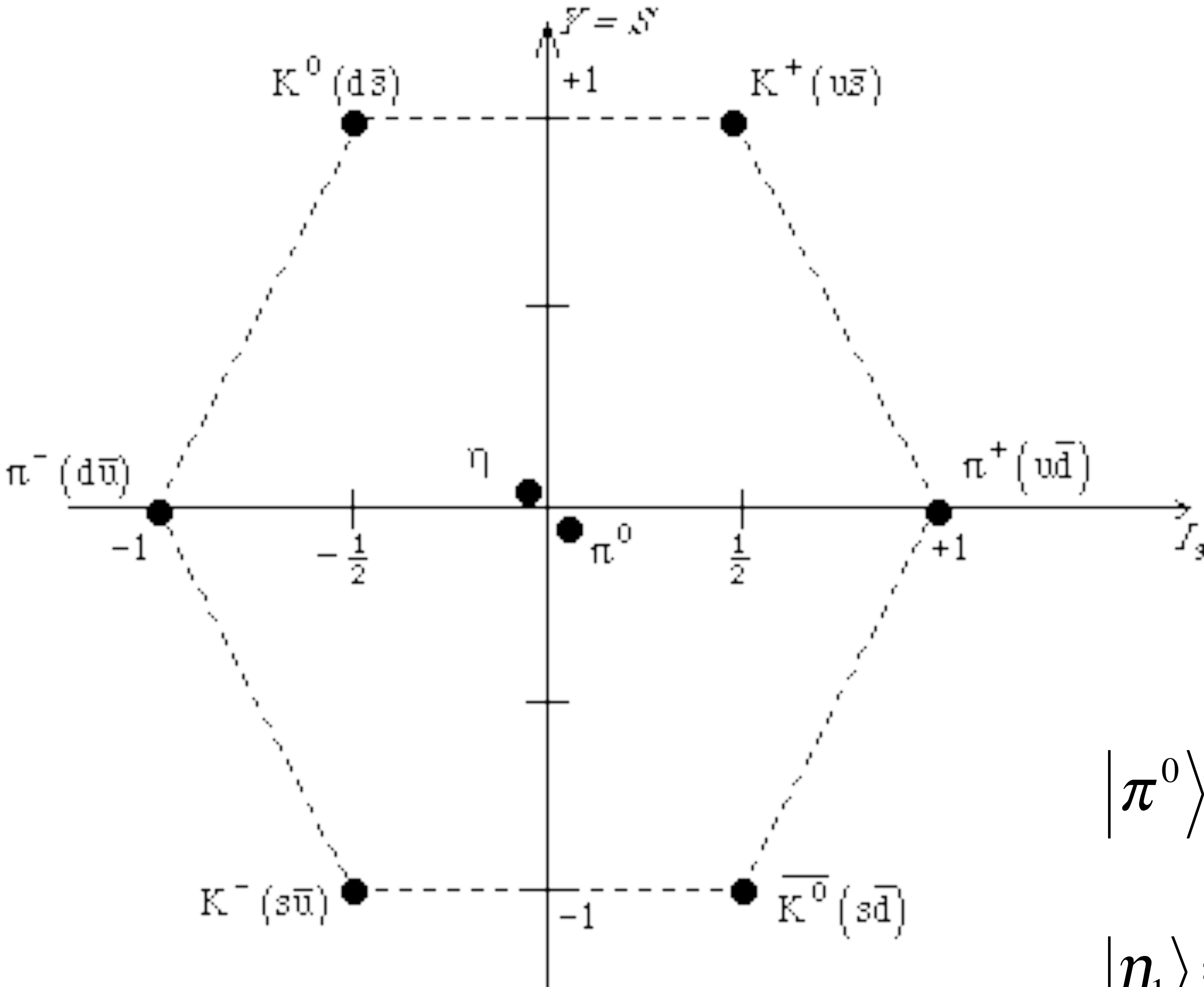
Examples:

- Mesons: $3 \times \underline{3} = 8 + 1$
  - Works for ground state as well as excited states.
- Baryons: $3 \times 3 \times 3 = (6 + \underline{3}) \times 3$

$$= 10 + 8 + (3 \times \underline{3})$$

$$= 10 + 8_S + 8_A + 1_A$$

- Note: The $\underline{3}$ in $(6 + \underline{3})$ is different from the quark triplet. It is the antisymmetric di-quark triplet.

