Physics 214 UCSD/225a UCSB

Lecture 8

- Finish neutrino Physics
  - Neutrinos going through Matter
    - Large Mixing MSW effect
    - Using Matter effect to understand the mass hierarchy.
  - Summary of everything we know
  - Are neutrinos their own anti-particles ?
    - Nuclear double beta decay
- Aside on number of light neutrinos from LEP.

References for Neutrino Physics

- B.Kayser hep-ph/0506165
  - Most of what I've done comes from there.
- 3 NuSAG reports to HEPAP
  - 1st for Majorana neutrinos
  - 2nd for sin theta13
  - 3rd for where the field is going.
    - Have used that the least in these lectures.
- All of this is linked into the course web page.

Neutrinos going through Matter

- Elastic scattering in the forward direction modifies the wave propagation by introducing something akin to an index of refraction. $e^{ipL} \rightarrow e^{inpL}$

$$n = 1 + \frac{2\pi N_e f(0)}{p^2}$$

- In EWK theory, one can show that:
  - Effect has opposite sign for neutrinos and anti-neutrinos.
  - $n = 1 - \sqrt{2}\, G_F N_e / p$
- We will not go through the derivation of what this does to the oscillation amplitude. I refer you to Kayser's paper for that. Instead, I'll simply quote the result and then discuss the impact on nature.

Impact of Matter on Oscillation

Oscillation probability in vacuum:

$$\Pr ob(\nu_e \to \nu_\mu) = \sin^2 2\theta \left[\sin^2 \frac{(m_1^2 - m_2^2)L}{4E} \right]$$

Oscillation probability in matter:

$$\Pr ob(\nu_e \to \nu_\mu) = \sin^2 2\theta_M \left[\sin^2 \frac{\Delta m_M^2 L}{4E} \right]$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

$$\Delta m_M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

$$x = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2}$$

Plug in some numbers:

- Example L=1000km and atmospheric neutrino oscillation:

  $|x| \sim E/12GeV$

  $\Rightarrow$ Effect sizeable for neutrino energy >10GeV.

- Example solar neutrino oscillation
  - $\sqrt{2}\, G_F N_e \sim 0.75\ 10^{-5}\ eV^2/MeV$

