PHYSICS 2D | QUIZ 6 | FALL QUARTER 2009
PROF. HIRSCH | | DECEMBER 4th, 2009

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p/\gamma$; $c = 3\times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \to -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \to -v$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1+v/c}\ /\ \sqrt{1-v/c}$ (approaching)

Momentum: $\vec{p} = \gamma m\vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\, MeV/c^2$ Proton: $m_p = 938.26\, MeV/c^2$ Neutron: $m_n = 939.55\, MeV/c^2$

Atomic mass unit: $1\, u = 931.5\, MeV/c^2$; electron volt: $1\text{eV} = 1.6\times 10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67\times 10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda,T)d\lambda$; Wien's law: $\lambda_m T = \dfrac{hc}{4.96k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_BT)}$

Planck's law: $u_\lambda(\lambda,T) = N_\lambda(\lambda)\times \overline{E}(\lambda,T) = \dfrac{8\pi}{\lambda^4}\times\dfrac{hc/\lambda}{e^{hc/\lambda k_BT} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\ eV A$; $k_B = (1/11{,}600)eV/K$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi$, $\phi \equiv$ work function; Bragg equation: $n\lambda = 2d\sin\vartheta$

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$; $\dfrac{h}{m_e c} = 0.0243A$; Coulomb constant: $ke^2 = 14.4\ eV A$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$; Drag force: $D = 6\pi a\eta v$

Rutherford scattering: $\Delta n = \dfrac{C}{\sin^4(\phi/2)}$; $\hbar c = 1{,}973\ eV A$

Hydrogen spectrum: $\dfrac{1}{\lambda_{mn}} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$; $R = 1.097\times 10^7\ m^{-1} = \dfrac{1}{911.3A}$

Electrostatic force, energy: $F = \dfrac{kq_1q_2}{r^2}$; $U = \dfrac{kq_1q_2}{r}$. Centripetal force: $F_c = \dfrac{mv^2}{r}$

Bohr atom: $E_n = -\dfrac{ke^2Z}{2r_n} = -\dfrac{Z^2E_0}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = 13.6eV$; $K = \dfrac{m_ev^2}{2}$; $U = -\dfrac{ke^2Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_eke^2} = 0.529A$; $L = m_evr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j\cos(k_jx - \omega_jt)$, or $y(x,t) = \int dk\, a(k)\, e^{i(kx - \omega(k)t)}$; $\Delta k\Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \dfrac{d\omega}{dk}$; $v_p = \dfrac{\omega}{k}$; Heisenberg: $\Delta x\Delta p \sim \hbar$; $\Delta t\Delta E \sim \hbar$

Schrodinger equation: $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2\Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\dfrac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

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Time – independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+U(x)\psi(x)=E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well: $\psi_n(x)=\sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n=\frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e}=3.81 eV A^2$ (electron)

Harmonic oscillator: $\Psi_n(x)=H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n=(n+\frac{1}{2})\hbar\omega$; $E=\frac{p^2}{2m}+\frac{1}{2}m\omega^2 x^2=\frac{1}{2}m\omega^2 A^2$; $\Delta n=\pm 1$

Expectation value of $[Q]$: $<Q> = \int \psi^*(x)[Q]\psi(x)\,dx$; Momentum operator: $p=\frac{\hbar}{i}\frac{\partial}{\partial x}$

Eigenvalues and eigenfunctions: $[Q]\ \Psi = q\ \Psi$ (q is a constant) ; uncertainty: $\Delta Q=\sqrt{<Q^2>-<Q>^2}$

Step potential: reflection coef : $R=\frac{(k_1-k_2)^2}{(k_1+k_2)^2}$, $T=1-R$; $k=\sqrt{\frac{2m}{\hbar^2}(E-U)}$

Tunneling: $\psi(x)\sim e^{-\alpha x}$; $T=e^{-2\alpha\Delta x}$; $T=e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x)=\sqrt{\frac{2m[U(x)-E]}{\hbar^2}}$

Schrodinger equation in 3D: $-\frac{\hbar^2}{2m}\nabla^2\Psi+U(\vec{r})\Psi(\vec{r},t)=i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(\vec{r},t)=\psi(\vec{r})e^{-i\frac{E}{\hbar}t}$

3D square well: $\Psi(x,y,z)=\Psi_1(x)\Psi_2(y)\Psi_3(z)$; $E=\frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2})$

