3

The Quantum Theory of Light

3-2 Assume that your skin can be considered a blackbody. One can then use Wien's displacement law, $\lambda_{\max} T = 0.289\,8 \times 10^{-2}\ \text{m} \cdot \text{K}$ with $T = 35\ ^0\text{C} = 308\ \text{K}$ to find

$$\lambda_{\max} = \frac{0.289\,8 \times 10^{-2}\ \text{m} \cdot \text{K}}{308\ \text{K}} = 9.41 \times 10^{-6}\ \text{m} = 9\,410\ \text{nm}\ .$$

3-4 (a) From Stefan's law, one has $\frac{P}{A} = \sigma T^4$. Therefore,

$$\frac{P}{A} = \left(5.7 \times 10^{-8}\ \text{W}/\text{m}^2 K^4\right)\left(3000\ \text{K}\right)^4 = 4.62 \times 10^6\ \text{W}/\text{m}^2\ .$$

(b) $$A = \frac{P}{4.62 \times 10^6\ \text{W}/\text{m}^2} = \frac{75\ \text{W}}{4.62 \times 10^6\ \text{W}/\text{m}^2} = 16.2\ \text{mm}^2.$$

3-5 (a) Planck's radiation energy density law as a function of wavelength and temperature is given by $u(\lambda,\ T) = \frac{8\pi hc}{\lambda^5\left(e^{hc/\lambda_B T} - 1\right)}$. Using $\frac{\partial u}{\partial \lambda} = 0$ and setting $x = \frac{hc}{\lambda_{\max} k_B T}$, yields an extremum in $u(\lambda,\ T)$ with respect to λ. The result is

$$0 = -5 + \left(\frac{hc}{\lambda_{\max} k_B T}\right)\left(e^{hc/\lambda_{\max} k_B T}\right)\left(e^{hc/\lambda_{\max} k_B T} - 1\right)^{-1} \text{ or } x = 5\left(1 - e^{-x}\right).$$

(b) Solving for x by successive approximations, gives $x \cong 4.965$ or

$$\lambda_{\max} T = \left(\frac{hc}{k_B}\right)(4.965) = 2.90 \times 10^{-3}\ \text{m} \cdot \text{K}\ .$$

3-10 The energy per photon, $E = hf$ and the total energy E transmitted in a time t is Pt where power $P = 100\ \text{kW}$. Since $E = nhf$ where n is the total number of photons transmitted in the time t, and $f = 94\ \text{MHz}$, there results $nhf = (100\ \text{kW})t = \left(10^5\ \text{W}\right)t$, or

$$\frac{n}{t} = \frac{10^5\ \text{W}}{hf} = \frac{10^5\ \text{J/s}}{6.63 \times 10^{-34}\ \text{J s}\left(94 \times 10^6\ \text{s}^{-1}\right)} = 1.60 \times 10^{30}\ \text{photons}/\text{s}\ .$$

3-14 (a) $$K = hf - \phi = \frac{hc}{\lambda - \phi} = \frac{1\,240\ \text{eV nm}}{350\ \text{nm}} - 2.24\ \text{eV} = 1.30\ \text{eV}$$

(b) At $\lambda = \lambda_c$, $K = 0$ and $\lambda = \frac{hc}{\phi} = \frac{1\,240\ \text{eV nm}}{2.24\ \text{eV}} = 554\ \text{nm}$

3-16 (a) $\phi = \frac{hc}{\lambda} - K$, $\phi = \frac{1\,240 \text{ eV nm}}{300 \text{ nm}} - 2.23 \text{ eV} = 1.90 \text{ eV}$

(b) $V_s = \frac{1\,240 \text{ eV nm}}{400 \text{ nm e}} - 1.90 \text{ eV}/\text{e} = 1.20 \text{ V}$

3-17 The energy of one photon of light of wavelength $\lambda = 300 \text{ nm}$ is

$$E = \frac{hc}{\lambda} = \frac{1\,240 \text{ eV nm}}{300 \text{ nm}} = 4.13 \text{ eV}.$$

(a) As lithium and beryllium have work functions that are less than 4.13 eV, they will exhibit the photoelectric effect for incident light with this energy. However, mercury will not because its work function is greater than 4.13 eV.