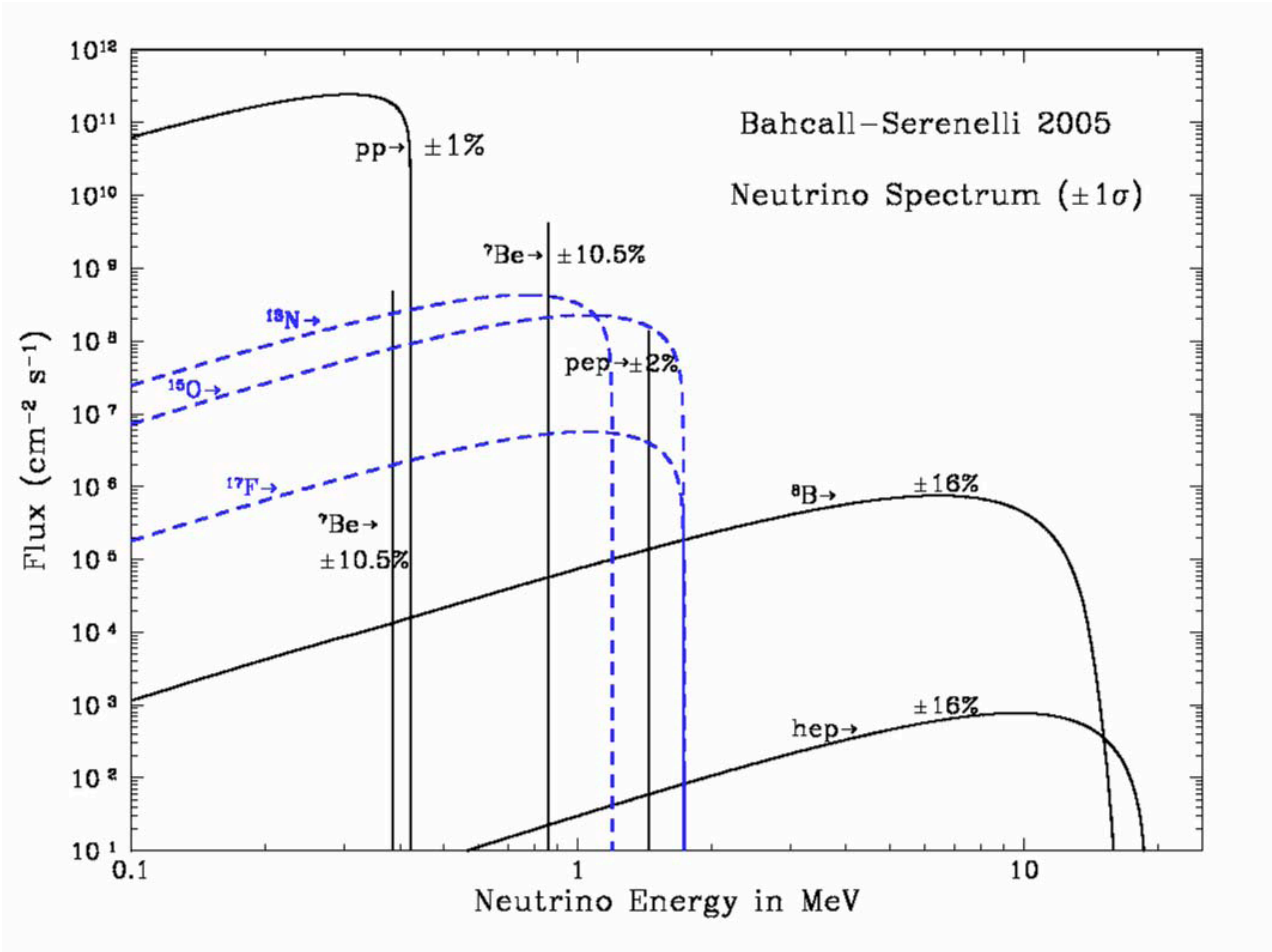
  $|x| \sim E/5MeV$

  => Effect sizeable for neutrinos from ^8B but not for neutrinos from ^7Be or pp.

