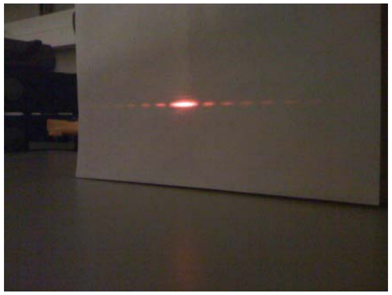
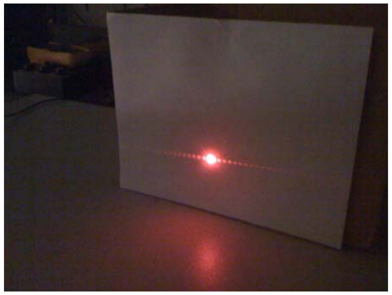
Experiment 6: Diffraction and Interference with Coherent Light

Goal: examine the diffraction and interference patterns caused by laser (coherent) light.

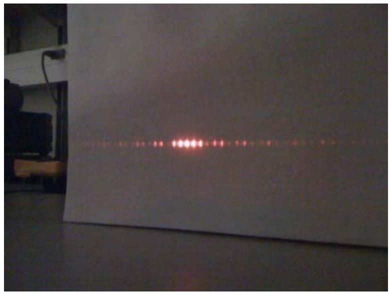
1. Determination of Wavelength Using Single-slit Diffraction



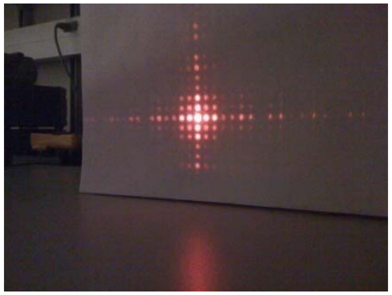
2. Determination of Thin Wires' Diameter



3. Determination of Wavelength Using
Multi-slit Interference Pattern



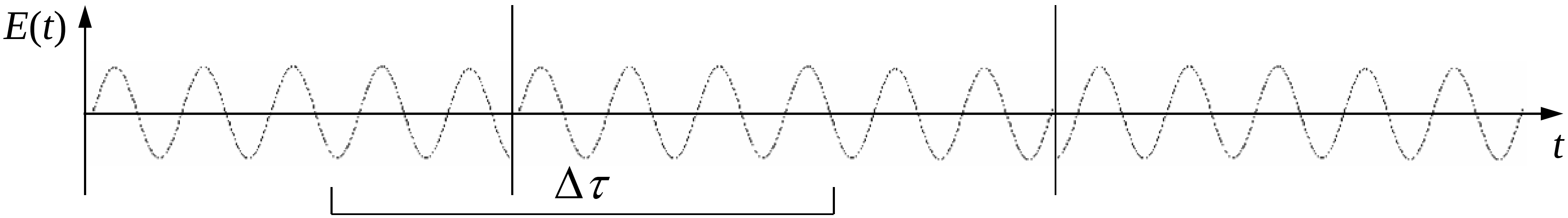
4. Two-Dimensional Array



Coherent and incoherent light

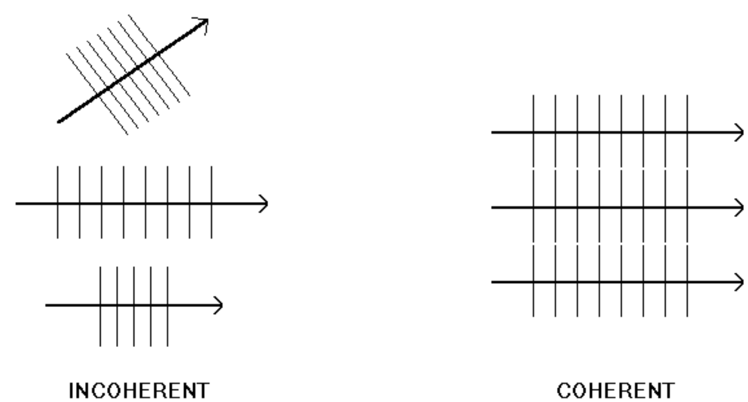
ordinary sources of light produce incoherent light

a beam of incoherent light is composed of a large number of incoherent waves



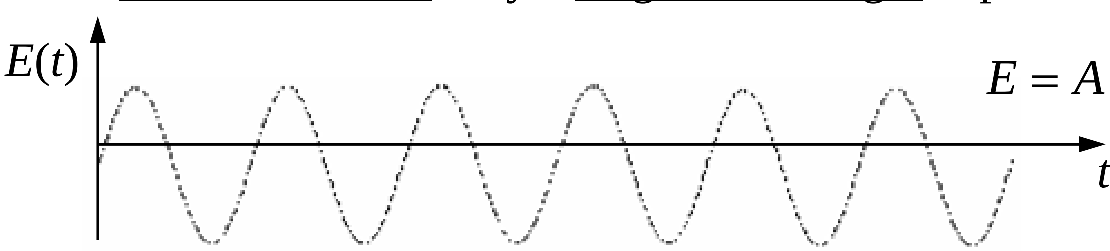
short trains of waves with different phases

incoherent waves have various wavelengths, directions and phases with respect to each other