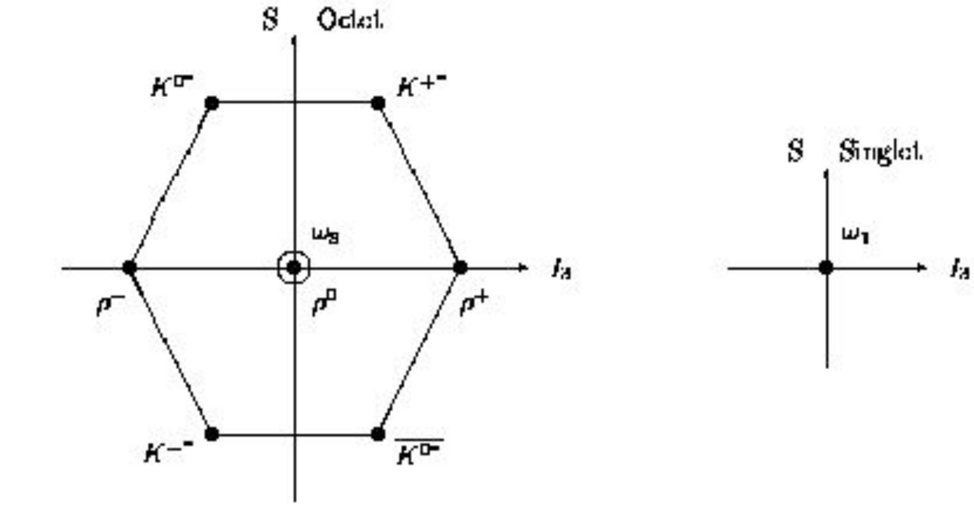


$$|\pi^0\rangle = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$

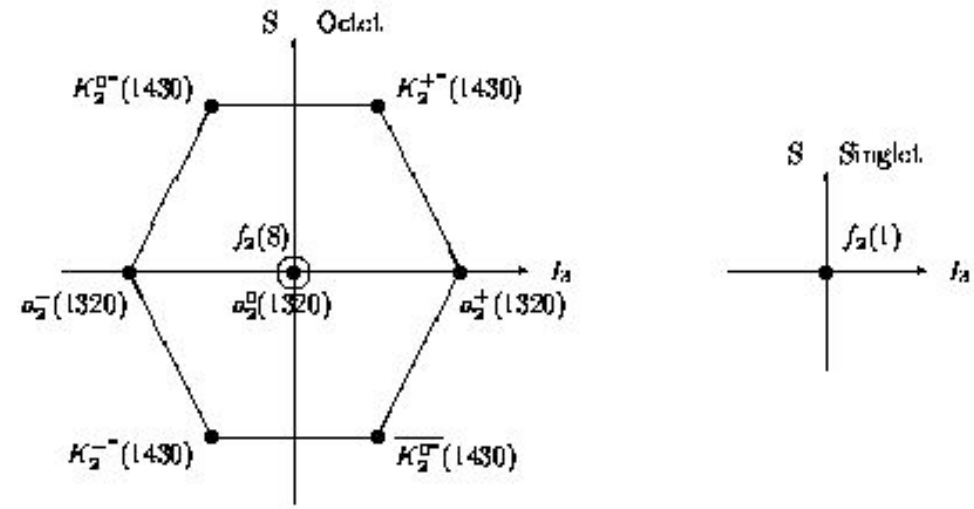
$$|\eta_1\rangle = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}}$$

