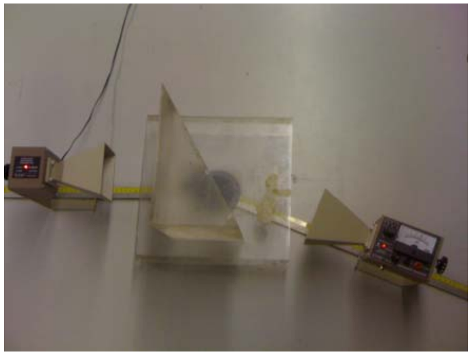
Experiment 4: Refraction and Interference with Microwaves

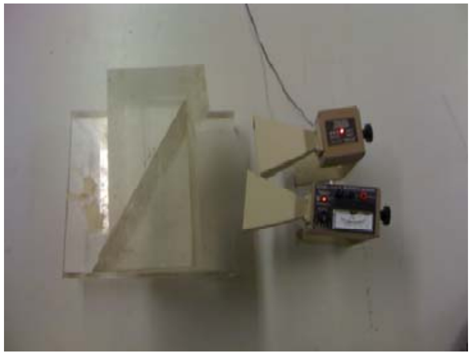
Goal: measure interference of microwaves and determine fundamental values of both the microwave and the mediums through which it travels.

1. Measure the wavelength in free space with interference

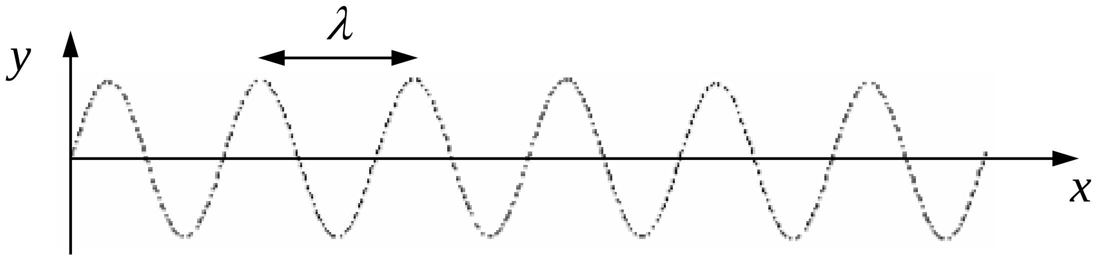
2. Determine the index of refraction of a prism using Snell's law



3. Measuring the wavelength in material with interference



Waves



displacement	$y = A\cos\left(\frac{2\pi}{\lambda}x - 2\pi ft + \phi\right) = A\cos\left(kx - \omega t + \phi\right)$
wave frequency	f
wavelength	λ
phase velocity	$v_p = \lambda f$
phase	ϕ

angular frequency	$\omega = 2\pi f$
wavenumber	$k = \frac{2\pi}{\lambda}$
phase velocity	$v_p = \frac{\omega}{k}$