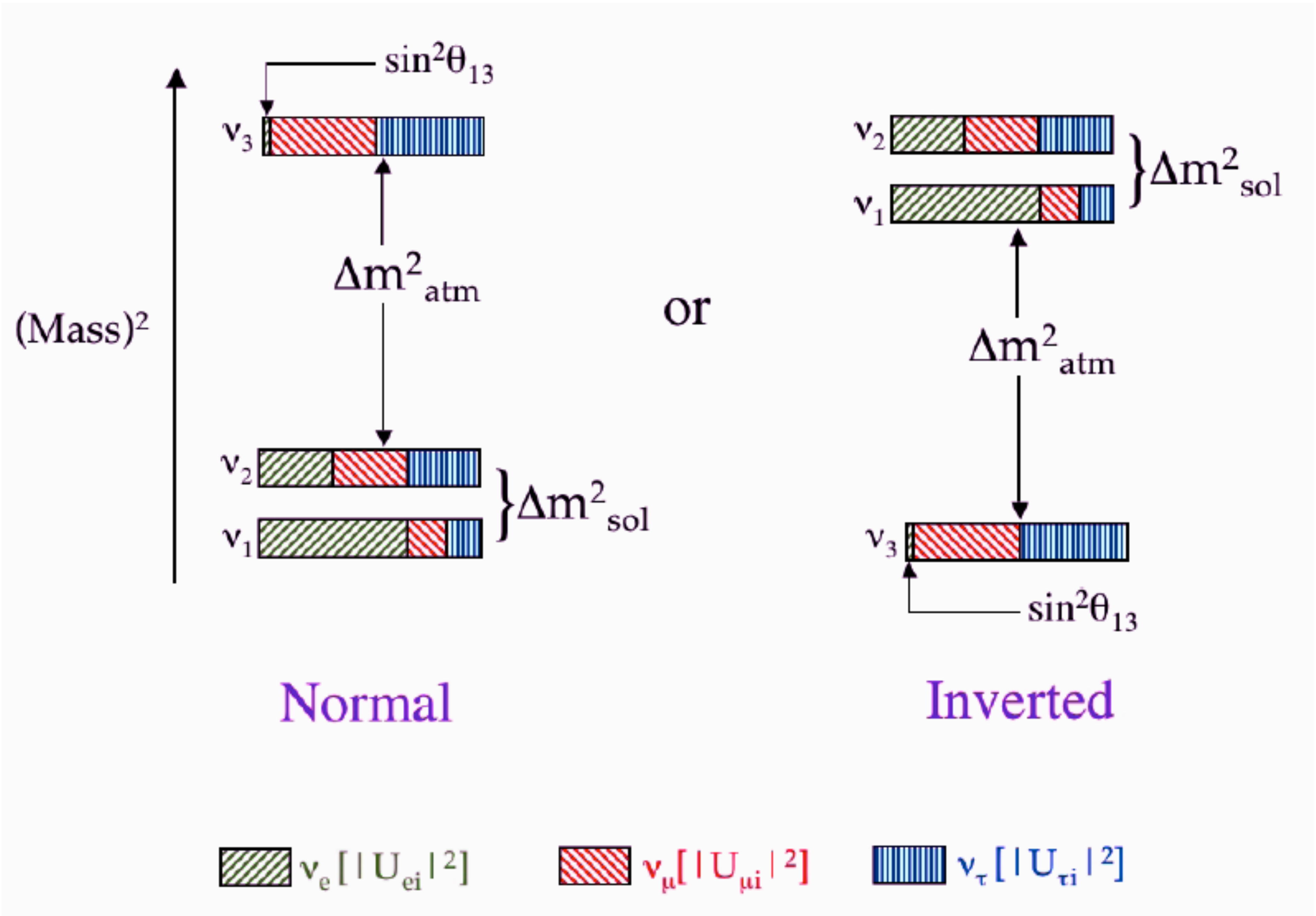
|x| ~ E/5MeV

Importance of matter effect depends on the part of the spectrum an experiment is sensitive to.

For ^8B neutrinos matter effect completely dominates!



Two Possible Mass Hierarchies



Determining the Mass hierarchy from matter effect

- Measure appearance of electrons and positrons with two different beams:

  $\nu_\mu \rightarrow \nu_e$ and $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$

- For beams of E < 2GeV, we can approximate:

$$\sin^2 2\theta_M = \sin^2 2\theta_{13}\left[1 \pm S\frac{E}{6GeV}\right]$$

$$\frac{P(\nu_\mu \rightarrow \nu_e)}{P(\overline{\nu}_\mu \rightarrow \overline{\nu}_e)} =$$

>1 for S = + , i.e. "normal"

<1 for S = - , i.e. "inverted"