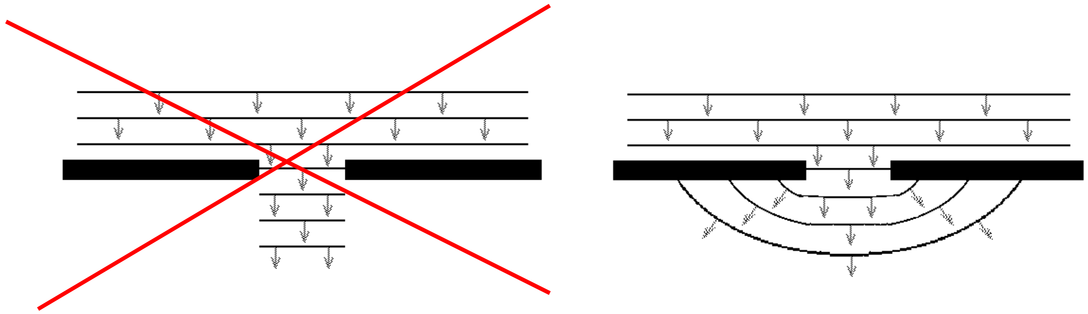
lasers produce coherent light

a beam of coherent light is a single train of waves
it is monochromatic, only a single wavelength is present



$$E = A\cos(kz - \omega t + \phi)$$

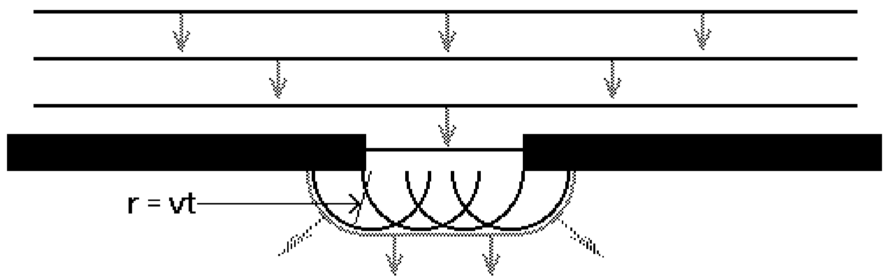
Huygens' Principle



waves passing
through a slit

A <u>wave front </u>is a surface of equal phase

<u>Huygens' Principle</u>: every point on a wave front acts as a point source of spherical waves

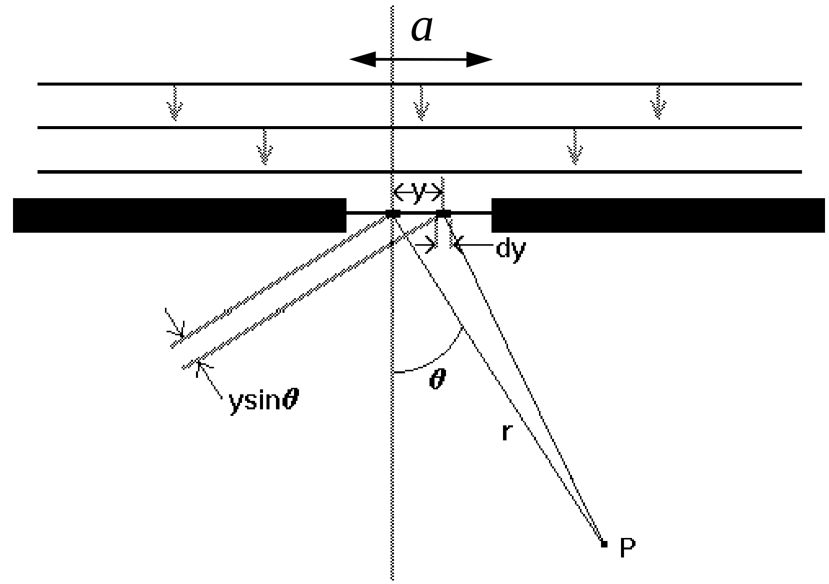


Huygens' construction

To predict the wave front time t, draw circles (spheres in 3D) of radius $r = vt$ having centers along the wave front at $t = 0$. Their envelope will be the wave front at time t.

Diffraction on a single slit

find the amplitude of the waves
as a function of the angle θ
between the observer and the aperture
at large distance from an aperture $r >> a$



Huygens' principle: consider wave front formed by many little generators of spherical (or circular) waves across the aperture