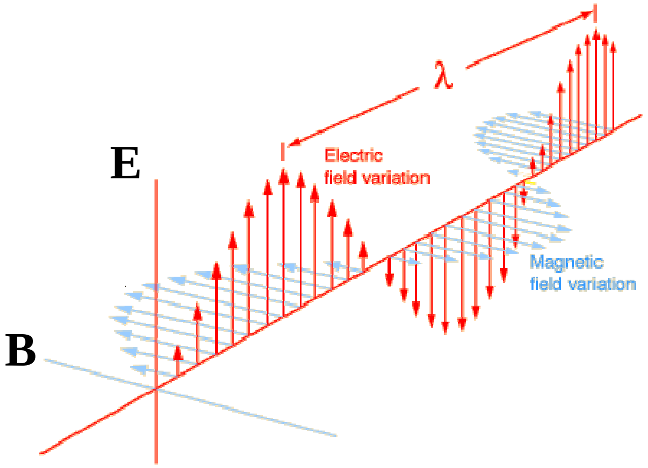
Electromagnetic waves

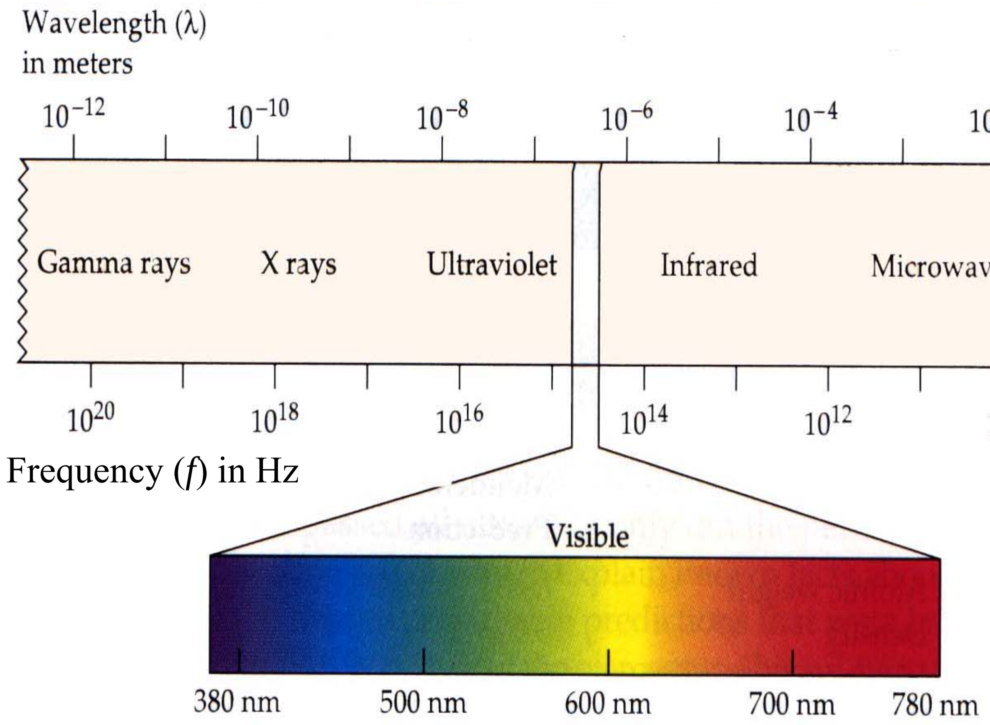
electromagnetic wave equation in vacuum
for a wave propagating along x

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}$$

$$E = A\cos(kx - \omega t + \phi)$$

$$v_p = \frac{\omega}{k} = \lambda f = c = 2.998 \times 10^8 \text{ m/s}$$





frequency $f = \dfrac{c}{\lambda}$

photon energy $E = hf = \dfrac{hc}{\lambda}$

all electromagnetic radiation propagates at a velocity $c = 3 \times 10^8$ m/sec in vacuum

Index of Refraction

refractive index *n* is the factor by which an electromagnetic wave is slowed upon entering a material medium from a vacuum region

$$\boxed{v = \frac{c}{n}}$$

the number of waves per second incident from a vacuum region onto the surface is the same as the number per second entering the medium → the frequency *f* of the waves must be the same in both regions

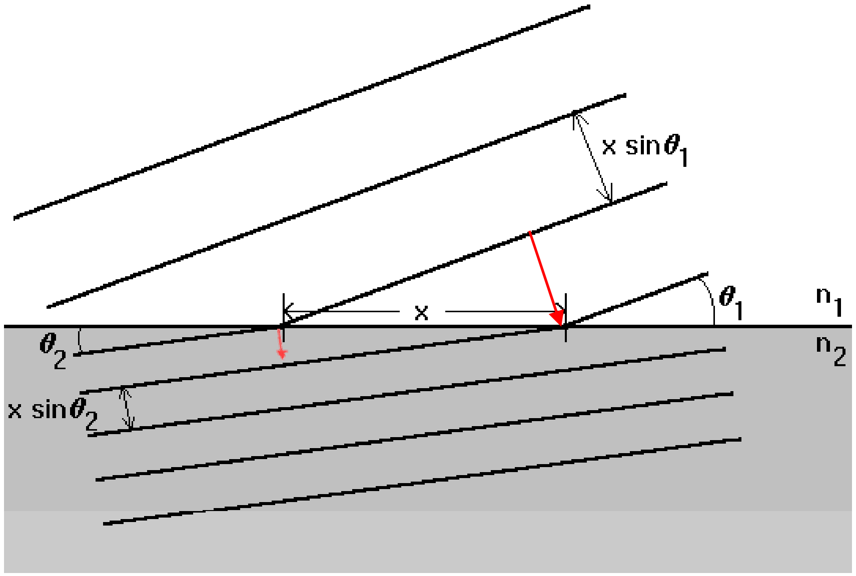
$$f_{vacuum} = \frac{c}{\lambda_0}, \qquad f_{medium} = \frac{v}{\lambda} = \frac{c}{n\lambda} \qquad\qquad f_{vacuum} = f_{medium}$$