$$|\eta_8\rangle = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}$$

The vector mesons $J^{PC} = 1^{--}$



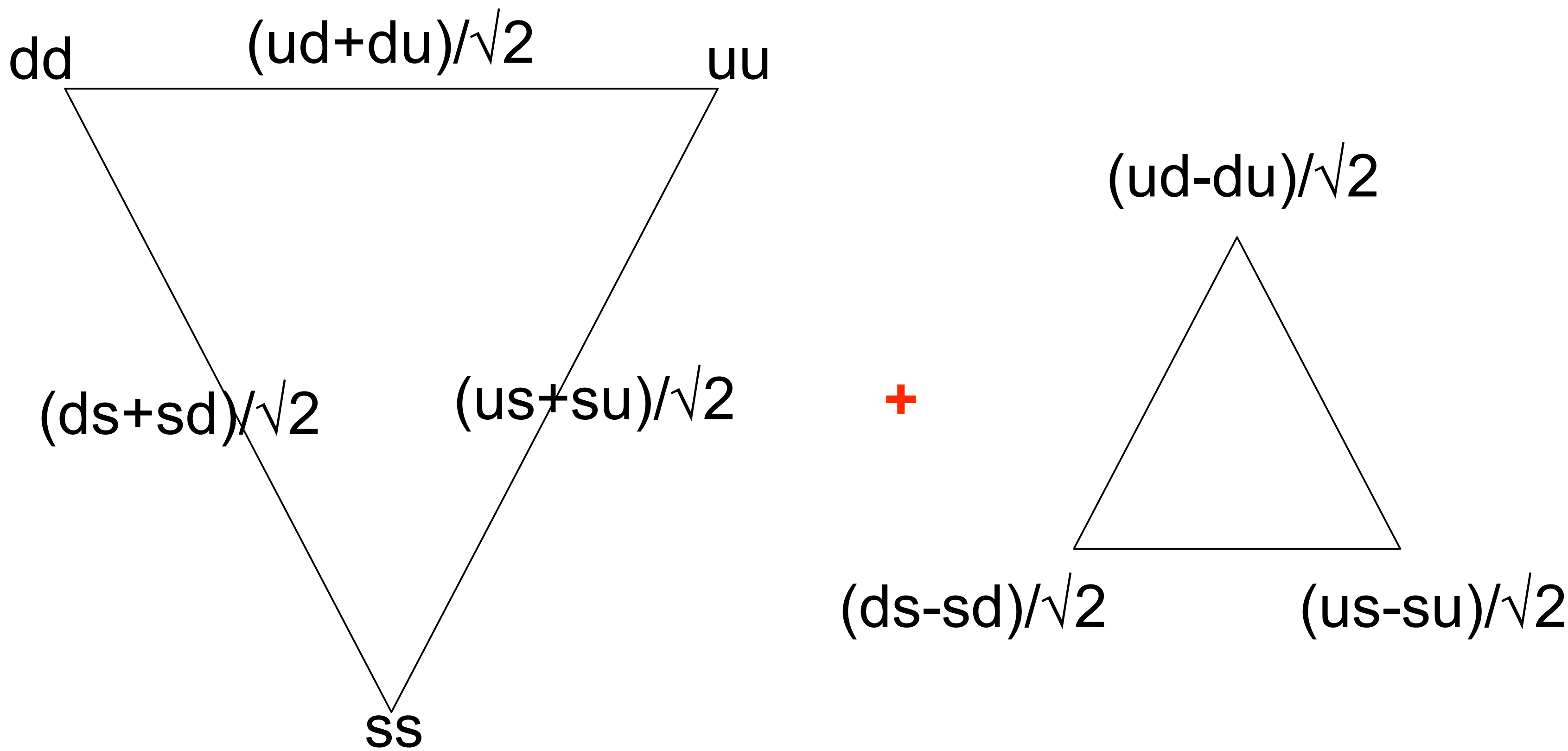
The tensor mesons $J^{PC} = 2^{++}$



Aside:

- SU(3) flavor is strongly dynamically broken in nature.
  - Masses within a multiplet depend on s-quark content.
  - Physical states with T=0 mix across singlet and octet. For vector mesons the physical states are indeed flavor orthogonal rather than flavor symmetric.
- Flavor SU(3) most important to order particles into multiplets, and to show that color must exist, and have (at least some) SU(3) properties.

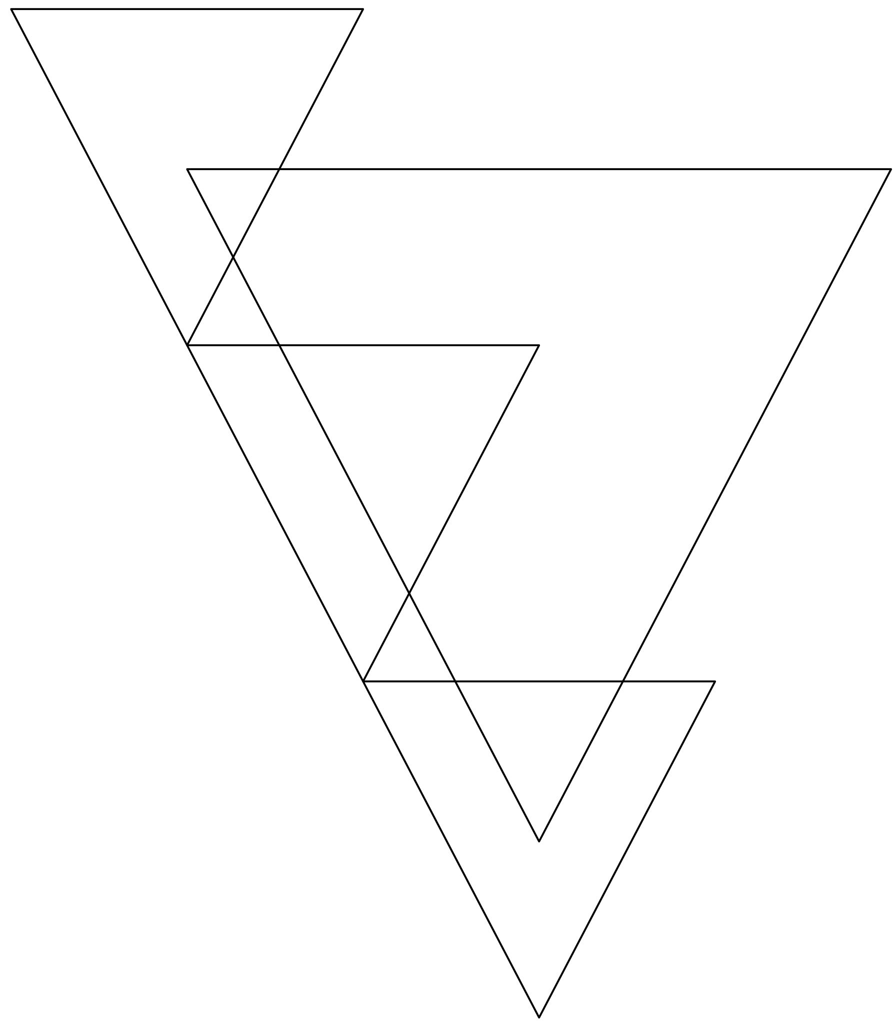
$3 \times 3 = 6 + \underline{3}$ for di-quarks