(b) The maximum kinetic energy is given by $K = \frac{hc}{\lambda - \phi}$, so

$K(\text{Li}) = \frac{1\,240 \text{ eV nm}}{300 \text{ nm}} - 2.3 \text{ eV} = 1.83 \text{ eV}$, and $K(\text{Be}) = \frac{1\,240 \text{ eV nm}}{300 \text{ nm}} - 3.9 \text{ eV} = 0.23 \text{ eV}$.

3-18 (a) $K_{\max} = eV_s = s(0.45 \text{ V}) = 0.45 \text{ eV}$

(b) $\phi = \frac{hc}{\lambda - K} = \frac{1\,240 \text{ eV nm}}{500 \text{ nm}} - 0.45 \text{ eV} = 2.03 \text{ eV}$

(c) $\lambda_c = \frac{hc}{\phi} = \frac{1\,240 \text{ eV nm}}{2.03 \text{ eV}} = 612 \text{ nm}$

3-20 $K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi \Rightarrow \phi = \frac{hc}{\lambda} - K_{\max}$;

First Source: $\phi = \frac{hc}{\lambda} - 1.00 \text{ eV}$.

Second Source: $\phi = \frac{hc}{\frac{\lambda}{2}} - 4.00 \text{ eV} = \frac{2hc}{\lambda} - 4.00 \text{ eV}$.

As the work function is the same for both sources (a property of the metal),

$$\frac{hc}{\lambda} - 100 \text{ eV} = \frac{2hc}{\lambda} - 4.00 \text{ eV} \Rightarrow \frac{hc}{\lambda} = 3.00 \text{ eV} \text{ and}$$

$$\phi = \frac{hc}{\lambda} - 1.00 \text{ eV} = 3.00 \text{ eV} - 1.00 \text{ eV} = 2.00 \text{ eV}.$$

3-25 $E = 300 \text{ keV}$, $\theta = 30°$

(a) $\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta) = (0.002\,43 \text{ nm})[1 - \cos(30°)] = 3.25 \times 10^{-13} \text{ m}$

$= 3.25 \times 10^{-4} \text{ nm}$

(b) $E = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{E_0} = \frac{\left(4.14\times10^{-15}\text{ eVs}\right)\left(3\times10^{8}\text{ m/s}\right)}{300\times10^{3}\text{ eV}} = 4.14\times10^{-12}\text{ m}$; thus,

$\lambda' = \lambda_0 + \Delta\lambda = 4.14\times10^{-12}\text{ m} + 0.325\times10^{-12}\text{ m} = 4.465\times10^{-12}\text{ m}$, and

$E' = \frac{hc}{\lambda'} \Rightarrow E' = \frac{\left(4.14\times10^{-15}\text{ eV s}\right)\left(3\times10^{8}\text{ m/s}\right)}{4.465\times10^{-12}\text{ m}} = 2.78\times10^{5}\text{ eV}$.

(c) $\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e$, (conservation of energy)

$$K_e = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right) = \frac{\left(4.14\times10^{-15}\text{ eV s}\right)\left(3\times10^{8}\text{ m/s}\right)}{\frac{1}{4.14\times10^{-12}} - \frac{1}{4.465\times10^{-12}}} = 22\text{ keV}$$