$$\boxed{\lambda_0 = n\lambda}$$

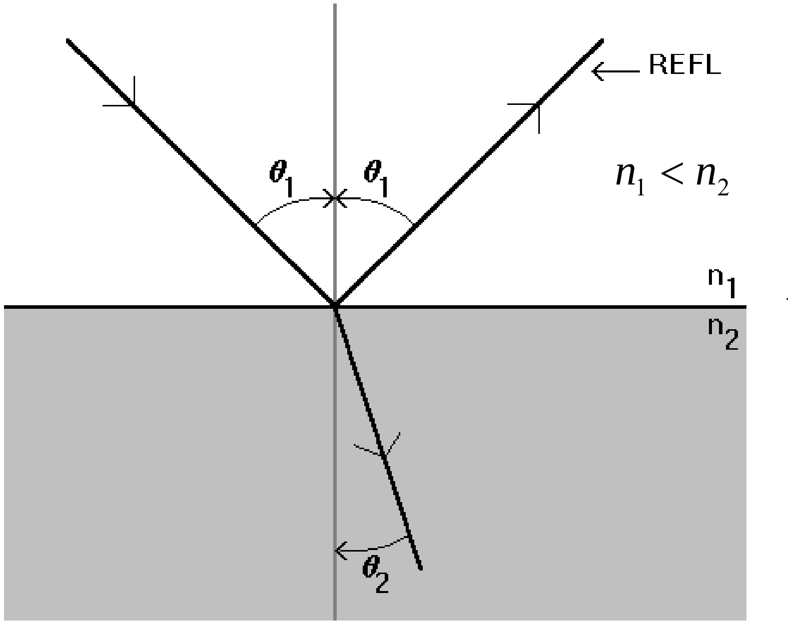
wavelength in vacuum

wavelength in medium

Snell’s Law



waves incident on an interface between n_1 and n_2



plane wave propagates along a ray direction
perpendicular to the plane

at a time t_1, the wave enters
a portion of the surface of width x,
at time t_2 it has just crossed through it

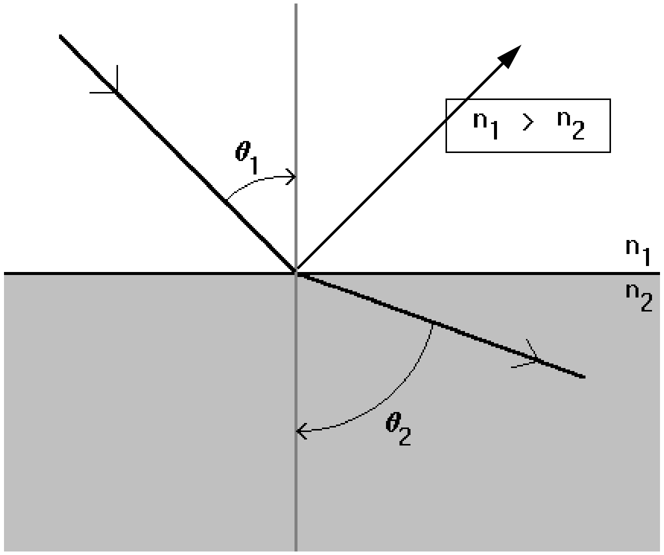
$$\frac{c}{n_1}\left(t_2 - t_1\right) = x\sin\left(\theta_1\right) \qquad \frac{c}{n_2}\left(t_2 - t_1\right) = x\sin\left(\theta_2\right)$$