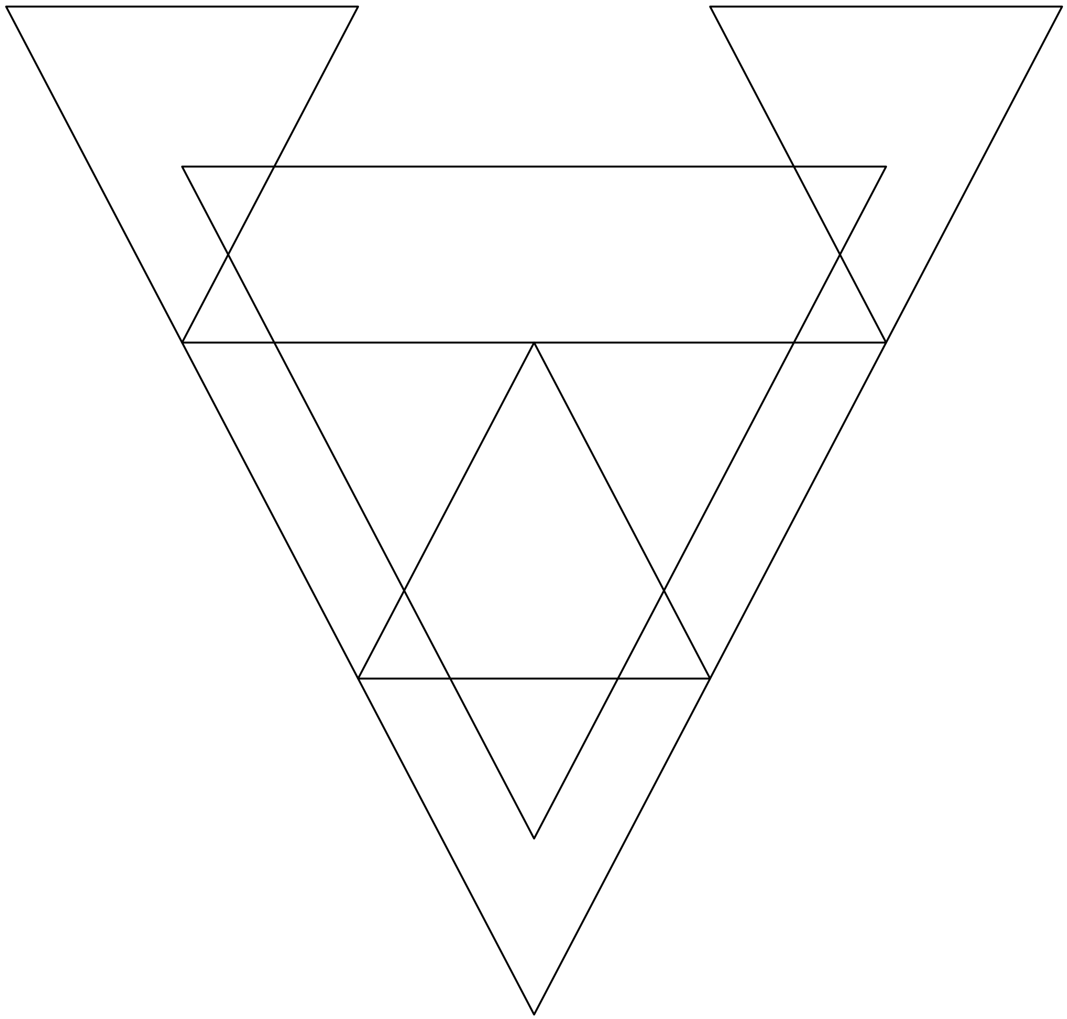
Symmetric Sextet

Antisymmetric triplet

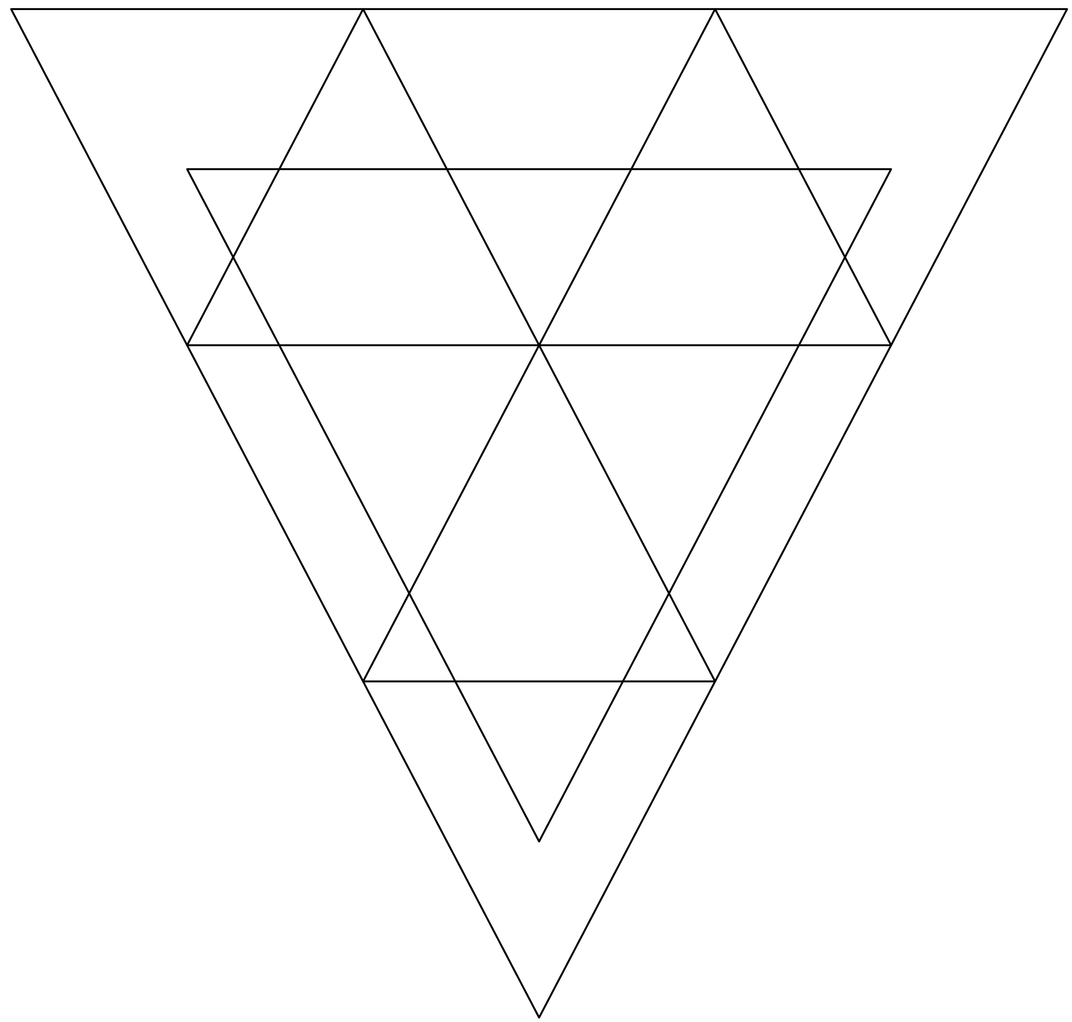