$$dA_P = A_0 \cos(kr - \omega t)dy$$

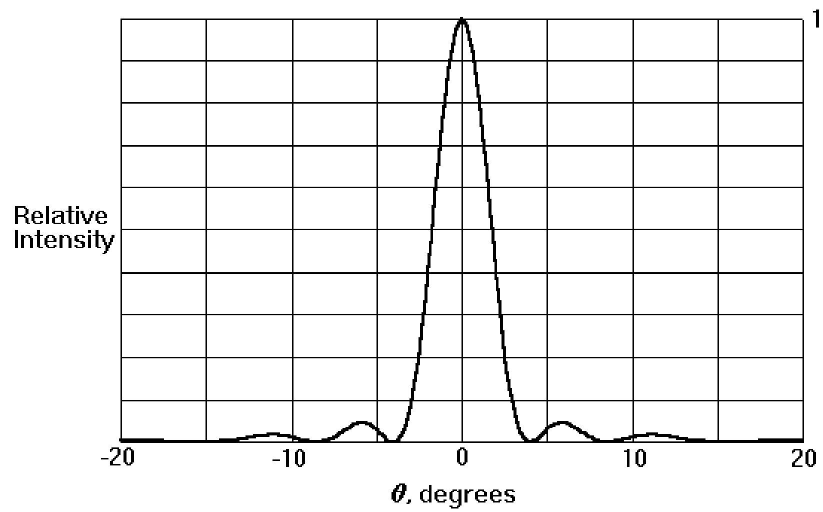
$$dA_P = A_0 \cos\left[k\left(r - y\sin\theta\right) - \omega t\right]dy$$

$$A_P(\theta) = A_0 \int_{-a/2}^{a/2} \cos\left[k\left(r - y\sin\theta\right) - \omega t\right]dy$$

$$A_P(\theta) = A_0 a \frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta} \cos(kr - \omega t)$$

intensity is proportional to the square of the amplitude

$$I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2, \text{ where } \beta = \frac{ka}{2}\sin\theta$$



Diffraction on a single slit

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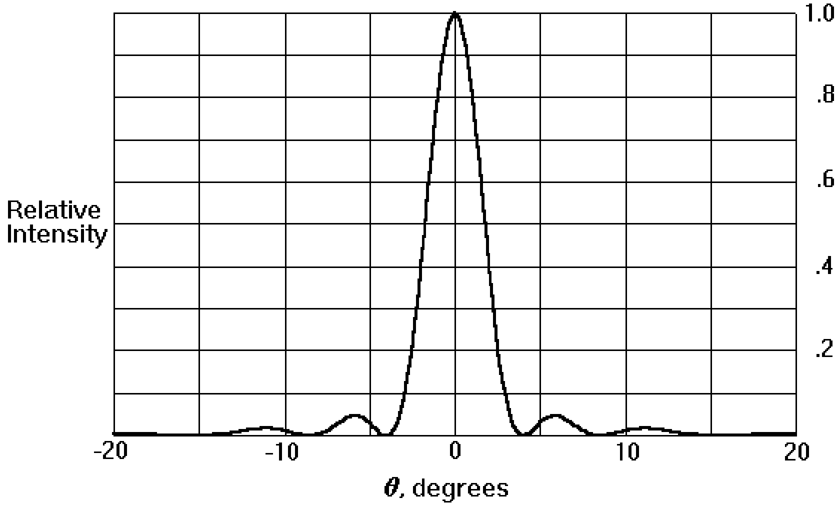


pattern for waves passing through narrow slit of width *a*

1. The intensity goes exactly to zero at "null angles"

$$I = 0 \text{ for } \frac{ka}{2}\sin\theta = \pi, 2\pi, \ldots$$

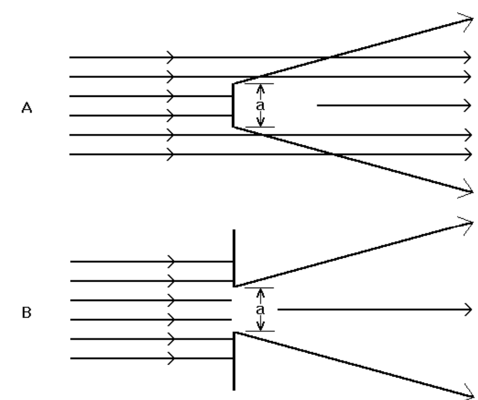
$$k = \frac{2\pi}{\lambda}$$

$$I = 0 \text{ for } \sin\theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \ldots$$

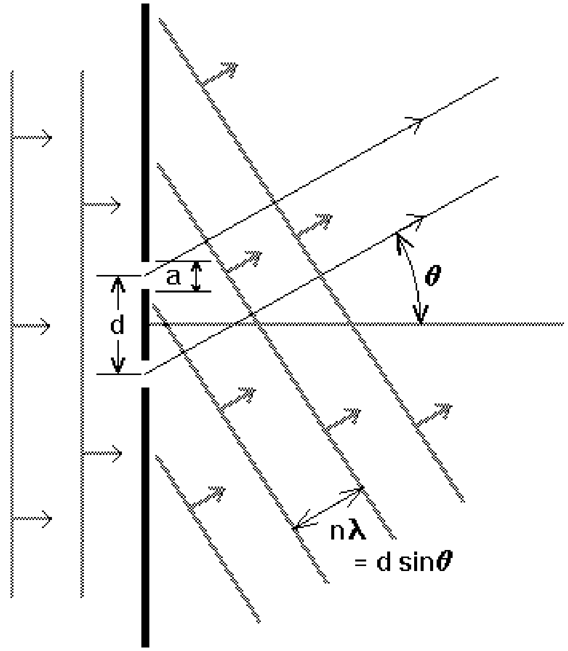
2. The wave pattern does spread out

$$\delta\theta \approx \frac{\lambda}{a}$$

<u>Babinet's Principle</u>: Except for the intensity of the central spot, the diffraction pattern produced by an opaque object is the same as that produced by an aperture of the same size and shape in an otherwise opaque screen