$$n_1 x\sin\left(\theta_1\right) = n_2 x\sin\left(\theta_2\right)$$

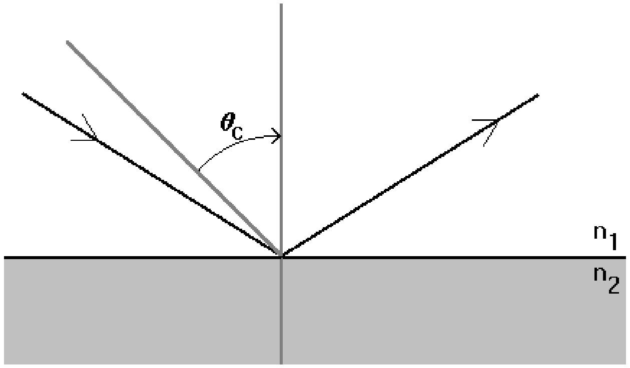
$$\boxed{n_1 \sin\left(\theta_1\right) = n_2 \sin\left(\theta_2\right)}$$

Snell’s Law

Total internal reflection



waves incident on an interface between n_1 and n_2



the critical angle for the case of total internal reflection

consider waves passing from a medium into another region of smaller refractive index

$$\boxed{n_1 \sin(\theta_1) = n_2 \sin(\theta_2)}$$

$$\theta_1 < \theta_2 \quad \text{for} \quad n_2 < n_1$$

increase θ_1 until $\sin(\theta_2) = 1$
then the transmitted ray is moving just parallel to the interface

$$\boxed{\sin\theta_1 = \frac{n_2}{n_1}}$$

<u>the critical angle</u>

increase θ_1 further → <u>total internal reflection</u>

the interface acts like a perfect mirror

↑

general principle behind fiber optic cables

Constructive and destructive interference of waves

consider two waves with same λ

same λ means same k and same ω

$k = 2\pi/\lambda \qquad \omega = ck = 2\pi c/\lambda$

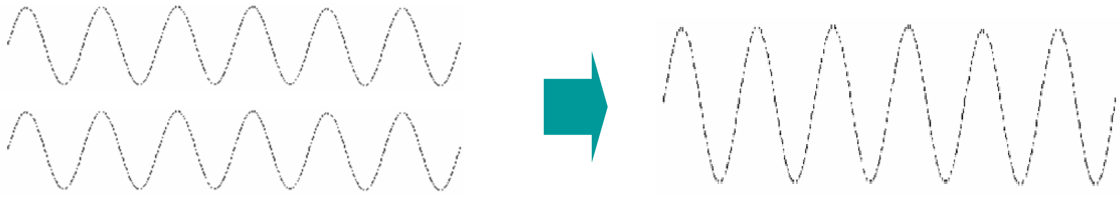
$$E_1 = A\cos(kx - \omega t + \phi_1)$$

$$E_2 = A\cos(kx - \omega t + \phi_2)$$

phase difference

waves in phase $\phi_1 - \phi_2 = 0 + 2\pi n$

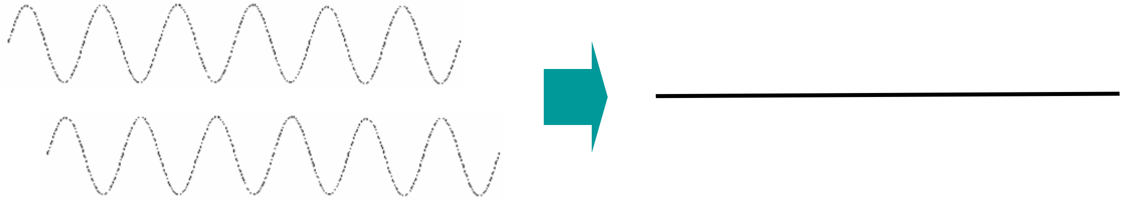
$$E = E_1 + E_2 = E_1 + E_1 = 2A\cos(kx - \omega t + \phi_1)$$



