Experiment 5: Measurements Magnetic Fields

Introduction

In this laboratory you will use fundamental electromagnetic Equations and principles to measure the magnetic fields of two magnets.

1 Physics

1.1 Faraday's Law and Magnetic Flux

In SI units (or Meters-Kilogram-Seconds, MKS, units), Faraday's law is:

$$V = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = -\frac{d\phi}{dt} \qquad \textbf{(1)}$$

where V is the integral $\int \mathbf{E}\cdot d\mathbf{l}$ around a closed contour, or in a practical sense, the voltage around a circuit, while ϕ is the total magnetic flux passing through the surface contained by the circuit (defined as the integral of the magnetic field over the area of the circuit, A). The minus sign in Equation 1 comes from the fact that the induced current opposes the change in magnetic flux. This is known as Lenz's Law. Since we are going to be concerned in this lab with the magnitudes of these values we will not be concerned with the sign in Equation 1.

1.2 Lorentz Force

The general Lorentz force on a point charge which is in an electromagnetic field is:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \textbf{(2)}$$

Since we are only going to observe a force in a purely magnetic field we can immediately set E = 0. In addition, if we are looking at a steady flow of charged particles it will be better for our purposes to look at the chunk of force produced by a small current element $I\,d\mathbf{l}$ due to a magnetic field $\boldsymbol{B}$. That differential force is:

$$d\mathbf{F} = I\,d\mathbf{l}\times\mathbf{B} \qquad \textbf{(3)}$$

where the appropriate units are: V (Volts), ϕ (Tesla-m^2=Weber), $\boldsymbol{F}$ (Newtons), I (Amperes), $\boldsymbol{B}$ (Tesla), and length (m). $I\,d\mathbf{l}$ has replaced $q\mathbf{v}$ in this Equation. Take note that these two quantities have the same units and that they both express a small amount of moving charge.

2 Mathematic Analysis

2.1 Flip Coil Method

Having written down the essential mathematical relationships between magnetic field, potential, and current, we must now ask how we can use these to measure a constant magnetic field. Consider Faraday's law first. It relates a circuit voltage to the time derivative of magnetic flux though it; yet our problem is the measurement of a steady field which has no time variation at all. Furthermore, even if the field were varying, we are interested in its magnitude, not its rate of increase.

Let us arrive at a technique by considering the above two objections in reverse order. How do we find the magnitude of ***B*** (or ϕ) from its derivative? One way is to integrate the Equation. However, how does one integrate a voltage from an electrical circuit? There are several ways, actually, but the simplest is one of the best. Imagine the electrical network shown in Figure 1.

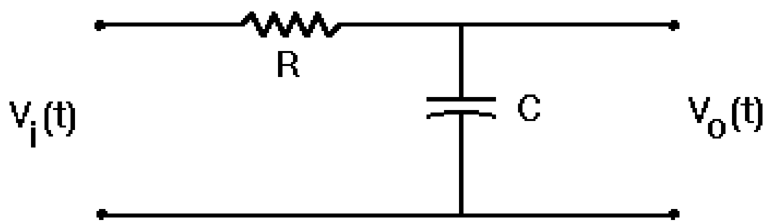

Figure 1 – *An RC integrator.*

We will impress a time varying voltage $V_i(t)$ at the input terminals at the left, and look at the resulting output voltage $V_0(t)$ at the right.

Recall first that the voltage across a capacitor is proportional to the total charge *Q* placed on it, where the constant of proportionality is the capacitance, *C*. Thus,

$$V = \frac{Q}{C} \tag{4}$$

Since current, *I*, is a rate of charge transfer, it follows that:

$$Q(t_1) = \int_0^{t_1} I dt \tag{5}$$

The preceding Equation assumes that ***Q*** = 0 at *t* = 0. Then,

$$V(t_1) = \frac{1}{C}\int_0^{t_1} I dt = V_o(t_1) \tag{6}$$

since our output voltage is just the voltage across the capacitor. Now, Ohm's law tells us that:

$$I = \frac{V_i - V_o}{R} \tag{7}$$

and so by direct substitution into Equation 6,

$$\begin{aligned} V_o(t_1) &= \frac{1}{RC}\int_0^{t_1}\left[V_i(t) - V_o(t)\right]dt \\ &= \frac{1}{RC}\int_0^{t_1} V_i(t)dt - \frac{1}{RC}\int_0^{t_1} V_o(t)dt \end{aligned} \tag{8}$$

If the product *RC*, which itself has the dimensions of a time, is much larger than the integration time t_1, the second right-hand term becomes small compared to V_0 itself. (i.e., we approximate $\int_0^{t_1} V_0 dt$ by $V_o t_1$, and so $V_0 t_1 / RC << V_0$). Then,

$$V_o(t_1) = \frac{1}{RC}\int_0^{t_1} V_i(t)dt \tag{9}$$

our circuit acts an **integrator**.