Interference



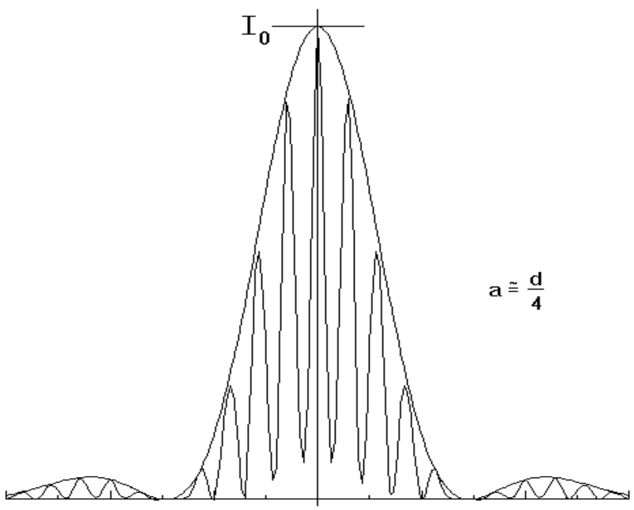
Intensity maxima will occur when all the slit contributions arrive in phase. This requires that the differences in distance must be an integral number of wavelengths

$$d \sin\theta = n\lambda \qquad n = 0, 1, 2, \ldots$$

However, each of the slits also produces its own diffraction pattern, i.e., a series of spots that satisfy

$$I = I_0 \left(\frac{\sin\beta}{\beta} \right)^2$$

$$I = 0 \quad \text{for } \sin\theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \ \ldots$$



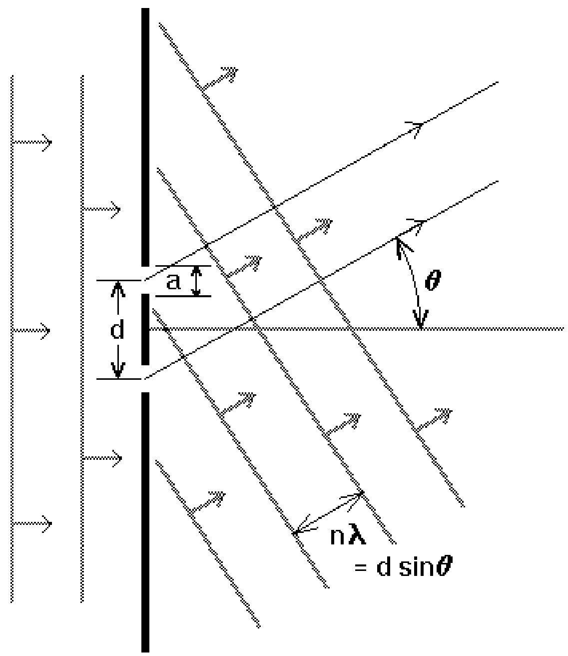
multi-slit interference superimposed on the single-slit diffraction pattern

The diffraction patterns of the individual slits and the interference pattern they produce in combination superimpose.

The closely spaced peaks arise from interference between the slits, while the broader envelope function that modulates the interference maxima is the diffraction pattern of a single slit.

Large features in diffraction or interference patterns correspond to small objects that is probed by the light, and small diffraction features correspond to large objects

Diffraction grating

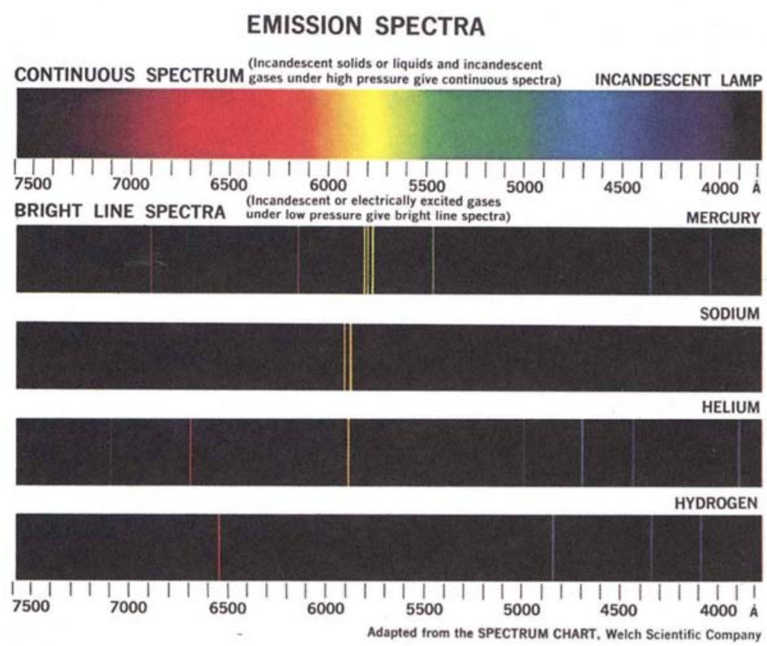


Intensity maxima will occur when all the slit contributions arrive in phase. This requires that the differences in distance must be an integral number of wavelengths