- Here S is the sign of Δm_{32}

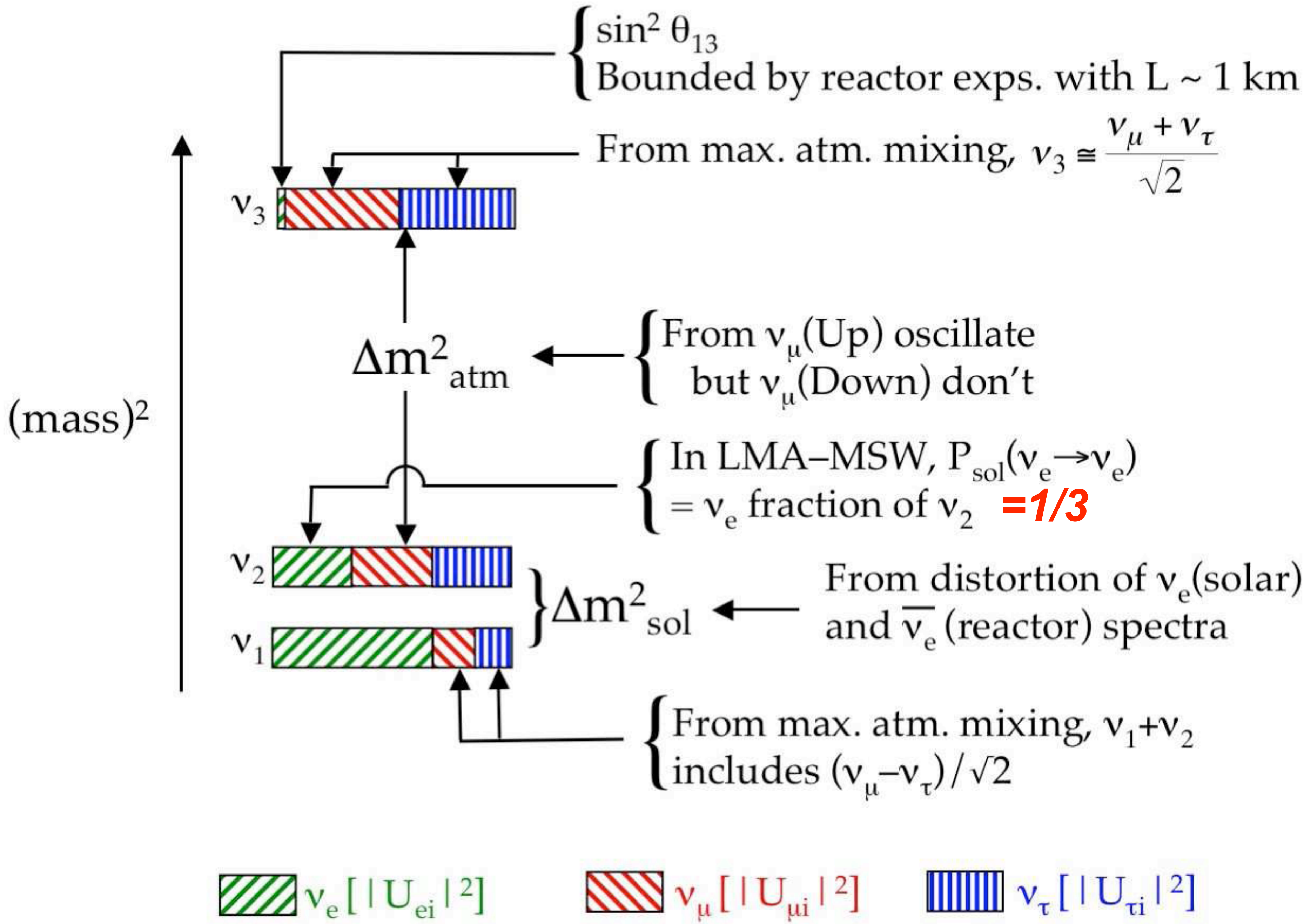
Aside on details

For the small mixing angle we can approximate:

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2} \approx \frac{\sin^2 2\theta}{(1-x)^2}$$

$$x = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2}$$

The sign of x depends on the mass hierarchy.
A small difference in neutrino vs anti-neutrino mixing will thus indicate the hierarchy.