Spherically symmetric potential: $\Psi_{n,\ell,m_\ell}(r,\theta,\phi)=R_{n\ell}(r)Y_\ell^{m_\ell}(\theta,\phi)$; $Y_\ell^{m_\ell}(\theta,\phi)=P_\ell^{m_\ell}(\theta)e^{im_\ell\phi}$

Angular momentum: $\vec{L}=\vec{r}\times\vec{p}$; $[L_z]=\frac{\hbar}{i}\frac{\partial}{\partial\phi}$; $[L^2]Y_\ell^{m_\ell}=\ell(\ell+1)\hbar^2 Y_\ell^{m_\ell}$; $[L_z]=m_\ell\hbar$

Radial probability density: $P(r)=r^2|R_{n\ell}(r)|^2$; Energy: $E_n=-\frac{ke^2}{2a_0}\frac{Z^2}{n^2}$

Ground state of hydrogen and hydrogen - like ions: $\Psi_{1,0,0}=\frac{1}{\pi^{1/2}}(\frac{Z}{a_0})^{3/2}e^{-Zr/a_0}$

Orbital magnetic moment: $\vec{\mu}=\frac{-e}{2m_e}\vec{L}$; $\mu_z=-\mu_B m_l$; $\mu_B=\frac{e\hbar}{2m_e}=5.79\times 10^{-5} eV/T$

Spin 1/2: $s=\frac{1}{2}$, $|S|=\sqrt{s(s+1)}\hbar$; $S_z=m_s\hbar$; $m_s=\pm 1/2$; $\vec{\mu}_s=\frac{-e}{2m_e}g\vec{S}$

Orbital + spin mag moment: $\vec{\mu}=\frac{-e}{2m}(\vec{L}+g\vec{S})$; Energy in mag. field: $U=-\vec{\mu}\cdot\vec{B}$

Two particles : $\Psi(\vec{r}_1,\vec{r}_2)=+/-\Psi(\vec{r}_2,\vec{r}_1)$; symmetric/antisymmetric

Screening in multielectron atoms: $Z\rightarrow Z_{eff}$, $1<Z_{eff}<Z$

Orbital ordering:

1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 6d ~ 5f

Justify all your answers to all problems

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Problem 1 (10 points)

An electron moves in a three-dimensional box of edge lengths L_1=1A, L_2=1A, L_3=2A.
(a) Find the quantum numbers and the energies (in eV) of the ground state and the first excited state.
(b) Find the quantum numbers and the energies (in eV) of the second excited state and of the third excited state. Are any of these states degenerate?
(c) Which is the lowest state that is not the ground state for which the wavefunction is not zero at the center of the box? Give its quantum numbers and justify your answer.
Hint: Use $\hbar^2/(2m_e) = 3.81\ eVA^2$

Problem 2 (10 points)

Consider an electron in the state of hydrogen described by the wavefunction
$\psi(r,\vartheta,\phi) = Cr^2 e^{-r/3a_0} \sin^2\vartheta\, e^{-i\phi}$
where C is a constant.
(a) Give the quantum numbers n, ℓ and m_ℓ for this state, and the values for the magnitude of its angular momentum and the z component of the angular momentum, expressed in terms of $\hbar$.
(b) In the absence of a magnetic field and ignoring spin, how many other states are there with the same energy as this state? Give their quantum numbers.
(c) Ignoring spin, by how much (in eV) does the energy of this electron change when a magnetic field of magnitude 3T pointing in the -z direction is turned on? Does the energy increase or decrease? Justify your answer.

Problem 3 (10 points)

The radial wave function for the n=2, ℓ=0 state of an electron in hydrogen is
$$R(r) = \frac{1}{(2a_0)^{3/2}}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$
(a) Find $\left\langle \frac{1}{r} \right\rangle$ in this state.
(b) Find the average potential energy for an electron in this state, expressed in terms of ke^2 and a_0 (k=Coulomb constant, a_0=Bohr radius).
(c) Find the average kinetic energy for an electron in this state, expressed in terms of ke^2 and a_0. Hint: remember that kin. energy + pot. energy = ...
(d) Compare the results in (b) and (c) with the results for the Bohr model for the n=2 orbit.

<u>Hint:</u> Use that $\int_0^\infty dx\, x^n e^{-\lambda x} = \frac{n!}{\lambda^{n+1}}$

Justify all your answers to all problems