Symmetric di-quarks to triquarks



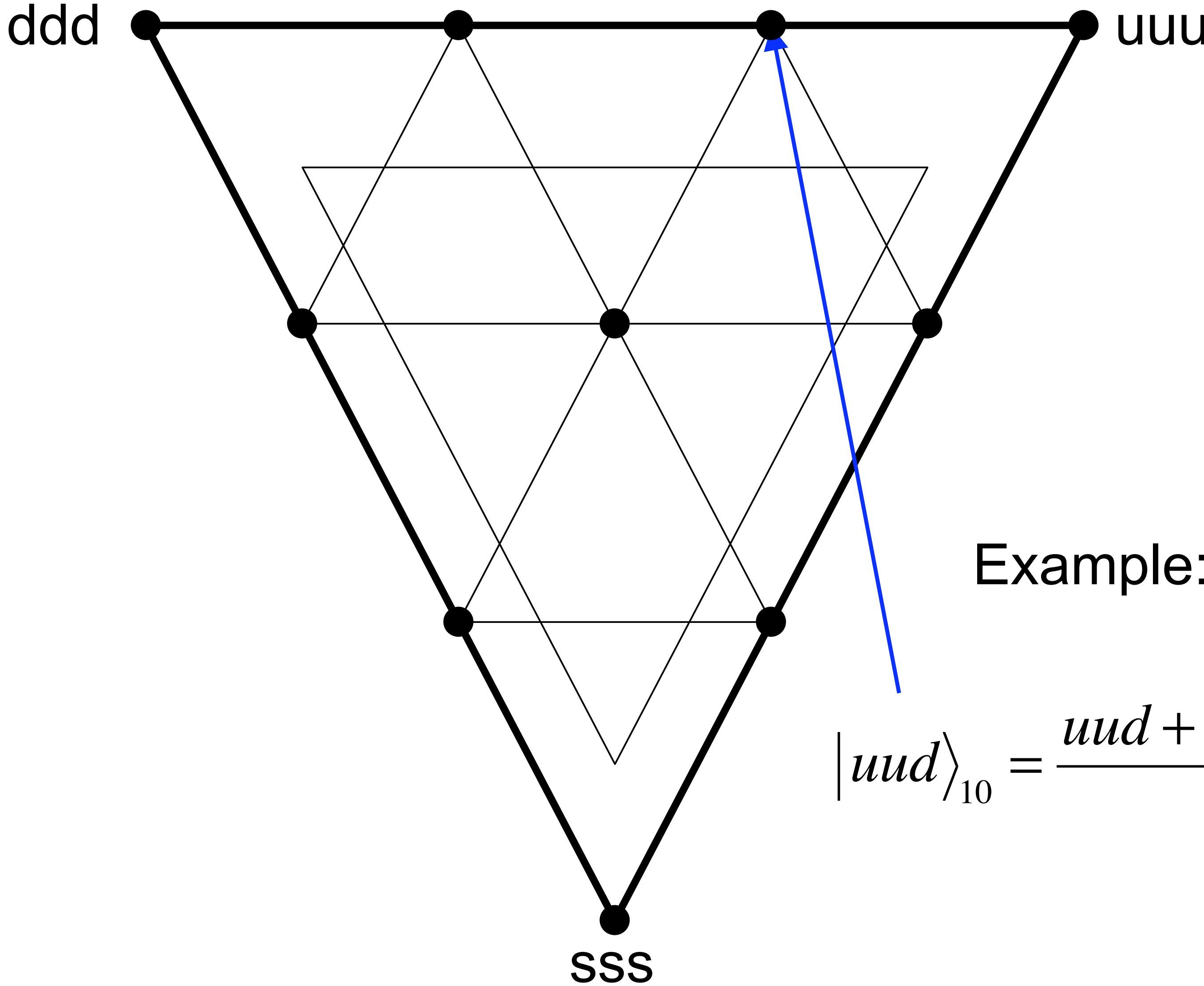
Symmetric di-quarks to triquarks