constructive interference

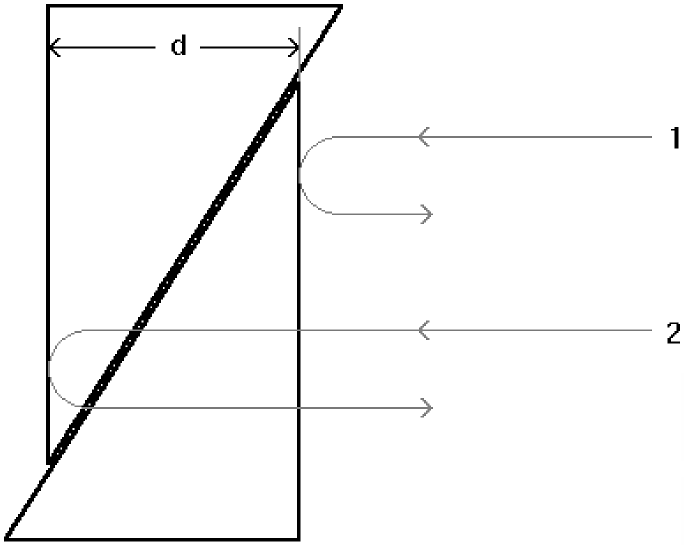
waves out of phase $\phi_1 - \phi_2 = \pi + 2\pi n$

$$E = E_1 + E_2 = E_1 - E_1 = 0$$



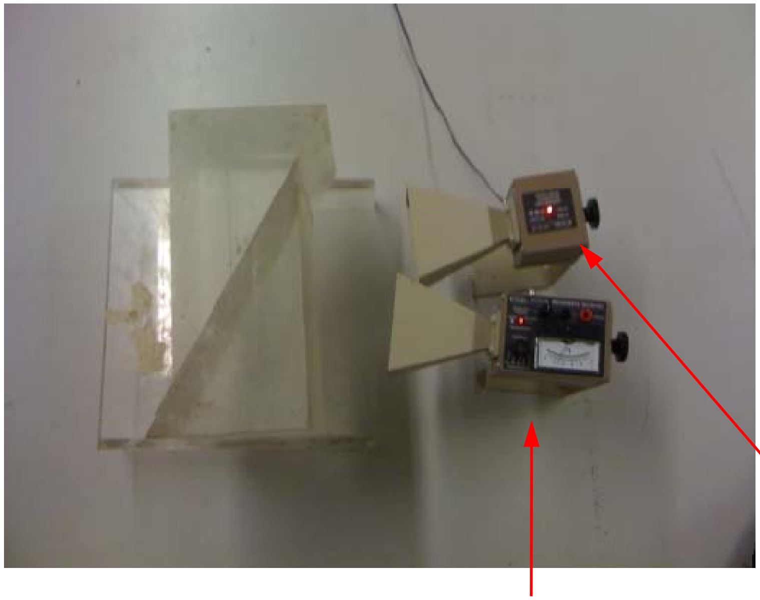
destructive interference

Measuring the wavelength in material with interference



the condition for constructive interference
of waves 1 and 2 is that
the number *m* of wavelengths
contained in the total path length 2*d*
be an integer

$$2d = m\lambda$$



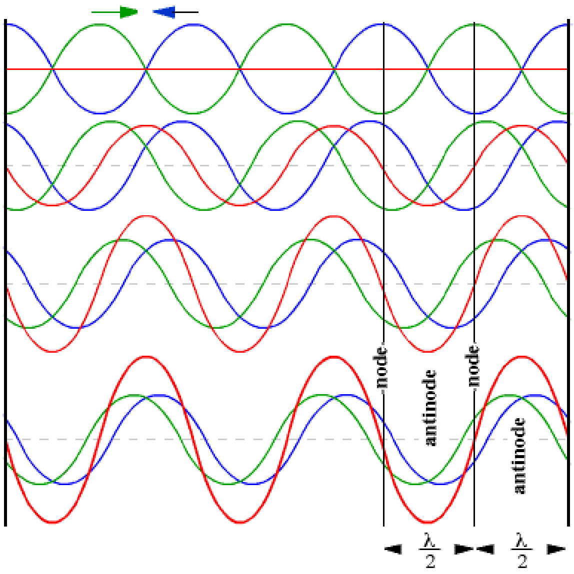
Construct a slab of variable thickness by using two solid prisms.