We can now connect such an integrator to a loop of wire located in a time-varying magnetic field increasing from $B = 0$, and obtain an output V_0, the magnitude of which is related to the flux through the loop by

$$V_o(t) = \frac{1}{RC}\int_0^{t} \frac{d\phi}{dt} dt = \frac{\phi(t)}{RC} \tag{10}$$

as long as $RC >> t$. If the loop has an area *A*, and the field strength *B* is **uniform** over this area[1], then $\phi = AB$, and

$$V_o(t) = \frac{A}{RC} B(t) \tag{11}$$

We can measure a magnetic field that has increased from zero in some suitable short time *t*. But how about measuring a steady field? You may already have anticipated the answer to this. We will simply take our loop which initially rests in a place where there is no field and thrust it suddenly into the

[1] If the field is uniform throughout the interior of the coil then you will use the area of the coil for *A*. If the area of the uniform magnetic field is smaller then you must use the area, over which the field is uniform.

magnetic region (or visa versa). By "suddenly", we mean in a time much shorter than the discharge time constant *RC*. The loop itself then sees a sudden rising field, and develops an appropriate voltage at its terminals. In fact, in most cases easier to have the coil at rest within the field, and then suddenly pull it out. The total *change* in *B*, which is the initial *B* itself, will appear as a change in V_0 $(= \Delta V_0)$ at the integrator output. Figure 2 illustrates our scheme, which is called the **flip-coil technique**.

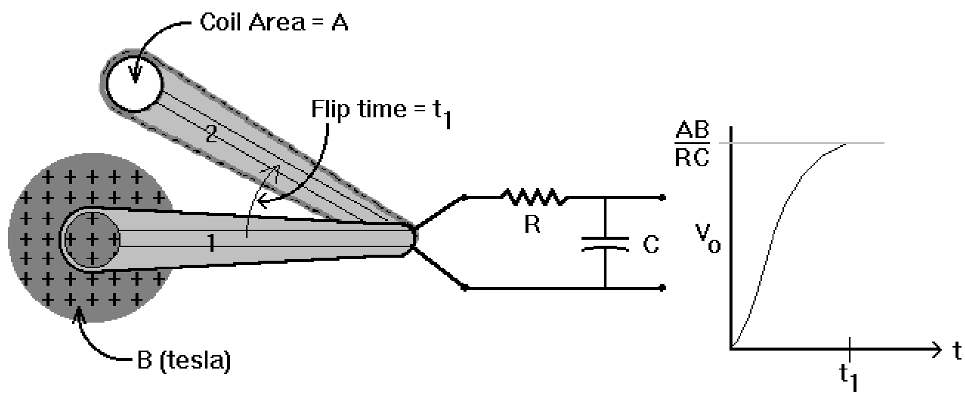

Figure 2 – *Illustration of the setup for moving a flip-coil out of a magnetic field, while integrating the induced currents with an RC circuit.*

Sometimes the value of V_0 we obtain (while still satisfying the requirement that $RC >> t_1$) is inconveniently low. This is easily remedied by wrapping several turns, say *n* of them, on the loop. The output increases by a factor of *n*, since this is just equivalent to connecting *n* separate single-turn flip coils in series and thus adding their output voltages. In this case, then,

$$\Delta V_o = \frac{nA}{RC} \Delta B \tag{12}$$

What happens, now, when the "flip" has occurred, and the integrator output voltage has jumped to ΔV_o? We can answer this by taking another look at the circuit, as shown in Figure 3.

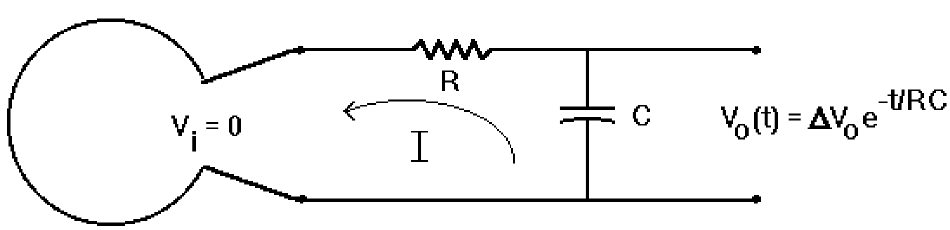

Figure 3 – *The RC integrator, discharging through the loop.*

There is no more input voltage, since ϕ = 0 in the coil. The capacitor simply discharges through *R* and the coil, whose resistance is usually close to zero. You may recall that when a capacitor discharges through a resistance, its voltage decays as

$$V(t) = V_o e^{-t/RC} \qquad \textbf{(13)}$$

We conclude then that the whole history of the integrator output voltage must look something like that shown in Figure 4.