Symmetric di-quarks to triquarks



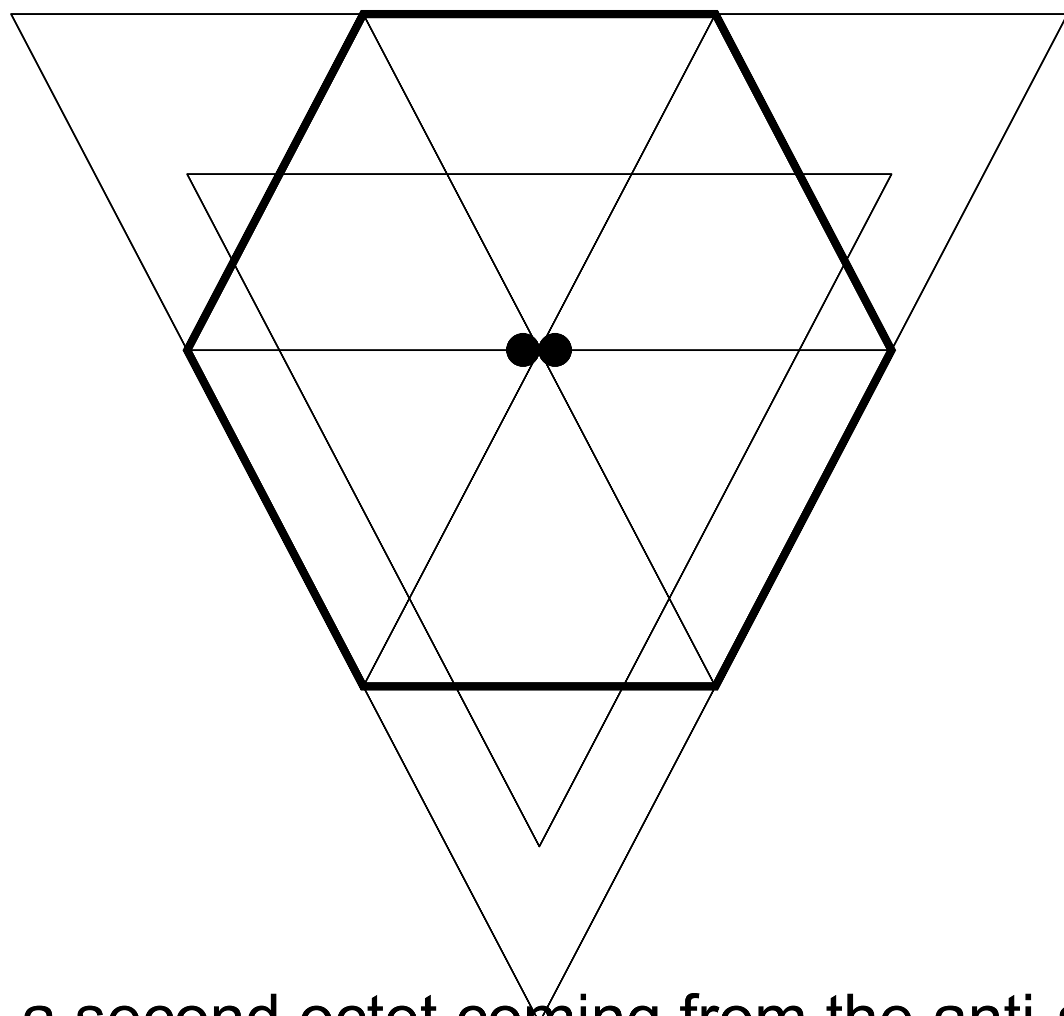
Symmetric 10-plet



Example:

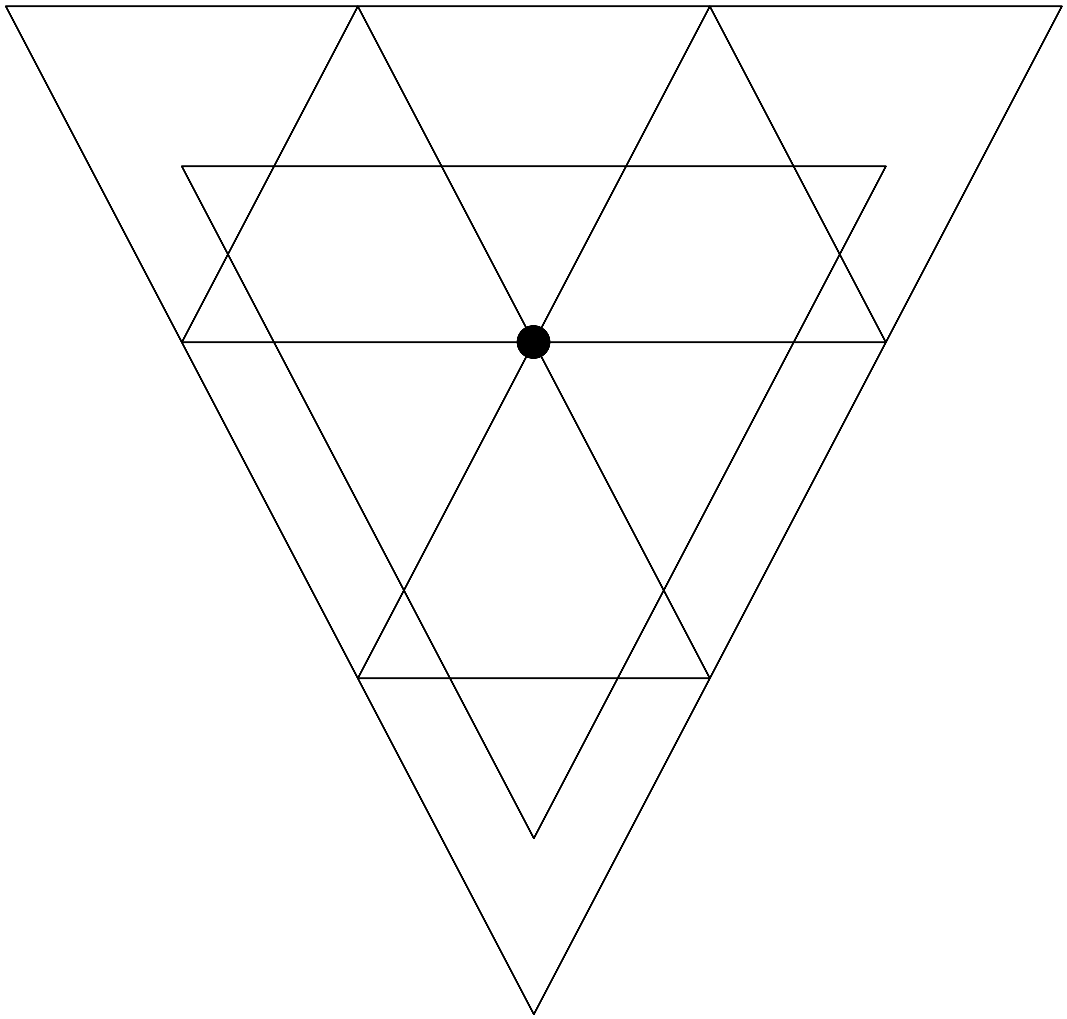
$$\left|uud\right\rangle_{10} = \frac{uud + udu + duu}{\sqrt{3}}$$

Octet with symmetric di-quarks



There's a second octet coming from the anti-symmetric Di-quark triplet x quark triplet.

Singlet



Conclusion from flavor SU(3) alone:

- We could have as ground state baryon multiplets:
  - One fully symmetric decuplet
  - Two octets
    - One is symmetric for the di-quarks
    - The other asymmetric for the di-quarks
  - One singlet

Next, we show that the lowest lying baryons are classified in
one octet and one decuplet
due to spin-statistics on the total wave function.

Total wave function symmetry must be anti-symmetric with interchange of any two fermions (Spin-statistics theorem)

- (baryon) = (flavor) (spin) (space) (color)
- (color) = antisymmetric singlet of SU(3)

  ⇒(flavor) (spin) (space) = symmetric
- Spin:
  - $2 \times 2 \times 2 = (3_s + 1_A) \times 2 = 4_s + 2_{s12} + 2_{A12}$
    - i.e. one spin 3/2 and two spin 1/2 multiplets possible.
- Ground state => L=0, symmetric for interchange of identical quarks.
- Need to combine symmetric states from spin and flavor. I.e. not all combos valid.

12 refers to interchange of quark 1 and quark 2, i.e di-quark symmetry under interchange.