By observing the variations in reflected intensity with slab thickness, find the wavelength λ of the microwaves in the material.

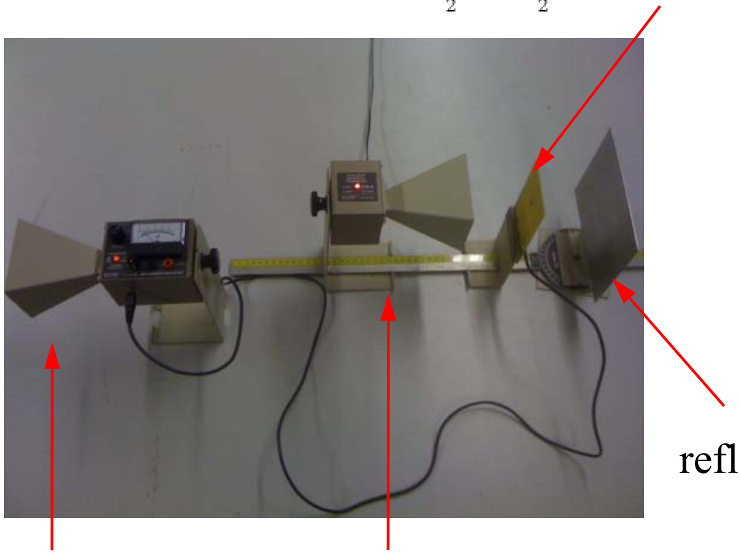
receiver

microwave generator (transmitter)

Measure the wavelength in free space with interference



a standing wave created by two waves each having the same wavelength λ traveling past each other at the same velocity



create a standing wave

↓

move the detector in order to measure the distance between several consecutive peaks in the standing wave

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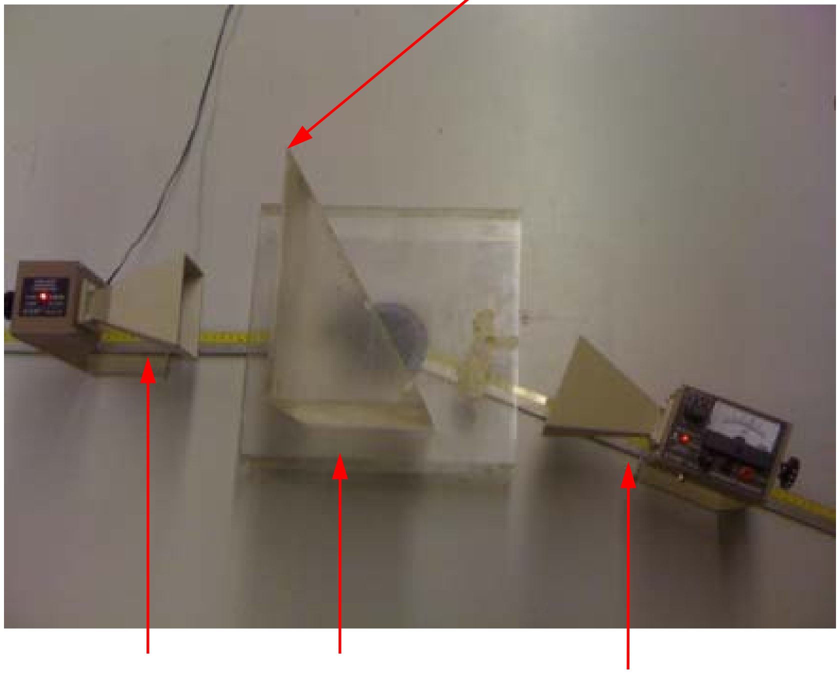
determine the wavelength of the microwave

Determine the index of refraction of a prism using Snell’s law

place a prism so that its face is perpendicular to the beam direction



generator prism receiver

change the angle of the receiver until you get a maximum signal

use the measured angles and Snell’s law to calculate the index of refraction of the prism

repeat the experiment with the prism mirror inverted with respect to the beam axis and average your results