$$d \sin\theta = n\lambda \qquad n = 0, 1, 2, \ldots$$

diffraction grating allows measuring of λ by measuring θ

spectrum measurement



elemental substance identifies itself uniquely by the light it emits

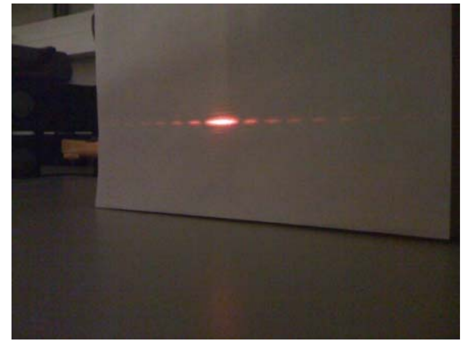
Experiment
1. Determination of Wavelength Using Single-slit Diffraction

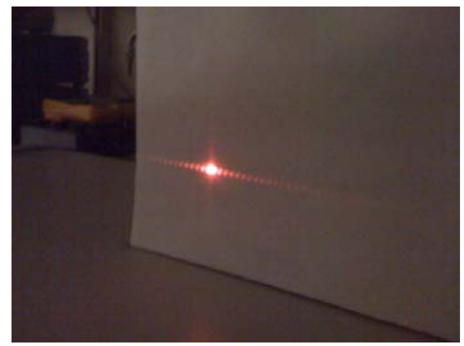
Place the slide with a single slit in front of the laser

Observe the diffraction pattern on a screen located a distance L away from the slide

Measure the distance u from the center of the diffraction pattern to the first minimum (measure the distance between the first minima on either sides of the central peak and divide by 2) and calculate the wavelength.

Do this for several different slit widths a.



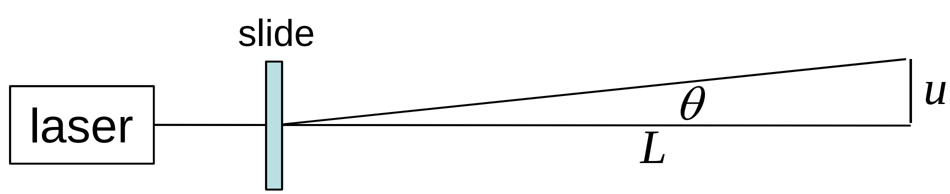


first minimum: $\sin\theta = \frac{\lambda}{a}$

$$\sin\theta = \frac{u}{\sqrt{L^2+u^2}} \longrightarrow \sin\theta \approx \frac{u}{L}$$

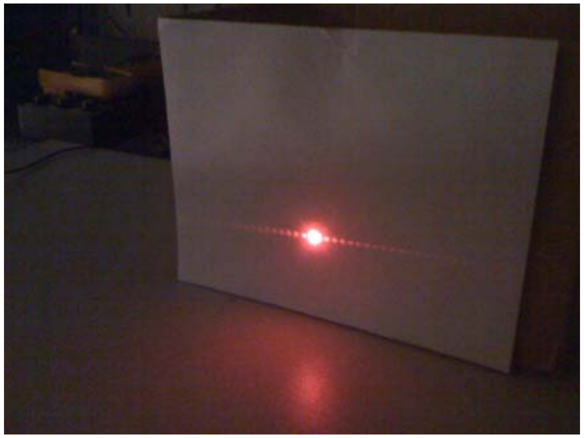
the small angle approximation

$$\frac{u}{L} = \frac{\lambda}{a} \text{ or } u = \lambda\frac{L}{a}$$

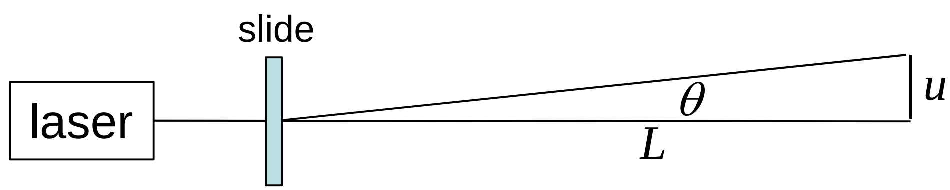


Experiment
2. Determination of Thin Wires' Diameter

Use of Babinet's Principle and the single-slit diffraction pattern to measure the diameters of thin filaments

