

**Formulas:**

Time dilation; Length contraction:  $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$  ;  $L = L_p / \gamma$  ;  $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation:  $x' = \gamma(x - vt)$  ;  $y' = y$  ;  $z' = z$  ;  $t' = \gamma(t - vx/c^2)$  ; inverse:  $v \rightarrow -v$

Spacetime interval:  $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation:  $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$  ;  $u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)}$  ; inverse:  $v \rightarrow -v$

Relativistic Doppler shift:  $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$  (approaching)

Momentum:  $\vec{p} = \gamma m \vec{u}$  ; Energy:  $E = \gamma mc^2$ ; Kinetic energy:  $K = (\gamma - 1)mc^2$

Rest energy:  $E_0 = mc^2$  ;  $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron:  $m_e = 0.511 \text{ MeV}/c^2$  Proton:  $m_p = 938.26 \text{ MeV}/c^2$  Neutron:  $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit:  $1 u = 931.5 \text{ MeV}/c^2$  ; electron volt:  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law:  $e_{tot} = \sigma T^4$ ,  $e_{tot}$  = power/unit area ;  $\sigma = 5.67 \times 10^{-8} \text{ W}/\text{m}^2 \text{K}^4$

$e_{tot} = cU/4$ ,  $U$  = energy density =  $\int_0^\infty u(\lambda, T) d\lambda$  ; Wien's law:  $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution:  $P(E) = C e^{-E/(k_B T)}$

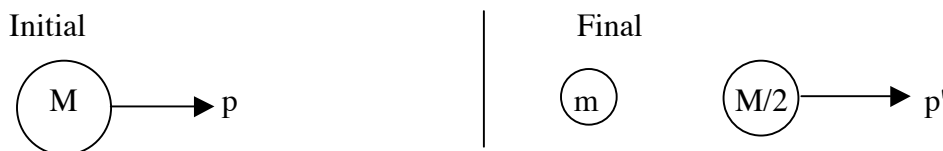
Planck's law:  $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$  ;  $N(f) = \frac{8\pi f^2}{c^3}$

Photons:  $E = hf = pc$  ;  $f = c/\lambda$  ;  $hc = 12,400 \text{ eV \AA}$  ;  $k_B = (1/11,600) \text{ eV}/\text{K}$

Photoelectric effect:  $eV_s = K_{max} = hf - \phi$ ,  $\phi \equiv$  work function

**Justify all your answers to all problems**

**Problem 1** (10 points)



A particle of mass  $M=100\text{MeV}/c^2$  is moving with momentum  $p=300\text{MeV}/c$ . It splits into two fragments, a particle of mass  $M/2=50\text{MeV}/c^2$  moving with momentum  $p'$  and a particle of mass  $m$  at rest.

(a) Find the momentum  $p'$  in units  $\text{MeV}/c$ .

(b) Find the mass  $m$  in  $\text{MeV}/c^2$ .

(c) Find the change in kinetic energy between initial and final states and verify that it equals the change in total mass. Was mass converted into kinetic energy or kinetic energy converted into mass?

**Problem 2** (10 points)

A tungsten filament at temperature  $T=3000\text{K}$  emits 100W power. Assume it is a black body.

- (a) At what wavelength does it emit maximum power?
- (b) How much total power will it emit if its temperature is raised to 6000K?
- (c) Consider only the radiation emitted at wavelengths larger than 100,000Å. In that range, approximately how much more power is emitted when the filament is at 6000K compared to when it is at 3000K? Hint: you may use the fact that  $e^{-x} \sim 1 - x$  for small  $x$ , provided you justify why it is valid to do so for this problem.

**Problem 3** (10 points)

Light of wavelength 3000Å is incident on a metal. The stopping voltage for the emitted photoelectrons is twice as large as when light of wavelength 3500Å is used.

- (a) Find the work function of this metal.
- (b) Find the cutoff wavelength.