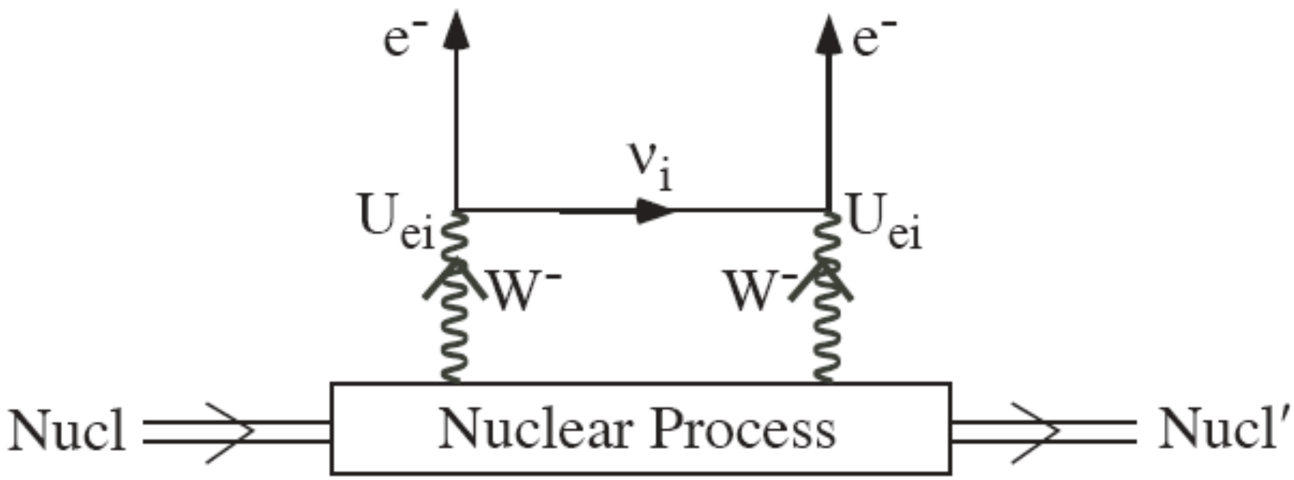


sin² θ13
Bounded by reactor exps. with L ~ 1 km
From max. atm. mixing, ν3 ≅ (νμ + ντ)/√2
ν3
Δm²atm
From νμ(Up) oscillate
but νμ(Down) don't
(mass)²
In LMA–MSW, Psol(νe→νe)
= νe fraction of ν2
=1/3
ν2
ν1
Δm²sol
From distortion of νe(solar)
and ν̄e (reactor) spectra
From max. atm. mixing, ν1+ν2
includes (νμ–ντ)/√2
νe [|Uei|²]
νμ [|Uμi|²]
ντ [|Uτi|²]

General Arguments

- L/E defines the mass splitting an experiment is sensitive to, i.e. either "atmospheric" or "solar" split.
- Pick a neutrino flavor for your source, and check if that flavor oscillates at that mass splitting.
  - If it doesn't then one of the two states has no component for that flavor.
  - If it does, then both states have some component of that flavor.
  - If it does maximally mix then the two flavors it mixes are equally present in both states.
- Solar neutrino is special for ^8B because it measures the ν_e content of ν_2 directly.
- Unitarity then demands that ν_1 has large ν_e component to compensate for ν_3 .

Are neutrinos their own anti-particles ?



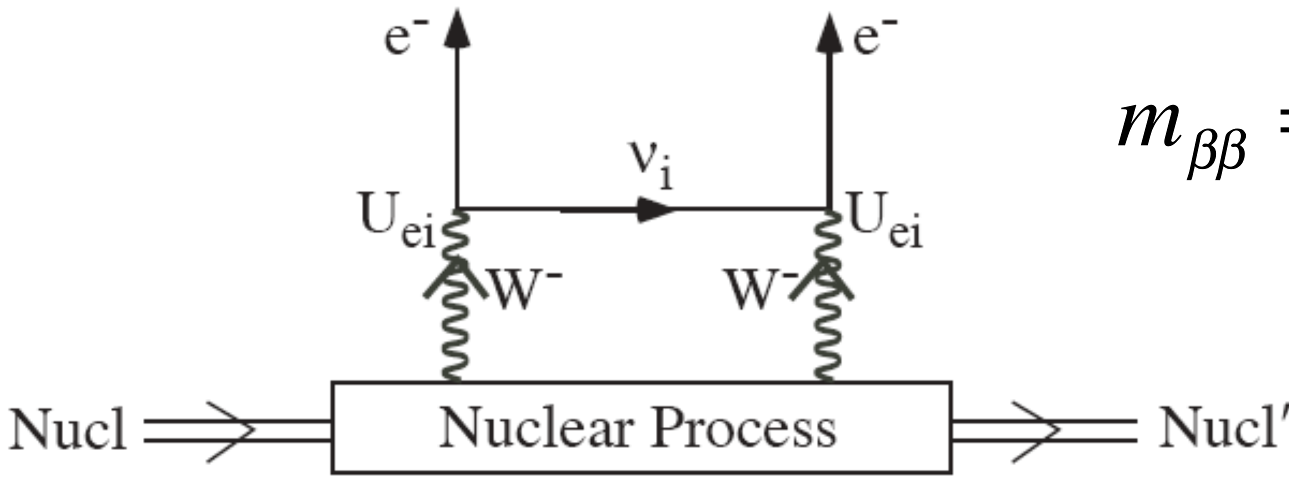
If neutrinos are their own anti-particles then a double beta decay without any neutrinos in the final state is possible !