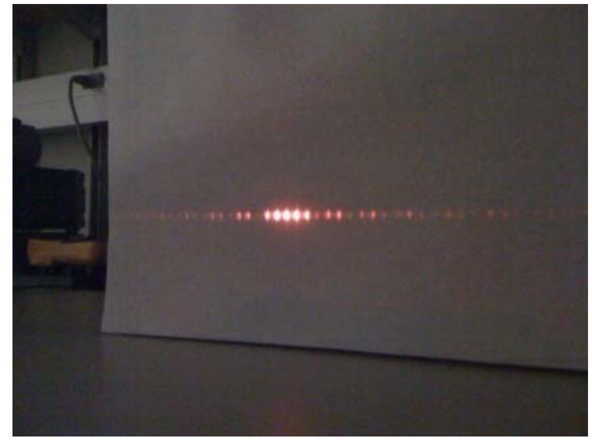


$$\frac{u}{L} = \frac{\lambda}{a} \text{ or } u = \lambda \frac{L}{a}$$



Experiment
3. Determination of Wavelength Using Multi-slit Interference Pattern

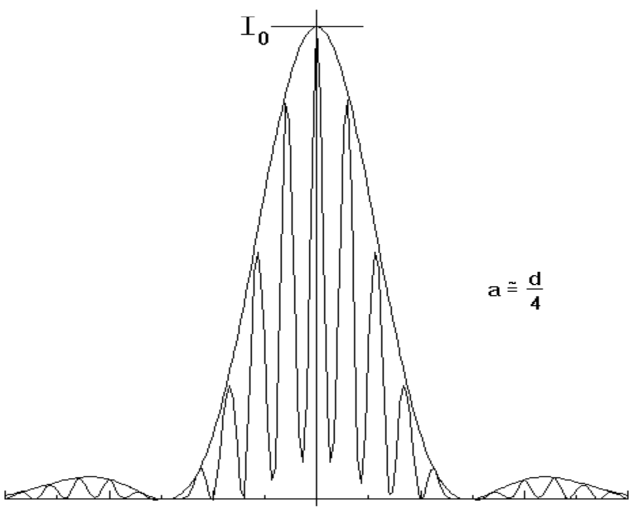
Measure the distance u between an interference maximum and the n^{th} interference maximum.
Determine the wavelength.



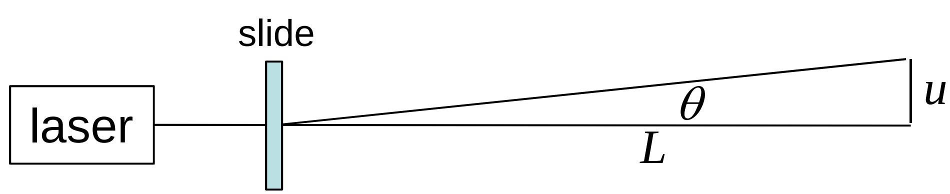
grating equation $d \sin\theta = n\lambda$

↓ the small angle approximation

$$\frac{u}{L} = \frac{n\lambda}{d} \text{ or } u = \frac{n\lambda L}{d}$$

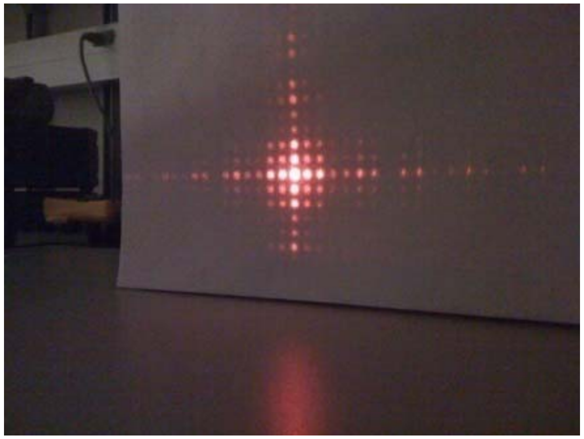


multi-slit interference superimposed on the single-slit diffraction pattern



Experiment
4. Two-Dimensional Array

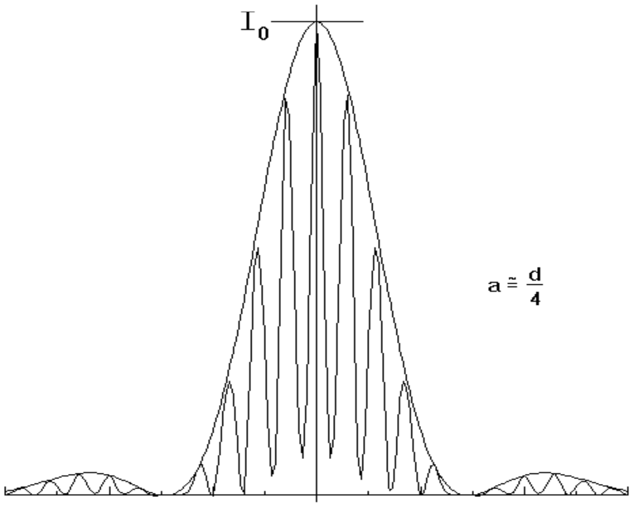
Measure the locations of the minima of the diffraction pattern to find the slit width, and the location of the interference maxima to find the mesh interval



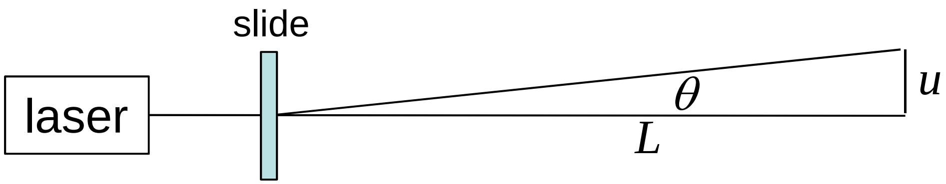
Grating equation $d \sin\theta = n\lambda$