3-27 Conservation of energy yields $hf = K_e + hf'$ (Equation A). Conservation of momentum yields $p_e^2 = p'2 + p^2 - 2pp'\cos\theta$. Using $p_{\text{photon}} = \frac{E}{c} = \frac{hf}{c}$ there results $p_e^2 = \left(\frac{hf'}{c}\right)^2 + \left(\frac{hf}{c}\right)^2 - 2\left(\frac{hf}{c}\right)\left(\frac{hf'}{c}\right)\cos\theta$ (Equation B). If the photon transfers all of its energy, $f' = 0$ and Equations A and B become $K_e = hf$ and $p_e^2\left(\frac{hf}{c}\right)^2$ respectively. Note that in general, $K_e = E_e - m_e c^2 = \left[p_e^2 c^2 + \left(m_e c^2\right)^2\right]^{1/2} - m_e c^2$. Finally, substituting $K_e = hf$ and $P_e^2 = \left(\frac{hf}{c}\right)^2$ into $K_e = \left[p_e^2 c^2 + \left(m_e c^2\right)^2\right]^{1/2} - m_e c^2$, yields $hf = \left[(hf)^2 + \left(m_e c^2\right)^2\right]^{1/2} - m_e c^2$ (Equation C). As Equation C is true only if *h*, or *f*, or m_e , or *c* is zero and all are non-zero this contradiction means that f' cannot equal zero and conserve both relativistic energy and momentum.

3-28 (a) From conservation of energy we have $E_0 = E' + K_e = 120\text{ keV} + 40\text{ keV} = 160\text{ keV}$. The photon energy can be written as $E_0 = \frac{hc}{\lambda_0}$. This gives

$$\lambda_0 = \frac{hc}{E_0} = \frac{1\,240\text{ nm eV}}{160\times10^{3}\text{ eV}} = 7.75\times10^{-3}\text{ nm} = 0.007\,75\text{ nm} .$$

(b) Using the Compton scattering relation $\lambda' - \lambda_0 = \lambda_c(1-\cos\theta)$ where $\lambda_c = \frac{h}{m_e c} = 0.002\,43\text{ nm}$ and $\lambda' = \frac{hc}{E'} = \frac{1\,240\text{ nm eV}}{120\times10^{3}\text{ eV}} = 10.3\times10^{3}\text{ nm} = 0.010\,3\text{ nm}$. Solving the Compton equation for $\cos\theta$, we find

$$-\lambda_c\cos\theta = \lambda' - \lambda_0 - \lambda_c$$
$$\cos\theta = 1 - \frac{\lambda' - \lambda_0}{\lambda_c} = 1 - \frac{0.010\,3\text{ nm} - 0.007\,5\text{ nm}}{0.002\,43\text{ nm}} = 1 - 1.049 = -0.049$$

The principle angle is $87.2°$ or $\theta = 92.8°$.

(c) Using the conservation of momentum Equations 3.30 and 3.31 one can solve for the recoil angle of the electron.

$$p = p' \cos\theta + p_e \cos\phi$$

$p_e \sin\phi = p' \sin\theta$; dividing these equations one can solve for the recoil angle of the electron

$$\tan\phi = \frac{p' \sin\theta}{p - p'\cos\theta} = \left(\frac{h}{\lambda'}\right)\frac{\sin\theta}{\frac{h}{\lambda_0} - \frac{h}{\lambda'\cos\theta}} = \left(\frac{hc}{\lambda'}\right)\frac{\sin\theta}{\frac{hc}{\lambda_0} - \frac{hc}{\lambda'\cos\theta}}$$
$$= \frac{120\ \text{keV}(0.998\ 8)}{160\ \text{keV} - 120\ \text{keV}(-0.049)} = 0.723\ 2$$

and $\phi = 35.9°$.

3-30 Maximum energy transfer occurs when the scattering angle is 180 degrees. Assuming the electron is initially at rest, conservation of momentum gives

$$hf + hf' = p_e c = \sqrt{(m_e c^2 + K)^2 - m^2c^4} = \sqrt{(511+50)^2} = 178\ \text{keV}$$

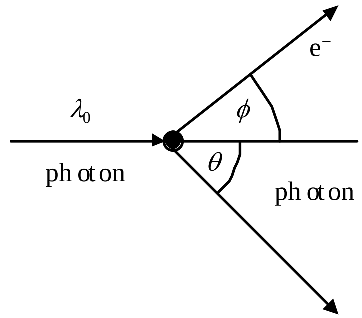
while conservation of energy gives $hf - hf' = K = 30\ \text{keV}$. Solving the two equations gives $E = hf = 104\ \text{keV}$ and $hf = 74\ \text{keV}$. (The wavelength of the incoming photon is $\lambda = \frac{hc}{E} = 0.012\ 0\ \text{nm}$.