The size of this effect depends on the mass of the neutrino, instead of the difference of mass^2.

Naively, the solar neutrino signal thus defines the sensitivity required to rule out a majorana neutrino.

Unfortunately, interference effects make it more complicated …

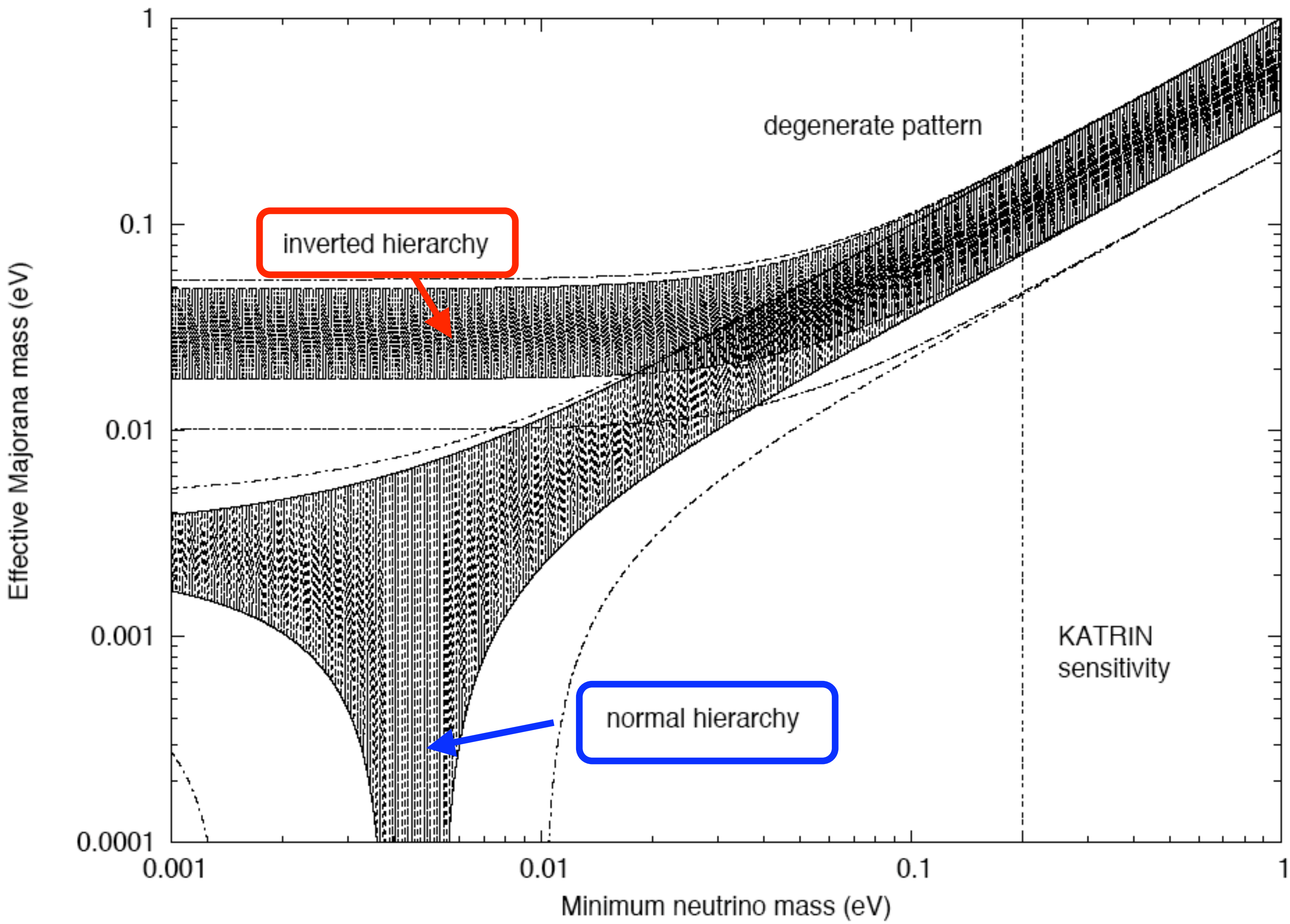
Are neutrinos their own anti-particles ?