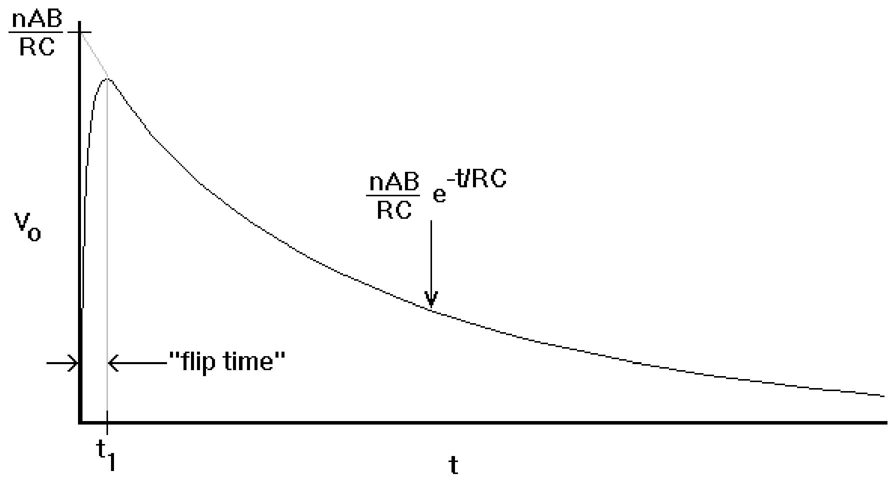

Figure 4 – *Time history of the output voltage for a flip coil moved rapidly out of a magnetic field region and integrated by an RC circuit followed by discharge of the capacitor.*

We can see now in what sense the flip of the coil must be sudden. If the discharge rate of the capacitor is anywhere near the charging rate during the flip of the coil, the output voltage will never reach its proper level. We must then make t_1 much less than the discharge time constant which is *RC*. This is simply restating our criterion for accurate integration by our integrator circuit.

There is one more Experimental "fact of life" to take into account in this system. It is that the device which measures $V_0(t)$ has something less than infinite resistance itself. The input resistance of the oscilloscope you will use is 10^6 ohms, for example. The complete circuit for our measurement is then:

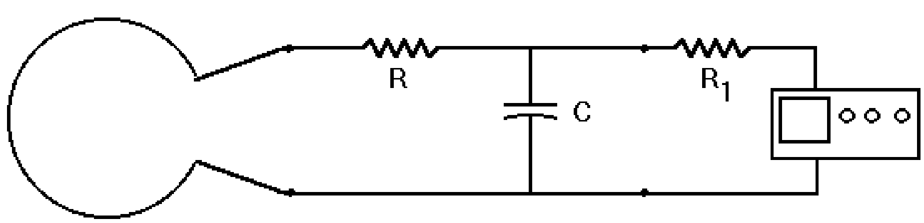

Figure 5 – *The flux integrator, with the oscilloscope input resistance shown as R_1.*

Since R and R_1 are in parallel as far as the discharge of the capacitor is concerned, the decay time constant is R_2C, where:

$$R_2 = \frac{R_1 R}{R_1 + R} \tag{14}$$

However, this does not change the integration time constant. The expression which you will use to find the magnetic field still contains *RC* and *not* R_2C. Thus,

$$\Delta V_o = \frac{nA}{RC}\Delta B \tag{15}$$

where now the flip time t_1 must be kept small compared to R_2C.

> *Question 5.1*
>
> Assume that the flip time is approximately 50ms. Choose a capacitor from the following: 0.001, 0.01, 0.1 and 1.0μF. What is an appropriate choice for the resistance *R*?

2.2 Current Balance Technique

The measurement of ***B*** by measuring the force on a current in the field is quite straightforward. The only subtlety involved in such an Experiment lies in the means by which one makes sure that the force which is measured is applied only to a well-defined segment of conductor in a homogeneous region of the field. That is, the place where the magnetic field is uniform should be where the current is applied.

Suppose we were to try a measurement of the field between the poles of a magnet where the field has a typical distribution as shown in Figure 6.

