Physics 2D Quiz 1 Solutions Fall 2009

Problem 1

$v = 0.6\,c, \quad \gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}} = \dfrac{1}{\sqrt{1 - 0.36}} = 1.25$

(a) For twin B, 1 year has passed. That is the proper time, Δt_p

The time on earth is

$$\Delta t = \gamma \, \Delta t_p = 1.25 \text{ years.}$$

So twin B is 21.25 years old when she lights up the candles according to clocks on the earth.

(b) After 1.25 years, ~~the~~ the spaceship has traveled a distance

$$d = 0.6c \times 1.25 \text{ years.}$$

The light from the candle of twin B travels towards twin A at speed c. It reaches twin A after a time

$$t = \frac{d}{c} = \frac{0.6c \times 1.25 \text{ years}}{c} = 0.75 \text{ years.}$$

So the age of twin A when the light first reaches him is

$$\text{age}(A) = 20 \text{ years} + 1.25 \text{ years} + 0.75 \text{ years} = 22 \text{ years.}$$

(c) Because of the principle of relativity, the answer must be the same for twin B

So, age (B) = 22 years when the light from A reaches her.

Problem 1 (cont.)

Alternative way to solve for part (c):

Twin A lights candle after 1 year.

Light travels time t until it reaches spaceship; so it travels distance $D = ct$

Spaceship has traveled a time = 1 year + t, at speed $0.6c$

So $D = (1\text{ year} + t) \times 0.6c$. Equating, we find t

$$(1\text{ year} + t) \times 0.6\cancel{c} = \cancel{c}t \Rightarrow 0.6\text{ year} + 0.6t = t$$

$$\Rightarrow 0.4t = 0.6\text{ year} \Rightarrow \boxed{t = 1.5\text{ years}}$$

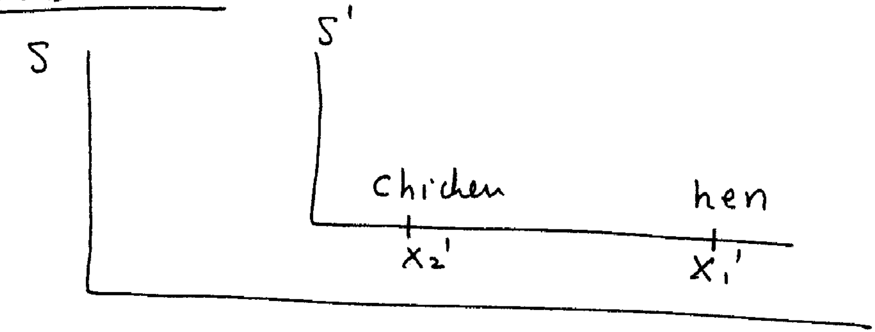
So according to the earth reference frame, the light from A reaches B after a time 1 year + t = = 1 year + 1.5 years = 2.5 years since the 20th birthday.

But the time for B is the proper time, so it is

$$\frac{2.5\text{ years}}{\gamma} = \frac{2.5}{1.25}\text{ years} = \boxed{2\text{ years.}}$$

So the light from A reaches B when B is 22 years old, as we had found earlier.

Problem 2



hen lays egg: $x_1' = 100\,m$, $t_1' = 0$

chicken is born: $x_2' = 0$, $t_2' = +0.1\,\mu s = 10^{-7}\,s$

$$t_1 = \gamma\left(t_1' + \frac{v}{c^2}x_1'\right) = \frac{\gamma v x_1'}{c^2}$$

$$t_2 = \gamma\left(t_2' + \frac{v}{c^2}x_2'\right) = \gamma t_2'$$

We want $t_1 = t_2 \Rightarrow \frac{v}{c^2}x_1' = t_2' \Rightarrow v = c^2 \frac{t_2'}{x_1'} \Rightarrow$

$$v = c \cdot \frac{t_2'}{x_1'} \cdot c = c \cdot \frac{10^{-7}\,\cancel{s}}{100\,\cancel{m}} \cdot 3\times10^8 \frac{\cancel{m}}{\cancel{s}} = 0.3\,c$$

So $\boxed{v = 0.3\,c}$

(For $v > 0.3c$, chicken is born before hen lays egg in ground ref. frame)

(b) $(\Delta s')^2 = (c\Delta t')^2 - (\Delta x')^2 = (3\times10^8 \times 10^{-7})^2\,m^2 - 100^2\,m^2 =$

$= 30^2\,m^2 - 100\,m^2 = \boxed{-9{,}100\,m^2}$ spacelike since it is negative.

(c) Since spacetime interval is an invariant, $\boxed{(\Delta s)^2 = -9{,}100\,m^2 \text{ also.}}$

Or, we derive it as follows: $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$

$\Delta t = 0$, since $t_1 = t_2$. $\Delta x = \frac{L_p}{\gamma} = L_p\sqrt{1 - v^2/c^2}$, with $L_p = 100\,m$

$$\Rightarrow (\Delta s)^2 = -(\Delta x)^2 = -\frac{L_p^2}{\gamma^2} = -10{,}000\,m^2 \times (1 - (0.3)^2) =$$

$$= -10{,}000\,m^2 \times 0.91 = \boxed{-9100\,m^2}$$

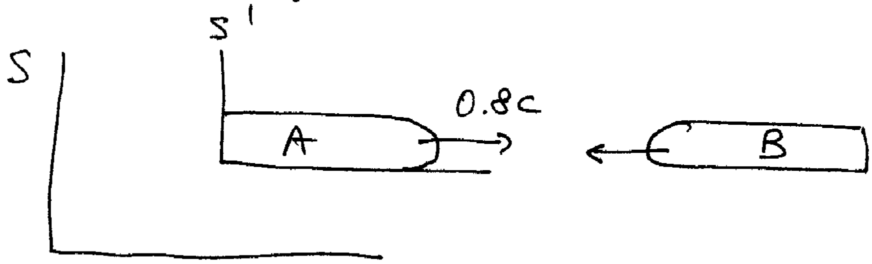
Problem 3

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.8^2}} = \frac{1}{0.6} = 1.6667$$ for ship relative to ground.

(b) Length of ship as measured from ground:

$$\boxed{L = \frac{L_p}{\gamma} = \frac{200\text{ m}}{1.667} = 120\text{ m}}$$

(a) Relative speed:



$u_x = -0.8c =$ speed of B with respect to ground

$v = 0.8c =$ speed of frame S' = reference frame of ship A

find u'_x = speed of B relative to frame S':

$$u'_x = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} = \frac{-0.8c - 0.8c}{1 + 0.8 \times 0.8} = -\frac{1.6c}{1+0.64} = -0.9756c$$

So relative speed of B with respect to A is $\boxed{0.9756c}$

(c) The γ for this relative speed is

$$\gamma = \frac{1}{\sqrt{1-0.9756^2}} = 4.555$$

so the length contraction is

$$\boxed{L = \frac{L_p}{\gamma} = \frac{200\text{ m}}{4.555} = 43.9\text{ m}}$$