$$m_{\beta\beta} = \left| \sum_i m_i U_{ei}^2 \right|$$

Normal: $m_1 < m_2 << m_3$
Inverted: $m_3 << m_1 < m_2$
Both: $|U_{11}| > |U_{12}| >> |U_{13}|$

For inverted, the m_3 part is negligible.
For normal, all three can contribute equally, and their phases may render $m_{\beta\beta}$ to be arbitrarily close to zero.



degenerate pattern
inverted hierarchy
normal hierarchy
KATRIN sensitivity
Effective Majorana mass (eV)
Minimum neutrino mass (eV)
1
0.1
0.01
0.001
0.0001
0.001
0.01
0.1
1

Summary - the case for $\sin 2\theta_{13}$

- We have learned that resolution of all the important open questions revolve around $\sin 2\theta_{13}$, in some sense:
  - Our ability to resolve the hierarchy of neutrino masses from matter effects is aided by a large $\sin 2\theta_{13}$
  - Size of CP violation depends directly on $\sin\theta_{13}$
  - Whether or not we will conclusively show that neutrinos are NOT their own anti-particles depends on the hierarchy and thus $\sin 2\theta_{13}$.

Let's take a closer look at measurement strategies.

Towards determining $\sin\theta_{13}$

- Two strategies:
  - Electron anti-neutrino disappearance at reactors

$$P[\overline{\nu_e} \rightarrow \text{Not } \overline{\nu_e}] \cong \sin^2 2\theta_{13} \sin^2\Delta_{31} + \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2\Delta_{21}$$

  - (Anti-)Electron-neutrino appearance in long baseline (anti-)muon-neutrino beams.

$$P[\overset{(-)}{\nu}_\mu \rightarrow \overset{(-)}{\nu}_e] \cong \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \Delta_{31}$$
$$+ \sin 2\theta_{13} \cos\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \sin\Delta_{31} \sin\Delta_{21} \cos(\Delta_{32} \pm \delta)$$
$$+ \sin^2 2\theta_{12} \cos^2 \theta_{23} \cos^2 \theta_{13} \sin^2 \Delta_{21}$$

Reactor anti-neutrino disappearance:

$$P[\overline{\nu_e} \rightarrow \text{Not } \overline{\nu_e}] \cong \sin^2 2\theta_{13} \sin^2 \Delta_{31} + \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$

For $\sin^2 2\theta_{13} > 0.01$, the first term dominates near $|\Delta_{31}| = \pi/2$.

Given a MINOS precision of 10% on Δm^2_{31}
This approach could yield an unambiguous result as long as $\sin^2(2\theta_{13})$ is not too small.

Nova and off-axis muon neutrinos.

$$P[\overset{(-)}{\nu}_\mu \rightarrow \overset{(-)}{\nu}_e] \cong \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \Delta_{31}$$
$$+ \sin 2\theta_{13} \cos \theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \pm \delta)$$
$$+ \sin^2 2\theta_{12} \cos^2 \theta_{23} \cos^2 \theta_{13} \sin^2 \Delta_{21}$$

Disappearance will get you some resolution of this mess:

$$P[\nu_\mu \rightarrow \text{Not } \nu_\mu] \cong \sin^2 2\theta_{23} \sin^2 \Delta_{atm}$$

In addition, you'd clearly want the reactor program come back with an unambiguous measurement of $\sin^2(2\theta_{13})$.

And then there is neutrino astrophysics …

Which I will not be talking about !

Amusing factoid, there’s a supernova warning system worldwide, because neutrinos are expected to arrive first at earth.

If a signal is seen, and the location can be determined, one can then point telescopes in the direction of the signal, before it reaches earth.

Number of light neutrino families.