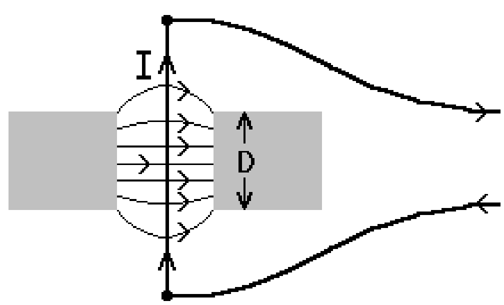

Figure 6 – *Current loop passing through the magnetic field created by a dipole magnet.*

We draw "field lines" to represent ***B***, as has become conventional, where the density of lines is a measure of the local field strength. It is characteristic of this field distribution that the magnitude of *B*, i.e., the line density, becomes smaller in the "fringing" region, and falls gradually from its maximum value between the poles to zero far outside.

A wire carrying current *I* will then feel a total force:

$$F_x = \int I_y B_z(y)dy \qquad \textbf{(16)}$$

However, this doesn't tell the Experimenter what ***B*** itself is at any point. We can only determine the integral $\int B_z dy$. If ***B*** were perfectly constant over the width of the pole faces and dropped abruptly to zero at the edges, then things would be simpler, and the integral would be $B_z D$, where D is the pole diameter. In this case, the field would then be given by

$$B_z = \frac{F_x}{I_y D} \qquad \textbf{(17)}$$

The strategy you could employ in this Experiment would have *I* flow only in the homogeneous region of the field. A moment's thought exposes this as nonsense, however, since the current has to enter from the outside and return there again.

A more realistic and workable strategy would be to arrange the wire so that the forces experienced by the wire as it enters and leaves through the inhomogeneous part of the field (i.e., the fringing field) cancel each other out, leaving only a net force from a section of the wire in the uniform field. How can these entering and leaving forces be made to cancel? Recall the differential Lorentz force from Equation 2:

$$d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B}$$

which can be resolved into:

$$dF_x = I_y B_z dl \qquad \textbf{(18)}$$

in the coordinates of our example. B_z does not change sign, and therefore, if we want to generate two forces having opposite signs in the fringing field, we can only do so by using two oppositely directed currents. You should be ready by now to appreciate the unique properties of the arrangement of wire in the region of the magnet poles shown in Figure 7 below.

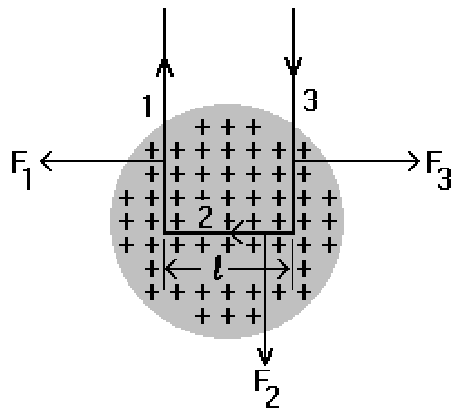

Figure 7 – *Forces on a hairpin circuit with current I, placed in a magnetic field as seen by looking along a line of B towards a pole face.*

The wire has the form of a hairpin, with three distinct straight segments, labeled 1, 2, 3, in the field. It should be clear that since I is equal, but oppositely directed in legs 1 and 3, and since these pass through identical field distributions, their forces, F_1 and F_3 are also equal and opposite, and so add to zero.

All that remains is leg 2 of length L, which is short enough to be everywhere in a uniform field. Therefore, the total force on the hairpin is

$$F_2 = ILB \qquad \textbf{(19)}$$

and one may now measure B by, for example, hanging the hairpin from a balance so as to measure F_2, and sending some carefully measured current I through it. This is precisely what you will do in this Experiment.

Question 5.2

Solve for B in Equation 19. What is the propagated error for the magnetic field, ΔB?

3 The Experiment

3.1 Flip Coil Method

You will determine the magnetic field of the circular magnet using the flip coil technique. Construct an integrator circuit (refer to Figure 5) for use with the flip coil. Note that according to Equation 15 the output voltage is proportional to $1/RC$, so a small value of RC is preferred. However, from the discussion leading up to Equation 14, the flip time must be kept small compared to R_2C. Since $R_2C < RC$, this latter condition may be quite difficult to obtain if the value of RC is chosen too small.