3-31 (a)

$$E' = \frac{hc}{\lambda'},\ \lambda' = \lambda_0 + \Delta\lambda$$
$$\lambda_0 = \frac{hc}{E_0} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3\times10^8\ \text{m/s})}{0.1\ \text{MeV}} = 1.243\times10^{-11}\ \text{m}$$
$$\Delta\lambda = \left(\frac{h}{m_e c}\right)(1-\cos\theta) = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(1-\cos 60°)}{(9.11\times10^{-34}\ \text{kg})(3\times10^8\ \text{m/s})} = 1.215\times10^{-12}\ \text{m}$$
$$\lambda' = \lambda_0 + \Delta\lambda = 1.364\times10^{-11}\ \text{m}$$
$$E' = \frac{hc}{\lambda'} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3\times10^8\ \text{m/s})}{1.364\times10^{-11}\ \text{m}} = 9.11\times10^4\ \text{eV}$$

(b)

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e$$
$$K_e = 0.1\ \text{MeV} - 91.1\ \text{keV} = 8.90\ \text{keV}$$



(c) Conservation of momentum along x: $\frac{h}{\lambda_0} = \left(\frac{h}{\lambda'}\right)\cos\theta + \gamma m_e v\cos\phi$. Conservation of momentum along y: $\left(\frac{h}{\lambda'}\right)\sin\theta = \gamma m_e v\sin\phi$. So that

$$\frac{\gamma m_e v\sin\phi}{\gamma m_e v\cos\phi} = \left(\frac{h}{\lambda'}\right)\sin\theta\left[\left(\frac{h}{\lambda_0}\right) - \left(\frac{h}{\lambda'}\right)\cos\theta\right]$$

$$\tan\phi = \frac{\lambda_0\sin\theta}{(\lambda' - \lambda_0)\cos\theta}$$

Here, $\theta = 60°$, $\lambda_0 = 1.243\times10^{-11}\ \text{m}$, and $\lambda' = 1.364\times10^{-11}\ \text{m}$. Consequently,

$$\tan\phi = \frac{\left(1.24\times10^{-11}\ \text{m}\right)(\sin 60°)}{(1.36 - 1.24\cos 60°)\times10^{-11}\ \text{m}} = 1.451$$

$$\phi = 55.4°$$

3-37 When waves are scattered between two adjacent planes of a single crystal, constructive wave interference will occur when the path length difference between such reflected waves is an integer multiple of wavelengths. This condition is expressed by the Bragg equation for constructive interference, $2d\sin\theta = n\lambda$ where d is the distance between adjacent crystalline planes, θ is the angle of incidence of the x-ray beam of photons, n is an integer for constructive interference, and λ is the wavelength of the photon beam which is in this case, 0.062 6 nm. Ignoring the incident beam that is not scattered, the first three angles for which maxima of x-ray intensities are found are $1\lambda = 2d\sin\theta_1$ or

$$\sin\theta_1 = \frac{\lambda}{2d} = \frac{0.626\times10^{-10}\ \text{m}}{8\times10^{-10}\ \text{m}}$$

$$\theta_1 = 0.078\ 3\ \text{radians} = 4.49°$$

$2\lambda = 2d\sin\theta_2$ or

$$\sin\theta_2 = \frac{\lambda}{d} = \frac{0.626\times10^{-10}\ \text{m}}{4.0\times10^{-10}\ \text{m}} = 0.156\ 5,\ \theta = 9.00°$$

$3\lambda = 2d\sin\theta_3$ or

$$\sin\theta_3 = \frac{3\lambda}{2d} = \frac{3\left(0.626\times10^{-10}\ \mathrm{m}\right)}{8\times10^{-10}\ \mathrm{m}} = 0.234\,75\,,\ \ \theta_3 = 13.6°$$