Intensity maxima $\sin\theta = \frac{n\lambda}{d}$

$$I = 0 \quad \text{for} \quad \sin\theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \ \dots$$



multi-slit interference superimposed on the single-slit diffraction pattern



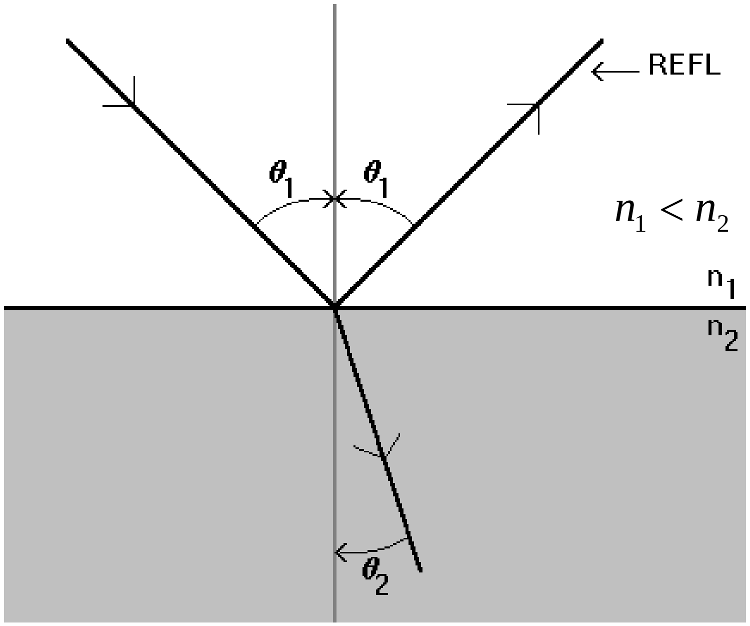
Experiment 7:   Lenses and the Human Eye

Goal: examine the operation of lenses

1. Finding focal lengths of lenses

2. The eye as a variable lens

Snell's Law

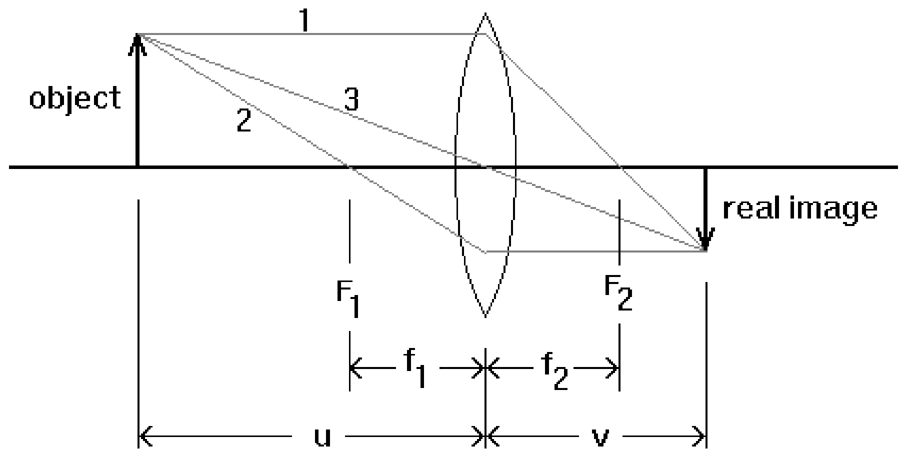


$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Snell's Law

enable the formation of images
by refraction from curved surfaces

Convergent Lenses



for a double convex lens
with symmetric lens concavity
$f_1 = f_2 = f$

Principal rays
Parallel Ray (1): A ray parallel to the axis on the incident side passes through the focus on the other side.

Focal Ray (2): A ray through the focus on the incident side emerges parallel.

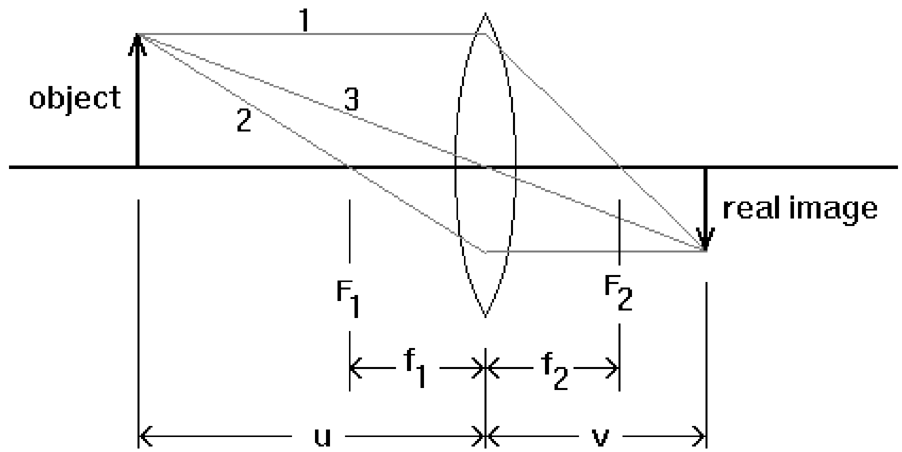
Center Ray (3): A ray directed towards the center of the lens on the incident side emerges undeflected.