If the pole faces of the permanent magnets are smaller than the diameter of the flip coil so that the magnetic field will not be uniform across the coil, then the area used in Equation 10 (shown again below) will be that of the area of uniform magnetic field. The method measures the total flux change in the coil (c.f. Equation 10/11 discussion and Figure 6). If we assume that the field is approximately uniform over the area of the pole faces and zero elsewhere, then Equation 10,

$$V_0(t) \approx \frac{A}{RC}\big(B(t) - B(0)\big)$$

still applies but A must now be the area of the pole faces rather than that of the coils.

Take at least 10 measurements of the resulting values of V_0 to determine a value for the magnetic field strength. You may use the standard deviation of the mean of your measurements as your uncertainty.

Question 5.3 If you have taken the following measurements of the peak voltage from the oscilloscope, what are the average, standard deviation and the standard deviation of the mean? Measurements: 0.10, 0.12, 0.11, 0.13, 0.14, 0.10, 0.09 and 0.08 mV.

Hints:

- *The flip coil axis must be aligned with the magnetic field to avoid introducing a factor of* $\cos(\theta)$ *(where* θ *is the misalignment angle) into Equation 14. To insure accurate coil alignment, simply rest the coil against one of the pole faces while "flipping" it.*

- *If the scope trigger is set on "auto" the spot will be sweeping across the screen continuously. If the sweep is slow enough it should be possible to time the "flip" so that it occurs near the center of the display, where it can be easily measured.*

3.2 Current Balance Technique

Measure the magnetic field of the wide horse-shoe shaped magnet by measuring the force on the magnet as a function of the current in the hairpin between the pole faces. You will use the setup in Figure 8. Choose several hairpins (at least 3) of different lengths, L. For each different hairpin choose several (at least 4) different currents. Measure the mass difference from the scale and calculate the Lorentz force.

Using ORIGIN plot F versus nIL. You should only have one plot. Some of the hairpins have current flowing on each side of the plastic faceplate. You will need to double the value of the current to account for twice as much current flowing through the uniform magnetic field.

Hints:

- Measure the mass of the magnetic before turning on the power supply. The difference in mass will correspond to the Lorentz force.

- When you turn the power supply on and off, make sure that the current is turned to zero. You could damage the power supply otherwise.

Question 5.4 What is the slope of the curve which you are asked to produce below?

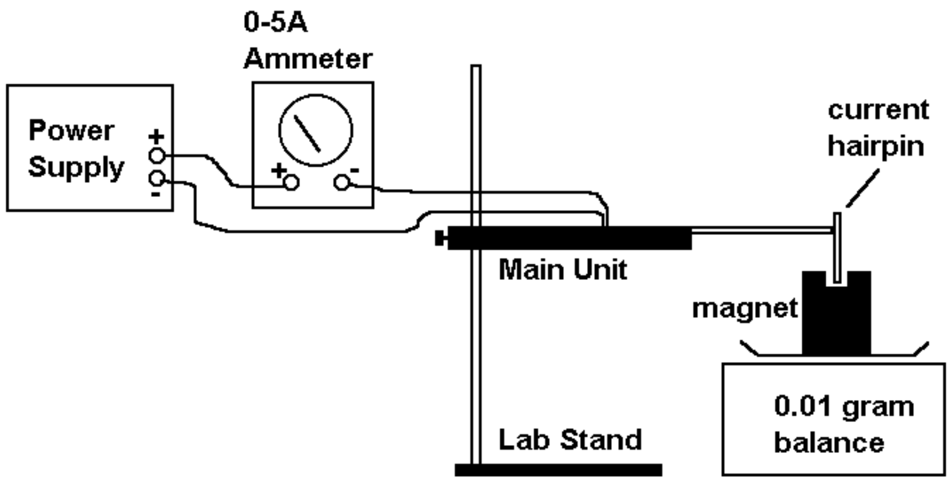

Figure 8 – *Schematic view of the Experimental setup for the force measurement*

Analysis

Compute the following:

- Mean and standard deviation of the mean of voltage data;
- Determination of square magnet's field using flip coil.

Conclusions

Highlight the themes of the lab and the physics the experiment verifies. You should discuss the errors you encounter in the lab and how you could improve the lab if you had to repeat it. If your results are unexpected or your t-values are high, you should identify possible explanations.

Discuss the results of both methods of determining magnetic fields. Which one do you find more reliable? Why?

Hints on reports

- Write neatly—if your TA cannot read it, you could lose points.
- Be organized—if your TA cannot find it, you could lose points.
- Report your data, including plots—if your data is not in your report, your TA does know you did it.
- Record uncertainty.
- Propagate uncertainty.
- Write your final answers with proper significant figures.