Symmetric Flavor-spin combos

Flavor SU(3)

- 10 = symmetric
- 8 = S12
- 8 = A12
- 1 = symmetric

Spin SU(2)

- 4 = symmetric
- 2 = S12
- 2 = A12

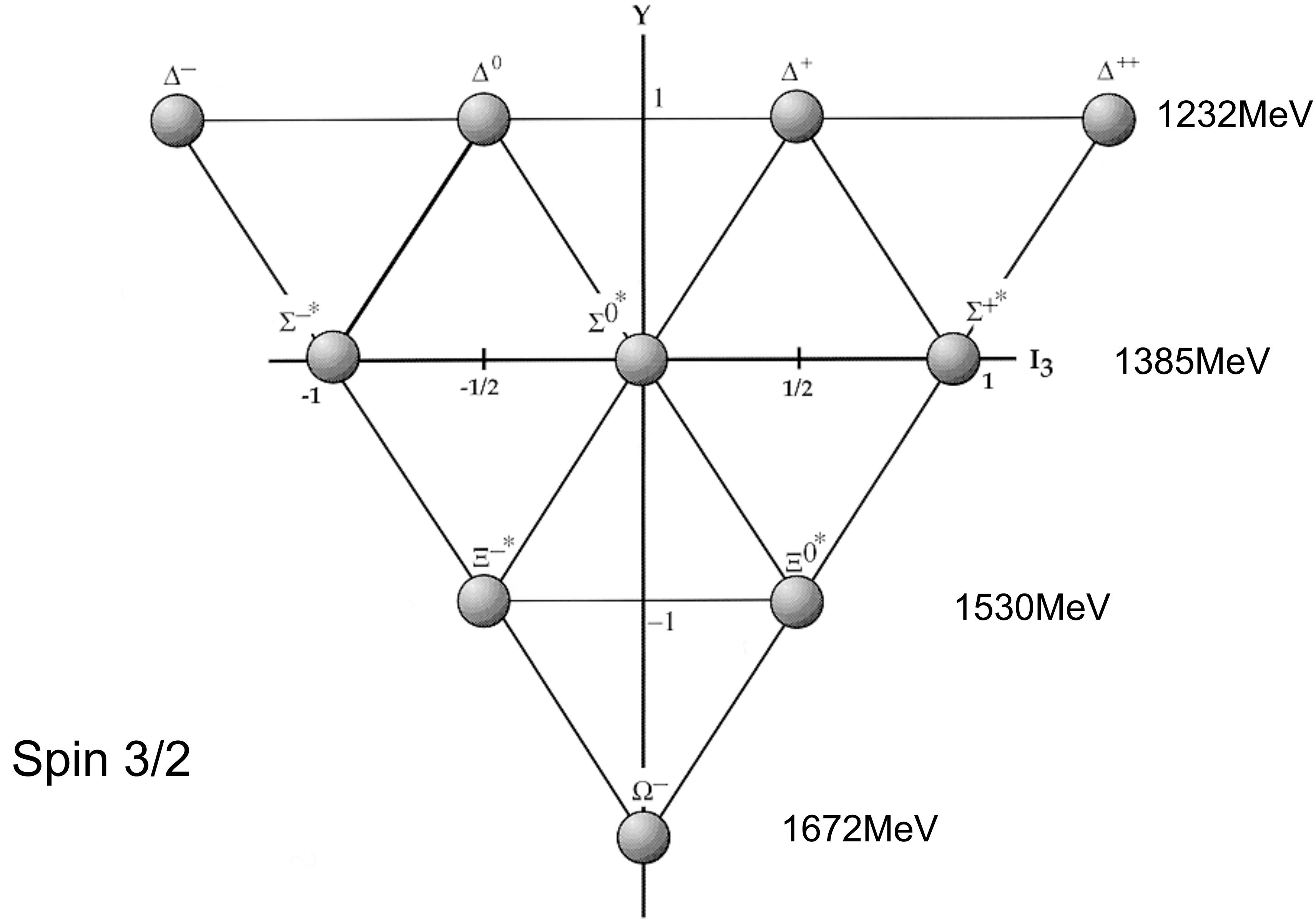
possible options are thus:
Spin 3/2 decouplet, i.e. (flavor,spin) = (10,4)
Spin 1/2 octet symmetric 1<->2, i.e. $(8_{S12},2_{S12})$
Spin 1/2 octet antisymmetric 1<->2, i.e. $(8_{A12},2_{A12})$

E.g., the symmetric flavor-spin octet is thus:
$(1/\text{sqrt}(2)) * [(8_{S12},2_{S12}) + (8_{A12},2_{A12})]$

Color is necessary

- If color did not exist then the Flavor SU(3) Decouplet would have to be combined with the anti-symmetric spin 1/2 dublet in order to make the total wave function anti-symmetric.
- This would predict the uuu, ddd, sss baryons to have spin 1/2 instead of spin 3/2.

Observing the spin of these baryons
thus proves the existence of color.



SU(3)- Dekuplett von Baryonen

Spin 1/2