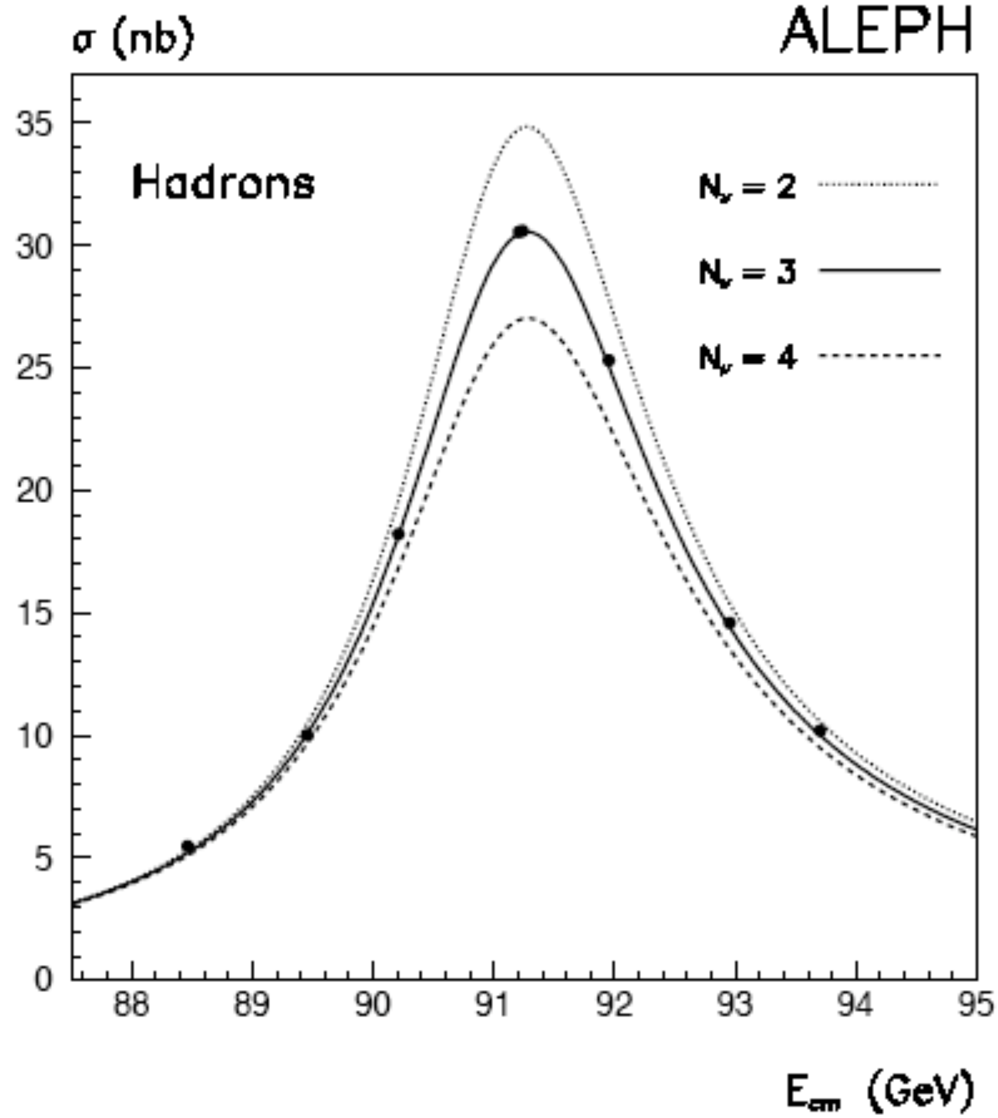
- LEP studied e+ e- sitting near the Z resonance.
- They measured:
  - Γ_{total} the total Z width
  - M_Z the Z mass
  - σ_{peak} the cross section at the peak.

$$\sigma_{peak} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_f}{\Gamma_Z^2}(1-\delta_{rad})$$

- They take from theory:
  - Γ_{ee} ,$\Gamma_{hadrons}$, $\Gamma_{\nu\nu}$
- They then use: $\Gamma_{total} = 3\Gamma_{ee} + \Gamma_{hadrons} + N_\nu\ \Gamma_{\nu\nu}$ to obtain the number of neutrino families.

- (Z. Phys. C62 (1994) 539-550)

Truth in advertizing: the details on how this was done for the result shown here are slightly different from what was done in the first ALEPH paper. Previous page describes the early not the final paper.



Basic ideas are the same.

$$N_\nu = 2.983 \pm 0.034$$