

**Formulas:**

Time dilation; Length contraction :  $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$  ;  $L = L_p / \gamma$  ;  $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation :  $x' = \gamma(x - vt)$  ;  $y' = y$  ;  $z' = z$  ;  $t' = \gamma(t - vx/c^2)$  ; inverse :  $v \rightarrow -v$

Spacetime interval :  $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation :  $u_x' = \frac{u_x - v}{1 - u_x v / c^2}$  ;  $u_y' = \frac{u_y}{\gamma(1 - u_x v / c^2)}$  ; inverse :  $v \rightarrow -v$

Relativistic Doppler shift :  $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$  (approaching)

Momentum :  $\vec{p} = \gamma m \vec{u}$  ; Energy :  $E = \gamma mc^2$  ; Kinetic energy :  $K = (\gamma - 1)mc^2$

Rest energy :  $E_0 = mc^2$  ;  $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron :  $m_e = 0.511 \text{ MeV}/c^2$  ; Proton :  $m_p = 938.26 \text{ MeV}/c^2$  ; Neutron :  $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit :  $1 u = 931.5 \text{ MeV}/c^2$  ; electron volt :  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law :  $e_{tot} = \sigma T^4$  ,  $e_{tot}$  = power/unit area ;  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$

$e_{tot} = cU/4$  ,  $U$  = energy density =  $\int_0^\infty u(\lambda, T) d\lambda$  ; Wien's law :  $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution :  $P(E) = C e^{-E/(k_B T)}$

Planck's law :  $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$  ;  $N(f) = \frac{8\pi f^2}{c^3}$

Photons :  $E = hf = pc$  ;  $f = c/\lambda$  ;  $hc = 12,400 \text{ eV \AA}$  ;  $k_B = (1/11,600) \text{ eV/K}$

Photoelectric effect :  $eV_s = K_{max} = hf - \phi$  ,  $\phi$   $\equiv$  work function; Bragg equation :  $n\lambda = 2d \sin \vartheta$

Compton scattering :  $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$  ;  $\frac{h}{m_e c} = 0.0243 \text{ \AA}$  ; Coulomb constant :  $ke^2 = 14.4 \text{ eV \AA}$

Force in electric and magnetic fields (Lorentz force) :  $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$  ; Drag force :  $D = 6\pi a \eta v$

Rutherford scattering :  $\Delta n = \frac{C}{\sin^4(\phi/2)}$  ;  $\hbar c = 1,973 \text{ eV \AA}$

Hydrogen spectrum :  $\frac{1}{\lambda_{mn}} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right)$  ;  $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3 \text{ \AA}}$

Electrostatic force, energy :  $F = \frac{kq_1 q_2}{r^2}$  ;  $U = \frac{kq_1 q_2}{r}$  . Centripetal force :  $F_c = \frac{mv^2}{r}$

Bohr atom :  $E_n = -\frac{ke^2 Z}{2r_n} = -\frac{Z^2 E_0}{n^2}$  ;  $E_0 = \frac{ke^2}{2a_0} = 13.6 \text{ eV}$  ;  $K = \frac{m_e v^2}{2}$  ;  $U = -\frac{ke^2 Z}{r}$

$hf = E_i - E_f$  ;  $r_n = r_0 n^2$  ;  $r_0 = \frac{a_0}{Z}$  ;  $a_0 = \frac{\hbar^2}{m_e ke^2} = 0.529 \text{ \AA}$  ;  $L = m_e v r = n\hbar$  angular momentum

de Broglie :  $\lambda = \frac{h}{p}$  ;  $f = \frac{E}{h}$  ;  $\omega = 2\pi f$  ;  $k = \frac{2\pi}{\lambda}$  ;  $E = \hbar \omega$  ;  $p = \hbar k$  ;  $E = \frac{p^2}{2m}$

**Justify all your answers to all problems**

**Problem 1** (10 points)

In a Compton scattering experiment, X-rays incident along the x direction of wavelength  $\lambda=1\text{\AA}$  are scattered by free electrons and the outgoing X-rays have wavelength  $1.01\text{\AA}$ .

- Find the kinetic energy of the scattered electrons, in eV.
- Find the angle of the scattered X-rays relative to the incident direction (x-axis), in degrees.
- Find the x-component of the momentum of the scattered electrons. Give your answer in units eV/c.

**Problem 2** (10 points)

In a Rutherford scattering experiment with a Magnesium (Mg) foil ( $Z=12$ ) and  $\alpha$ -particle energy 6 MeV, it is found that for every 600 particles scattered at angle  $90^\circ$  there are 150  $\alpha$ -particles scattered at angle  $180^\circ$ . When the  $\alpha$ -particle energy is 7 MeV it is found that for every 700  $\alpha$ -particles scattered at angle  $90^\circ$  there are 150  $\alpha$ -particles scattered at angle  $180^\circ$ .

- For  $\alpha$ -particle energy 5 MeV, for every 500  $\alpha$ -particles scattered at angle  $90^\circ$ , how many  $\alpha$ -particles are scattered at angle  $180^\circ$ ?
- Find minimum and maximum sizes of the nucleus of Mg from this information (i.e. give the radii, in Angstroms).

**Problem 3** (10 points)

A hydrogen atom and a  $\text{Li}^{++}$  ion ( $Z=3$ ) have electrons in states that have the same total energy. The electron in hydrogen is in the lowest energy state (ground state).

- What state is the electron in the  $\text{Li}^{++}$  ion in? Give its quantum number.
- Compare the speed of these electrons. I.e. give the ratio of the speed of the electron in this state of  $\text{Li}^{++}$  to the speed of the electron in the ground state of H.
- What are the possible wavelengths of the photons emitted when this electron in  $\text{Li}^{++}$  makes a transition from this state to a lower energy state? Give your answer in Angstroms.

**Justify all your answers to all problems**