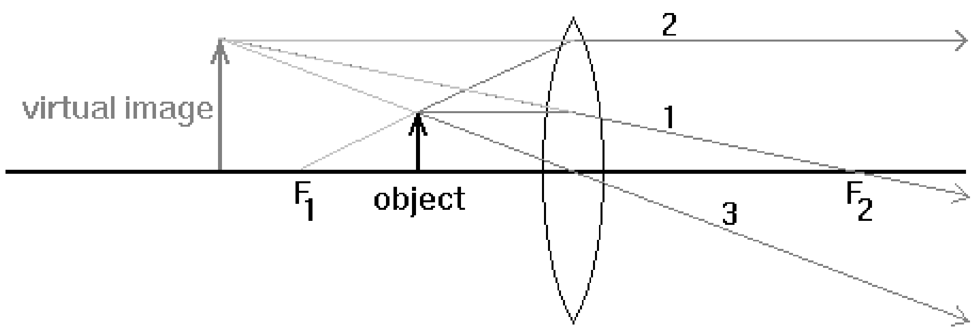
the lens formula $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (f > 0)$$

the magnification, i.e. ratio of image to object size $$m = \frac{-v}{u}$$

a negative m denotes an inverted image

Virtual Image Formation by Convergent Lens

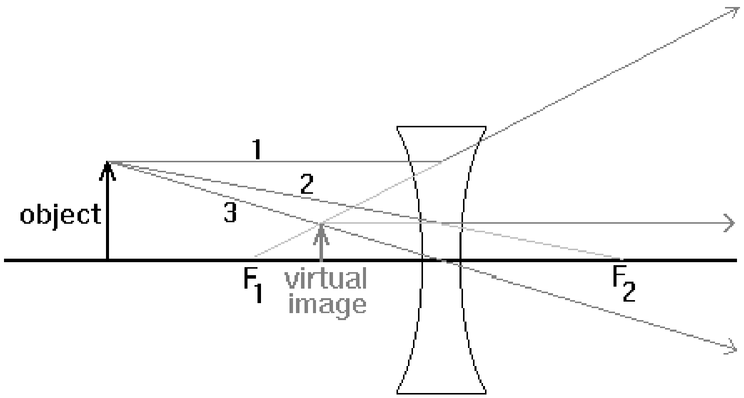




If the rays originating in an object point actually converge on an image point, so that they could be received on a screen, <u>the image is called real</u>.

If the rays do not actually converge but appear to come from the image point, <u>the image is called virtual</u>.

Divergent Lenses



the focal length f is negative

Principal rays

Parallel Ray (1): A ray parallel to the axis incident from the left side emerges as thought it was coming from the focus F1.

Focal Ray (2): A ray incident from the left heading for focus F2 emerges parallel.

Central Ray (3): A ray incident towards the center emerges undeflected.

the lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (f < 0)$$

Combination of Lenses and Power

When two thin lenses, of focal lengths f_1 and f_2, are put close together, the combination acts like a single lens of focal length f

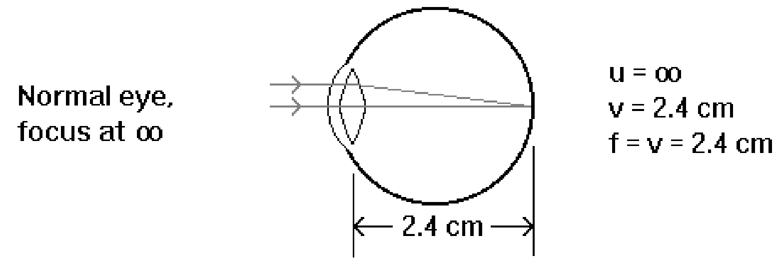
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Power of a lens $$P = \frac{1}{f[m]}$$

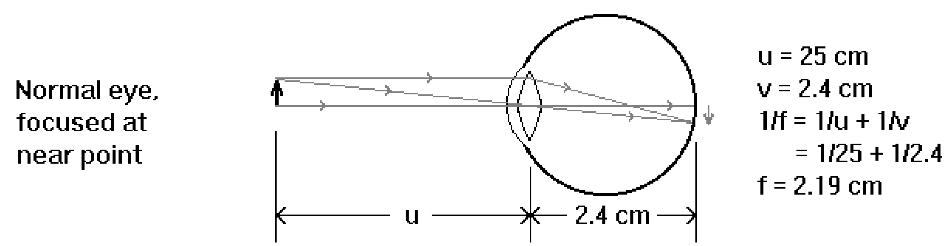
$$P = P_1 + P_2$$

The Human Eye

When the eye muscles are relaxed, the focal length *f* of the lens of the eye is $D \approx 2.4$ cm. An idealized eye will focus an object at infinity on the retina which is located at distance *D* behind the lens.



When the eye views a closer object, the eye muscles produce a shortening of *f* (so-called accommodation).
The closest point on which the eye can focus is called the near point u_{min}~25 cm



adjustments for focus at long distances or near points